A Conformal Finite Difference Time Domain (CFDTD) Algorithm
For Modeling Perfectly Conducting Objects

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Abstract: In this paper, the authors present a new and robust conformal Finite Difference Time Domain (FDTD) algorithm for the accurate modeling of perfectly conducting objects with curved surfaces and edges. We illustrate the application of this approach by analyzing a number of representative antenna and cavity problems. These include a quarter wave monopole mounted on a perfectly conducting elliptic disk, a circular patch antenna, and a cylindrical cavity. We validate the method by comparing the results for the pattern, impedance and resonant frequency, etc., with those derived by using other techniques.

I. INTRODUCTION

The finite difference time domain method (FDTD), though it has been widely used to model various electromagnetic phenomena [1,2,3,4], continues to present a challenge when used to analyze structures with curved surfaces and edges. Although, in the past, there have been many attempts to address this problem via the use of conformal FDTD methods [2-19], the problems of mesh generation and instabilities have continued to plague most of these approaches. In this paper we present a generalization of the conformal FDTD method, which substantially preserves the conventional FDTD update algorithm as introduced by Yee [1]. We show, through several examples, that the present CFDTD approach is accurate, numerically efficient, and stable.

We begin by providing a brief review of the various techniques that have been employed for analyzing objects with curved surfaces. These include:

(i) Methods based on a globally-distorted and body-fitted grid model
These methods use non-Cartesian grids that conform to smoothly-shaped structures [6,7], and are limited in their application to special classes of geometries.

(ii) Hybrid methods
These approaches combine the FDTD method with other numerical methods, such as the Finite Element Method (FEM) [8,9] and the Method of Moments (MoM) [10]. In these approaches, the problem of interfacing the FDTD regions with the MoM or FEM domains in a manner such that spurious reflections are minimized and the solution is stable, is a challenging one and must be addressed to obtain satisfactory results.
Locally distorted grid models

These methods have the advantage of preserving the basic Cartesian nature of the grid in the entire computational domain [11-17]. However, they frequently suffer from the problems of late time instability and difficulties with mesh generation.

II. THE CONFORMAL FDTD APPROACH

Let us now describe briefly the conformal FDTD technique presented in this paper. It begins by dividing the cells, that are partially-filled with metallic conductors (assumed to be PECs), into two parts, viz., the inside and outside subregions (see Fig. 1). The E-fields on the edges of the inside subregions are set to zero, whereas they are updated by using a modified algorithm in the outside subregions.

The magnetic fields inside the partially-filled cells are updated by using a slightly modified form of the conventional FDTD algorithm which accounts for the deformation of the cell. Unlike past approaches, however, we no longer employ the deformed cell area for updating the H-field— but use the entire cell area instead (see Fig. 2)—and this serves to eliminate the instability problems experienced in the past. The magnetic field component \( H_y \) is written as

\[
H_y^{(i,j,k+1)} = H_y^{(i,j,k)} + \frac{\Delta t}{\mu_r(i,j,k)} \left[ \frac{\Delta x(i,j,k) E_x^{(i,j,k)} - \Delta z(i,j,k) E_z^{(i,j,k)}}{dx(i) \times dz(k)} \right]
\]

In the next section we present the numerical results for a number of representative problems to illustrate the application of the method just described above.
III. NUMERICAL RESULTS

To illustrate the versatility of the modified Finite Difference Time Domain program described above, referred to herein as the CFDTD code, we now present the numerical results for three different problems. They include: (i) the computation of the far-zone pattern for a quarter-wave monopole mounted on a perfectly conducting elliptic disk; (ii) evaluation of the impedance of a circular patch antenna; and, (iii) computation of resonant frequencies for a cylindrical cavity. In all of these cases, a 10-layer unsplit PML is used to truncate the FDTD domain, and the time step is chosen to be

\[ \Delta t = \frac{0.995}{c \sqrt{\left(\frac{1}{dx}\right)^2 + \left(\frac{1}{dy}\right)^2 + \left(\frac{1}{dz}\right)^2}} \] (2)

3.1. Monopole on elliptic and circular ground planes

First, we consider a quarter-wave monopole, mounted on a perfectly conducting elliptic disk, as shown in Fig. 3. The accuracy of the fields in the shadow region of the monopole depends on our ability to model the ground plane accurately.

The major and minor axes of the elliptic disk \((b\) and \(a)) are assumed to be 1.7 and 1.5 meters, respectively. The excitation source is a Gaussian pulse with a 3 dB cutoff frequency of 300 MHz, modulated by a sine function. The antenna is excited at the gap between the monopole and the ground plane. The FDTD domain is discretized into \(79 \times 87 \times 30\) cells, with a uniform mesh, whose dimensions are: \(dx(i) = 0.05\ m\), \(dy(j) = 0.05\ m\) and \(dz(k) = 0.05\ m\). The normalized far zone field patterns for \(\phi = 0^\circ\) and \(\phi = 90^\circ\) at a frequency of 300 MHz are shown in Fig. 4 and Fig. 5, respectively. We observe that the CFDTD results are in good agreement with those obtained by using the MoM technique for both cuts.

