A Conformal Finite Difference Time Domain (CFDTD) Algorithm For Modeling Perfectly Conducting Objects

Wenhua Yu, Raj Mittra, Dean Arakaki, and Douglas H. Werner

Department of Electrical Engineering
The Pennsylvania State University
University Park, PA 16802
Email: wxy6@psu.edu

Abstract: In this paper, the authors present a new and robust conformal Finite Difference Time Domain (FDTD) algorithm for the accurate modeling of perfectly conducting objects with curved surfaces and edges. We illustrate the application of this approach by analyzing a number of representative antenna and cavity problems. These include a quarter wave monopole mounted on a perfectly conducting elliptic disk, a circular patch antenna, and a cylindrical cavity. We validate the method by comparing the results for the pattern, impedance and resonant frequency, etc., with those derived by using other techniques.

I. INTRODUCTION

The finite difference time domain method (FDTD), though it has been widely used to model various electromagnetic phenomena [1,2,3,4], continues to present a challenge when used to analyze structures with curved surfaces and edges. Although, in the past, there have been many attempts to address this problem via the use of conformal FDTD methods [2-19], the problems of mesh generation and instabilities have continued to plague most of these approaches. In this paper we present a generalization of the conformal FDTD method, which substantially preserves the conventional FDTD update algorithm as introduced by Yee [1]. We show, through several examples, that the present CFDTD approach is accurate, numerically efficient, and stable.

We begin by providing a brief review of the various techniques that have been employed for analyzing objects with curved surfaces. These include:

(i) *Methods based on a globally-distorted and body-fitted grid model*
These methods use non-Cartesian grids that conform to smoothly-shaped structures [6,7], and are limited in their application to special classes of geometries.

(ii) *Hybrid methods*
These approaches combine the FDTD method with other numerical methods, such as the Finite Element Method (FEM) [8,9] and the Method of Moments (MoM) [10]. In these approaches, the problem of interfacing the FDTD regions with the MoM or FEM domains in a manner such that spurious reflections are minimized and the solution is stable, is a challenging one and must be addressed to obtain satisfactory results.

(iii) *Locally distorted grid models*

These methods have the advantage of preserving the basic Cartesian nature of the grid in the entire computational domain [11-17]. However, they frequently suffer from the problems of late time instability and difficulties with mesh generation.

II. THE CONFORMAL FDTD APPROACH

Let us now describe briefly the conformal FDTD technique presented in this paper. It begins by dividing the cells, that are partially-filled with metallic conductors (assumed to be PECs), into two parts, *viz.*, the inside and outside subregions (see Fig. 1). The E-fields on the edges of the inside subregions are set to zero, whereas they are updated by using a modified algorithm in the outside subregions.

The magnetic fields inside the partially-filled cells are updated by using a slightly modified form of the conventional FDTD algorithm which accounts for the deformation of the cell. Unlike past approaches, however, we no longer employ the deformed cell area for updating the H-field--but use the entire cell area instead (see Fig. 2)--and this serves to eliminate the instability problems experienced in the past. The magnetic field component H_y is written as

$$H_y^{n+\frac{1}{2}}(i,j,k)=H_y^{n-\frac{1}{2}}(i,j,k)+\frac{\Delta t}{\mu_y(i,j,k)}\times\left[\frac{\Delta z(i,j,k)E_z^n(i,j,k)-\Delta z(i-1,j,k)E_z^n(i-1,j,k)}{dx(i)\times dz(k)}+\frac{\Delta x(i,j,k-1)E_x^n(i,j,k-1)-\Delta x(i,j,k)E_x^n(i,j,k)}{dx(i)\times dz(k)}\right] \quad (1)$$

In the next section we present the numerical results for a number of representative problems to illustrate the application of the method just described above.

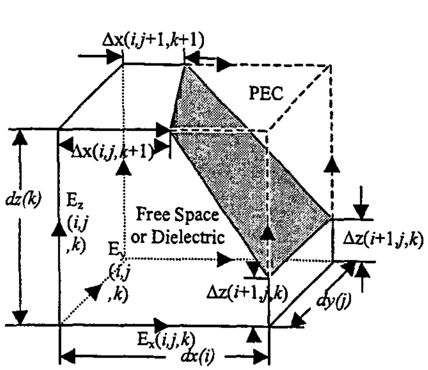

Fig. 1 FDTD cell partially filled with PEC

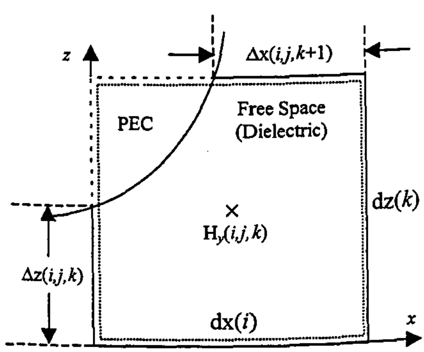

Fig.2 Intersection between the FDTD mesh and a PEC in the *x*-*z* plane

III. NUMERICAL RESULTS

To illustrate the versatility of the modified Finite Difference Time Domain program described above, referred to herein as the CFDTD code, we now present the numerical results for three different problems. They include: (i) the computation of the far-zone pattern for a quarter-wave monopole mounted on a perfectly conducting elliptic disk; (ii) evaluation of the impedance of a circular patch antenna; and, (iii) computation of resonant frequencies for a cylindrical cavity. In all of these cases, a 10-layer unsplit PML is used to truncate the FDTD domain, and the time step is chosen to be

$$\Delta t = \frac{0.995}{c\sqrt{\left(\frac{1}{dx}\right)^2 + \left(\frac{1}{dy}\right)^2 + \left(\frac{1}{dz}\right)^2}} \tag{2}$$

3.1. *Monopole on elliptic and circular ground planes*

First, we consider a quarter-wave monopole, mounted on a perfectly conducting elliptic disk, as shown in Fig. 3. The accuracy of the fields in the shadow region of the monopole depends on our ability to model the ground plane accurately.

The major and minor axes of the elliptic disk (*b* and *a*) are assumed to be 1.7 and 1.5 meters, respectively. The excitation source is a Gaussian pulse with a 3 dB cutoff frequency of 300 MHz, modulated by a sine function. The antenna is excited at the gap between the monopole and the ground plane. The FDTD domain is discretized into 79 × 87 × 30 cells, with a uniform mesh, whose dimensions are: $dx(i) = 0.05\ m$, $dy(j) = 0.05\ m$ and $dz(k) = 0.05\ m$. The normalized far zone field patterns for $\phi = 0^0$ and $\phi = 90^0$ at a frequency of 300 MHz are shown in Fig. 4 and Fig. 5, respectively. We observe that the CFDTD results are in good agreement with those obtained by using the MoM technique for both cuts.

Next we change the disk shape to be a circular one with a radius of 0.62 *m*, and use a non-uniform version of the CFDTD to calculate the far zone field pattern of the same monopole antenna at 1 GHz (see Fig. 6). We observe, once again, that the conformal FDTD results are in good agreement with those reported elsewhere in [21,22].

3.2 *Circular patch antenna*

Next, we compute the impedance of a microstrip patch antenna, circular in shape, as shown in Fig. 7. The computational domain for this problem is subdivided into 55 × 44 × 10 cells. The cell dimensions are chosen to be 0.002 *m*, 0.002 *m* and 0.000795 *m*, in the *x*, *y* and *z* directions, respectively. The simulation is run for 25,000 time steps (though 15,000 were found to be adequate), with a time step of $\Delta t = 2.29847\ ps$, and the solution is found to be entirely stable. The impedance of the circular patch antenna versus frequency is shown in Fig. 8. The results reported in [2,18,19] via the application of the symmetric and asymmetric PGY approaches, and those obtained by using the MoM [22], are also plotted in the same figure for the sake of comparison. The CFDTD results are seen to compare quite favorably with that obtained via the MoM; however, both the PGY (symmetric) and PGY (non-symmetric) methods exhibit a slight downward shift in the resonant frequency. We also point out that in the PGY simulations a time step of $\Delta t = 0.725$ *ps* [2] was used, which is one-third of that needed in the CFDTD simulations.

3.3 *Cylindrical cavities*

As a final example, we turn to the problem of computing the resonant frequencies of the dominant TE and TM modes of a circularly-cylindrical cavity. The dimensions of the cylindrical cavities and results are presented in Table 1, along with the dimensions of the cavity, with R and H denoting the radius and height, respectively. The spatial discretization for this problem was chosen to be 0.005 *m*, and 5000 time steps were used to derive the results. Once again, we observe very good agreement between the new CFDTD results and the analytical solutions. In addition, no instabilities were observed, for this closed region problem, even though the program was tested up to 40,000 time steps.

TABLE 1. Resonant Frequencies of Cylindrical Cavities

	Methods	Geometry I R×H= (0.1m×0.1m)	Geometry II R×H= (0.1m×0.08m)	Geometry III R×H= (0.1m×0.06m)	Geometry IV R×H= (0.06m×0.06m)
TE_{111}	CFDTD	1.734	2.062	2.639	3.133
	Analytic	1.738	2.071	2.650	3.148
	Error	0.23%	0.43%	0.42%	0.48%
TM_{011}	CFDTD	1.88	2.194	2.742	2.872
	Analytic	1.889	2.199	2.751	2.897
	Error	0.48%	0.23%	0.33%	0.86%

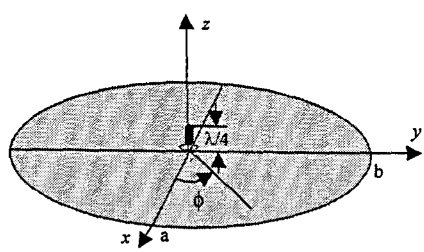

Fig. 3 Geometry for the radiation from a λ/4 monopole on an elliptic disk

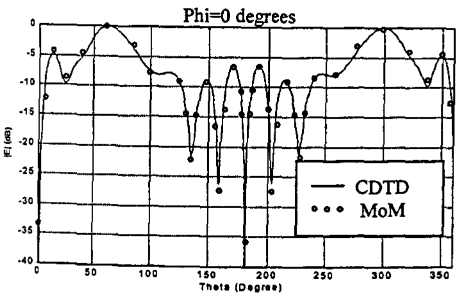

Fig.4 Normalized total electric field pattern for a λ/4 monopole on an elliptic disk with a=1.5 m and b=1.7 m at a frequency of 300 MHz.

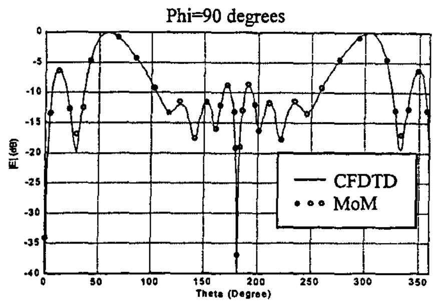

Fig.5 Normalized total electric field pattern for a λ/4 monopole on an elliptic disk with a=1.5 m and b=1.7 m at a frequency of 300 MHz.

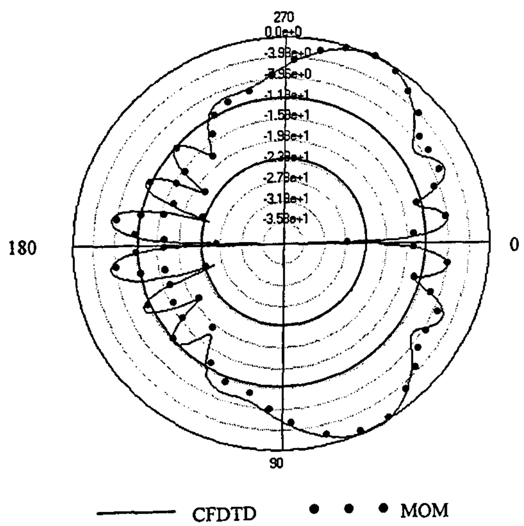

Fig. 6 Normalized total electric field pattern for a λ/4 monopole on a circular disk with a=b= 0.61 m at a frequency of 1 GHz.

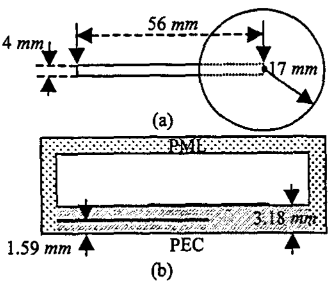

Fig. 7 Microwave coupled circular patch antenna. The microstrip is printed on a 1.59 *mm* substrate of $\varepsilon_r = 2.62$ over a ground plane, and the patch antenna is printed above a 1.59 *mm* superstrate of $\varepsilon_r = 2.62$. (a) top view; (b) side view.

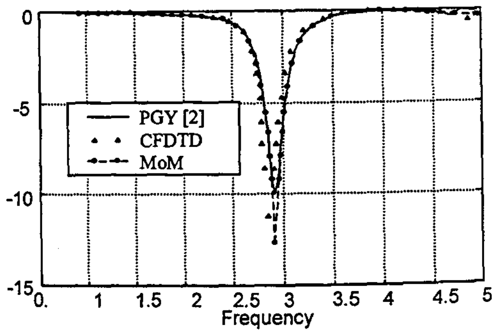

Fig. 8 Comparison of the reflection loss for a microstrip coupled circular patch antenna computed using CFDTD, MoM, and PGY.

REFERENCES:

[1] K. S. Yee, "Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media," *IEEE Trans. Antennas and Propagat.*, vol. AP-14, pp. 302-307, May 1966.

[2] A. Taflove (editor), *Advances in Computational Electromagnetics: The Finite-Difference Time-Domain Method*, Boston, MA: Artech House, 1998.

[3] T. Itoh (Editor), *Time-Domain Methods for Microwave Structures*, IEEE Press, 1998.

[4] A. Taflove, *Computational Electromagnetics: The Finite-Difference Time-Domain Method*, Boston, MA: Artech House, 1995.

[5] J. P. Berenger, "A Perfectly Matched Layer for the Absorption of Electromagnetic Waves," *J. Comput. Phys.*, vol.114, pp.185-200, 1994.

[6] M. Fusco, "FDTD Algorithm in Curvilinear Coordinates," *IEEE Trans. Antennas and Propagat.*, vol. AP-38, no.1, pp. 76-89, January 1990.

[7] J-F. Lee, R. Palandech, and R. Mittra, "Modeling Three-Dimensional Discontinuities in Waveguides Using a Nonorthogonal FDTD Algorithm," *IEEE Trans. Antennas and Propagat.*, vol. AP-40, no.2, pp. 346-352, February 1992.

[8] K. S. Yee and J. S. Chen, "The Finite-Difference Time-Domain (FDTD) and the Finite-Volume Time-Domain (FVTD) Methods in Solving Maxwell's Equations," " *IEEE Trans. Antennas and Propagat.*, vol. AP-45, no.3, pp. 355-363, March 1997.

[9] Monorchio, A., and R. Mittra, "Time-Domain (FE/FDTD) Technique for Solving Complex Electromagnetic Problems," *IEEE Microwave and Guided Wave Letters*, vol. 8, no.2, pp. 93-95, February 1998.

[10] A. R. Bretones, R. Mittra, and R. G. Martin, "A Hybrid Technique Combining the Method of Moments in the Time Domain and FDTD," *IEEE Microwave and Guided Wave Letters*, vol. 8, no.8, pp. 281-283, August 1998.

[11] T. G. Jurgens, A. Taflove, K. Umashankar, and T. G. Moore, "Finite-Difference Time-Domain Modeling of Curved Surfaces," *IEEE Trans. Antennas and Propagat.*, vol. AP-40, no. 4, pp. 357-366, April 1992.

[12] T. G. Jurgens, and A. Taflove, "Three-Dimensional Contour FDTD Modeling of Scattering from Single and Multiple Objects," *IEEE Trans. Antennas and Propagat.*, vol. AP-41, no.12, pp. 1703-1708, December 1993.

[13] J. B. Schneider, and K. L. Shlager, "FDTD Simulations of TEM Horns and the Implications for Staircased Representations," *IEEE Trans. Antennas and Propagat.*, vol. AP-45, no.12, pp. 1830-1838, December 1997.

[14] Y. Nishioka, O. Maeshima, T. Uno, and S. Adachi, "FDTD Analysis of Resistive-Loaded Bow-Tie Antennas Covered with Ferrite-Coated Conducting Cavity for Subsurface Radar," *IEEE Trans. Antennas and Propagat.*, vol. AP-45, no. 12, pp. 970-977, December 1997.

[15] C. J. Railton, and J. B. Schneider, "An Analytic and Numerical Analysis of Several Locally Conformal FDTD Schemes," *IEEE Trans. Microwave Theory Tech.*, vol. 47, no.1, pp. 56-66, Jan. 1999.

[16] S. Dey and R. Mittra, "A Locally Conformal Finite Difference Time Domain (FDTD) Algorithm for Modeling 3-D Objects with Curved Surfaces," *IEEE AP-S International Symposium*, Montreal, Canada, 1997.

[17] S. Dey and R. Mittra, "A Locally Conformal Finite Difference Time Domain (FDTD) Algorithm for Modeling Three-Dimensional Perfectly Conducting Objects," *IEEE Microwave and Guided Wave Letters*, vol.7, no.9, pp. 273-275, September 1997.

[18] S. Gedney, F. Lansing, "A parallel Plane Generalized Yee Algorithm for the Analysis of Microwave Circuit Devices," J. Numerical Modeling, (Electric Networks and Fields), vol. 8, 1995, pp. 265-276.

[19] S. Gedney, F. Lansing, and D. Rescoe, "A Full Wave Analysis of Passive Monolithic Integrated Circuit Devices Using a Generalized Yee Algorithm," *IEEE Trans. Microwave Theory Tech.*, vol. 44, 1996, pp. 1393-1400.

[20] Wenhua Yu, and R. Mittra, "A Technique for Improving the Accuracy of the Non-uniform Finite Difference Time Domain (FDTD) Algorithm," *IEEE Trans. Microwave Theory Tech.*, vol. 47, no. 3, pp. 353-356, March 1999.

[21] P. A. Tirkas and C. A. Balanis, "Finite-Difference Time-Domain Method for Antenna Radiation," *IEEE Trans. Antennas and Propagat.*, vol. AP-40, no.3, pp. 334-340, March 1992.

[22] C. A. Balanis, *Advanced Engineering Electromagnetics,* John Wiley & Sons, New York, 1989, chapters 13 and 14.

[23] Zeland Software, 39120 Argonaut Way, Suite 499, Freemont, CA 94538.