Next we change the disk shape to be a circular one with a radius of 0.62 m, and use a non-uniform version of the CFDTD to calculate the far zone field pattern of the same monopole antenna at 1 GHz (see Fig. 6). We observe, once again, that the conformal FDTD results are in good agreement with those reported elsewhere in [21,22].

3.2 Circular patch antenna

Next, we compute the impedance of a microstrip patch antenna, circular in shape, as shown in Fig. 7. The computational domain for this problem is subdivided into \(55 \times 44 \times 10\) cells. The cell dimensions are chosen to be \(0.002\ m, 0.002\ m\) and \(0.000795\ m\) in the \(x, y\) and \(z\) directions, respectively. The simulation is run for 25,000 time steps (though 15,000 were found to be adequate), with a time step of \(\Delta t = 2.29847\ ps\), and the solution is found to be entirely stable. The impedance of the circular patch antenna versus frequency is shown in Fig. 8. The results reported in [2,18,19] via the application of the symmetric and asymmetric PGY approaches, and those obtained by using the MoM [22], are also plotted in the same figure for the sake of comparison. The CFDTD results are seen to compare quite favorably with that obtained via the MoM; however, both the PGY (symmetric) and PGY (non-symmetric) methods exhibit a slight downward shift in the resonant frequency. We also
point out that in the PGY simulations a time step of $\Delta t = 0.725 \text{ ps}$ was used, which is one-third of that needed in the CFDTD simulations.

### 3.3 Cylindrical cavities

As a final example, we turn to the problem of computing the resonant frequencies of the dominant TE and TM modes of a circularly-cylindrical cavity. The dimensions of the cylindrical cavities and results are presented in Table 1, along with the dimensions of the cavity, with R and H denoting the radius and height, respectively. The spatial discretization for this problem was chosen to be 0.005 m, and 5000 time steps were used to derive the results. Once again, we observe very good agreement between the new CFDTD results and the analytical solutions. In addition, no instabilities were observed, for this closed region problem, even though the program was tested up to 40,000 time steps.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Geometry I R×H= (0.1m×0.1m)</th>
<th>Geometry II R×H= (0.1m×0.08m)</th>
<th>Geometry III R×H= (0.1m×0.06m)</th>
<th>Geometry IV R×H= (0.06m×0.06m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE111</td>
<td>CFDTD 1.734</td>
<td>2.062</td>
<td>2.639</td>
<td>3.133</td>
</tr>
<tr>
<td></td>
<td>Analytic 1.738</td>
<td>2.071</td>
<td>2.650</td>
<td>3.148</td>
</tr>
<tr>
<td></td>
<td>Error 0.23%</td>
<td>0.43%</td>
<td>0.42%</td>
<td>0.48%</td>
</tr>
<tr>
<td>TM011</td>
<td>CFDTD 1.88</td>
<td>2.194</td>
<td>2.742</td>
<td>2.872</td>
</tr>
<tr>
<td></td>
<td>Analytic 1.889</td>
<td>2.199</td>
<td>2.751</td>
<td>2.897</td>
</tr>
<tr>
<td></td>
<td>Error 0.48%</td>
<td>0.23%</td>
<td>0.33%</td>
<td>0.86%</td>
</tr>
</tbody>
</table>

Fig. 3 Geometry for the radiation from a $\lambda/4$ monopole on an elliptic disk
Fig. 4 Normalized total electric field pattern for a $\lambda/4$ monopole on an elliptic disk with $a=1.5$ m and $b=1.7$ m at a frequency of 300 MHz.

Fig. 5 Normalized total electric field pattern for a $\lambda/4$ monopole on an elliptic disk with $a=1.5$ m and $b=1.7$ m at a frequency of 300 MHz.

Fig. 6 Normalized total electric field pattern for a $\lambda/4$ monopole on a circular disk with $a=b=0.61$ m at a frequency of 1 GHz.
Fig. 7 Microwave coupled circular patch antenna. The microstrip is printed on a 1.59 mm substrate of \( \varepsilon_r = 2.62 \) over a ground plane, and the patch antenna is printed above a 1.59 mm superstrate of \( \varepsilon_r = 2.62 \). (a) top view; (b) side view.

Fig. 8 Comparison of the reflection loss for a microstrip coupled circular patch antenna computed using CFDTD, MoM, and PGY.

REFERENCES:


