A TECHNIQUE FOR ANALYZING RADIATION FROM CONFORMAL ANTENNAS MOUNTED ON ARBITRARILY-SHAPED CONDUCTING BODIES

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Abstract—This paper presents an efficient method to solve the problem of radiation from conformal aperture and microstrip antennas mounted on arbitrarily-shaped conducting bodies. The method, based on the surface equivalence and reciprocity principles, uses a combination of the Finite Difference Time Domain (FDTD) and Method of Moments (MoM) techniques to substantially improve the computational efficiency of the radiation pattern calculation. When the geometry and location of the radiating element are modified, only a small portion of the overall analysis requires re-simulation. This leads to a significant improvement in computational efficiency over presently-used techniques, and can substantially improve design efficiency when included in an optimization loop. The technique is first validated by solving two canonical problems, namely a thin slot which is oriented either axially or azimuthally on an infinitely long, perfectly conducting cylinder. These patterns are then compared to those produced by the same slots mounted on finite length cylinders. Finally, patterns are computed for a cavity-backed elliptical patch antenna mounted on an infinite-length PEC cylinder and compared to patterns computed by an alternate method.
1. INTRODUCTION

Low profile conformal antennas for use on mobile communications systems are receiving widespread attention due to the advantages of minimized aerodynamic friction and radar cross-sectional area, reduced risk of antenna structural damage, and simplified construction requirements over conventional types. Since this type of antenna has a complex configuration and is usually mounted on a large structure, the simulation of these antennas continues to be a challenge. Methods that will accommodate complex structures such as FDTD and the Finite Element Method (FEM) require an excessive number of volume elements for large bodies. Surface methods such as the Method of Moments (MoM) that can model large structures are limited to relatively simple material compositions such as those that are homogeneous or layered. Thus, any of these methods used alone cannot efficiently model a typical conformal antenna when mounted on an electrically large platform. To do this requires a hybrid approach involving a combination of these techniques.

Several combinations of simulation techniques have been used; the hybrid method used in this paper includes the MoM and FDTD. These methods were selected for their strengths in modeling large arbitrarily-shaped homogeneous conducting bodies and relatively small regions with a complex material structure, respectively. The reciprocity theorem is used to link the two methods by relating the aperture fields (or equivalent currents) calculated using the FDTD, and the surface currents (or equivalent magnetic fields) derived by using the MoM [1–6]. A reciprocity approach is also applied in [7]; however, the Finite Element Method (FEM) is used to compute the equivalent magnetic currents instead of the FDTD technique.

Researchers have also used a combination of the FEM and MoM methods to solve problems of similar configuration e.g., a cavity-backed patch antenna [8], and an interior region composed of an inhomogeneous medium with an arbitrarily shaped dielectric in the exterior region [9]. The hybrid technique employs the MoM method in regions where the Green’s function is known (outside of the cavity containing the patch antenna), while applying the FEM to the cavity region, which is inhomogeneously filled and is not amenable to the hybrid approach. The overall approach was verified in [8] through FDTD simulations in two dimensions [9], a volume integral/moment method solution [10], and the unimoment solution [11, 12]. This latter method incorporates
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The radiation condition into either the finite difference or finite element method with less computer storage and run time requirements.

The hybrid method used in this paper further reduces computational requirements for the calculation of radiation patterns, especially when the shape of the radiating element and location on the mounting structure are modified. For most configurations, the size and shape of the mounting structure is fixed; the proposed method exploits this constraint by requiring analysis on this part of the problem only once. Instead of re-simulating the entire structure to obtain the aperture field distribution for every element shape and location change, this approach requires re-solving the problem in the vicinity of the element only if the element shape is altered. The proposed hybrid method eliminates the need to carry out multiple, computationally-intensive matrix operations encountered in the use of other simulation methods such as the hybrid techniques mentioned above.

Since the FDTD technique is well-suited to the analysis of complex inhomogeneous structures, it is used to model the region which includes the cavity-backed conformal antenna and a sufficient volume around it to account for the effects of the surrounding environment on the radiating aperture field distribution. Through the surface equivalence principle, equivalent currents are found and the cavity region is then closed with a perfect electrical conductor (PEC). The resulting structure, which can be large relative to the radiating aperture and of arbitrary shape, is analyzed by the MoM technique, since this method approximates the surface with triangular element discretization. This enhances both computational accuracy and efficiency via the use of a closely-fitting surface mesh of the PEC object, as opposed to using a volume discretization, which would be required in the FDTD method.

2. APPROACH

This paper utilizes the reciprocity principle to divide the original problem into two parts [1–6]. First, we analyze the region containing the conformal antenna, which is typically inhomogeneous. The FDTD method is used for this analysis to derive equivalent electric and magnetic currents J and M on the surface S1 (see Fig. 1) of the radiating aperture of the antenna. Next, S1 is backed by a perfect electrical conductor (PEC) to short out the electric currents and the problem reduces to that of computing the radiation from these magnetic currents located on the closed PEC body. This calculation takes into
account the shape of the PEC body in the vicinity of the antenna.

The second step entails the application of the reciprocity principle to address the pattern computation problem. By invoking the reciprocity principle, we can write

\[ \int_{V_2} E_1 \cdot J_2 dV = - \int_{V_1} H_2 \cdot M_1 dV \]  \hspace{1cm} (1)

Our objective is to compute \( E_1 \) radiated by \( M_1 \) at a far-zone observation point \( P \). We proceed by solving for the surface electric currents flowing on the closed PEC body by illuminating it with an infinitesimal dipole source \( J_2 \) placed at location \( P \). This problem is well-suited for handling by the Method of Moments (MoM). From the electric currents, the equivalent surface magnetic fields \( H_2 \) are determined and by performing the integration on the right-hand side of (1), the desired field \( E_1 \) is computed.

Note that once \( H_2 \) is determined for the entire mounting structure, only \( M_1 \) requires recalculation if the radiating element changes shape or location. The volume integral on the right-hand side of (1) reduces to the surface area of the superstrate (see Fig. 1). This re-simulation requires only a small fraction of the time required for a full-wave analysis of the entire mounting structure.

### 3. NUMERICAL RESULTS

To illustrate the procedure, we first compute the radiation pattern produced by a thin slot mounted on an infinite-length PEC cylinder.
The slots are oriented in two directions: (i) along the cylinder’s axis (axial); and, (ii) in the circumferential (azimuthal) direction. For the axial orientation, the $|E_\phi|$ pattern is computed as the aperture fields in the slot are $\phi$-polarized (across the narrow width of the slot). For the reciprocity calculation, the far-zone dipole source must be polarized in the same direction; therefore, analytical formulas taken from [13] to determine surface currents generated by a $\text{TE}_z^z$ plane wave were used to obtain the necessary $H$ fields (from the surface electric currents). For the azimuthal slot, the polarization of the aperture field is in the $z$-direction; thus, surface currents generated by a $\text{TM}_z^z$ plane wave were used in the reciprocity pattern computation for $|E_\phi|$. The slot field variation was assumed to be cosinusoidal (dominant mode) and uniform along the length and width of the slot, respectively, for both orientations. The patterns calculated via reciprocity are compared to analytical formulations for these two configurations [13] and are found to be in exact agreement (see Figs. 2 and 3). These patterns are computed for slots mounted on a $0.2\lambda$ radius PEC cylinder. The size of the slots are $0.1\lambda$ by $0.25\lambda$ for the narrow and wide dimensions, respectively, for both orientations. The subtended angle for the azimuthal slot is $24.8^\circ$, which is identical to the example considered in [13].

Simulations were then carried out on finite-length cylinders. Since analytical formulas do not exist for surface currents generated by plane waves incident on finite-length cylinders, a numerical code based on the MoM technique was used to obtain these currents. For the axial slot, the polarization of the slot aperture and therefore the far-zone dipole source are $\phi$-directed. Hence, the truncation in the $z$-direction has a minimal effect on the pattern. This can be seen in Fig. 4, which plots the patterns produced by axial slots mounted on $1\lambda$, $2\lambda$, and infinite length PEC cylinders. The solid line represents the pattern for an axial slot mounted on an infinite cylinder. Thus, for a two-wavelength cylinder, the pattern has essentially converged to the infinite case. The pattern for the one-wavelength cylinder shows a higher back lobe level and deeper nulls caused by an enhanced standing wave presence created by the decreased distance to the truncated edge.

For the azimuthal slot, since the length of the cylinders had a significant impact on the shape of the patterns — an odd vs. an even number of wavelengths — they are presented separately (see Fig. 5). Again, the solid line shows the pattern produced by an azimuthal slot.
Figure 2. Radiation patterns: axial slot on infinite-length PEC cylinder. Computation by analytical (direct) vs. reciprocity (H*M) methods.
Figure 3. Radiation patterns: azimuthal slot on infinite-length PEC cylinder. Computation by analytical (direct) vs. reciprocity (H*M) methods.
Figure 4. $|E_\phi|$ vs. $\phi$ radiation patterns: axial slot on finite-length PEC cylinders.

mounted on an infinite-length cylinder. The large back lobes for the even wavelength case are due to the current pattern formed on the finite cylinder when illuminated by a $z$-polarized plane wave. Since the truncation direction ($z$) is in the same direction as the polarization of the plane wave, standing waves are formed with the number of peaks equaling the number of wavelengths. For the even wavelength case, the $\phi$-cut in the $z = 0$ plane passes through a local minimum in the current pattern on the lit (incident) side and a small local maximum on the shadow side of the cylinder. It is this shadow side peak that causes the large back lobe which appears in the patterns for the even case. For the odd case, the $\phi$-cut in the $z = 0$ plane passes through a local maximum on the lit side and a local minimum on the shadow side. This results in a correspondingly smaller back lobe.

While the patterns appear to converge to the infinite case more closely for the odd wavelength case, both cases tend toward the infinite case as the length of the cylinder increases. The standing wave effects diminish only when the cylinder length tends toward infinity, at which point both the odd and even wavelength cases essentially converge to the infinite length case.

Finally, the radiation pattern produced by a cavity-backed elliptical patch antenna covered by a superstrate and mounted on an infinite-
Figure 5. Radiation patterns: azimuthal slot on finite-length PEC cylinders.
length PEC cylinder is computed via a reciprocity approach. The radius of the cylinder is \(0.631\lambda_0 (0.5\lambda_0 + 0.2\lambda_d)\). The patch considered for this analysis is elliptical with major and minor axes of \(0.25\lambda_d\) and \(0.2\lambda_d\), respectively, and was placed inside a rectangular block of dielectric \((\varepsilon_r = 2.33)\) of length \(1\lambda_d\) (\(x'\)-direction) and width \(0.8\lambda_d\) (\(y'\)-direction) with a thickness of \(0.2\lambda_d\). The major axis of the ellipse was aligned with the cylinder axis. A diagram of the conformal antenna is shown in Fig. 6.

![Diagram of conformal antenna](image)

**Figure 6.** Elliptical patch antenna mounted on infinite length PEC cylinder.

A diagram of the microstrip patch antenna configuration, indicating all dimensions and the probe feed point, is shown in Fig. 7. The current distribution on the surface of the PEC cylinder is first calculated by analytical methods [13], followed by a computation of the electric fields (computed by a locally conformal FDTD code [14–17]) on the surface of the superstrate. The conformal method improves the accuracy of the calculation for curved conducting surfaces without a corresponding increase in computational resource requirements. This is accomplished by first approximating the conducting object at each cell by planes defined by the intersection points between the object and the cell edges. The electric fields at cell edges located outside the conducting material are then computed. In this way, the shape of the object can be closely approximated without the need for smaller cell
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Figure 7. Microstrip patch antenna geometry.

sizes. This enhances the accuracy of the calculation without requiring an increase in computational resources.

The patch antenna is linearly polarized along the cylinder z-axis (patch x'-axis), hence the $|E_\theta|$ field ($\theta = 90^\circ$) is the co-polarization direction. This polarization requires the computation of the surface electric currents produced on the cylinder by a z-polarized plane wave, which produces only z-directed currents. Therefore, only the x'-polarized aperture fields on the superstrate surface contribute to this pattern. For the cross-polarized radiation, only the y'-polarized aperture fields contribute to the pattern.

The resulting patterns were constructed by summing the complex dot-product of the electric current and electric field for each point on the surface of the superstrate and repeating this calculation while rotating the array of surface currents in the $\phi$-direction. This dot-product is effectively the same as $H_2 \ast M_1$ which appears on the right-
hand side of (1), due to the normal vector being common to both vector quantities.

For comparison purposes, radiation patterns using the Reciprocity Theorem applied in a different manner on a geometry related to the above configuration are computed. The associated geometry is described in the diagram of Fig. 8. In this figure, the z-axis is out of the page and the diagram shows the xy-plane (i.e., \( z = 0 \)) cut through the microstrip patch antenna (refer to Fig. 6 for a 3-D view). This geometry was selected to provide a means of comparison to the first method and to quantify the effects of a cavity on the radiation pattern.

In the alternate method, an MoM technique is used to compute the electric fields at the top surface of the dielectric produced by the patch antenna. The dielectric material is approximated by a flat dielectric slab of infinite extent to allow efficient modeling by the MoM technique. The fields on the surface of the superstrate \( S_1 \) in the original geometry are used to compute the pattern. The magnitude distribution of these fields is shown in Fig. 9. Although fields exist outside this region, they are not considered in the calculation. The radiation patterns are computed by using the Reciprocity relationship in (1), applied in the same manner as in the proposed method.

4. RECIPROCITY-BASED PATTERN COMPUTATIONS

Radiation patterns for the cavity-backed patch antenna mounted on an infinite length conducting cylinder and the associated geometry are computed in this section by using the proposed method and the alternate technique described at the end of Sec. 4 above, respectively.
Figure 9. Superstrate field distributions, $|E_x|$ and $|E_y|$ (MoM). The units for both the $x$ and $y$ axes correspond to cell numbers, where each cell has dimensions of $\lambda_d/100$ by $\lambda_d/100$. 
Figure 10. $E_\theta$ H*M patterns: cavity vs. no-cavity.

The patterns are compared and the effect of the cavity on both aperture polarizations is noted. Both the co- ($\theta$) and cross- ($\phi$) polarizations of the radiated field are computed.

The patterns for the co-polarization ($E_\theta$) radiated fields for the H*M computation on the original geometry (with cavity) and the H*M calculation on the associated geometry (no cavity) are shown in Fig. 10. Even without the cavity, use of the surface electric fields at the position of the cavity (computed by MoM) yields patterns that are identical with the computation for the original geometry (fields computed by the conformal FDTD).

Next, the cross-polarization radiation patterns ($E_\phi$) are presented. For this polarization, the patterns computed by both methods compare well. The plots appear in Fig. 11. The creeping waves launched from the cavity into the shadow region are less attenuated as compared to the $E_\theta$ case. The nulls at $\phi = 0^\circ$ and $180^\circ$ are caused by the two peaks in the $E_y'$ aperture distribution being equal in magnitude, but $180^\circ$ out of phase.

To determine the effect of the cavity on the two aperture polarizations, patterns for slots narrow in the $\phi$-direction are plotted for a conducting cylinder infinite in length and with a radius of $0.631\lambda_o$. The plot in Fig. 12 depicts surface electric currents $J_z$ and $J_\phi$ produced
by $E_{\theta}$- and $E_{\phi}$-polarized, normally incident plane waves, respectively. The plots are normalized to their respective maximums and plotted in dBA/m. If an azimuthal slot is narrow in the $\phi$-direction, its pattern can be plotted directly from the $J_z$ currents for the axial slot narrow in the $\phi$-direction. Therefore, the plot of $|J_z|$ and $|J_\phi|$ also represents the radiation patterns for slots narrow in the $\phi$-direction with aperture fields polarized in the $z$ and $\phi$ directions, respectively.

The above radiation patterns show that $E_{\phi}$ polarized creeping waves launched from an axial slot onto the conducting cylinder surface are attenuated less by the slot edges compared to $E_{\theta}$ polarized waves. This explains why the $E_{\phi}$ pattern for the cavity-backed elliptical patch antenna compares well with the $E_{\phi}$ patterns for the associated geometry; the creeping waves originating from the cavity are of the same approximate magnitude as those propagating on the surface of the dielectric in the associated geometry. In contrast, the $E_{\theta}$ creeping waves emitted by the cavity in the original geometry are significantly attenuated by the cavity edge compared to those free to propagate through and on the dielectric surface in the associated geometry. This can be seen in the current plots of Fig. 12, where the difference in current magnitudes at $\phi = 180^\circ$ is approximately 22 dB.
Figure 12. $|J_z|$ and $|J_\phi|$ (dBA/m) vs. $\phi$ for infinite length conducting cylinder with $r = 0.631\lambda_o$.

6. CONCLUSIONS

This paper has presented an efficient method for determining radiation patterns for arbitrarily shaped conformal antennas. Several examples were presented illustrating the procedure for computing a pattern using the reciprocity approach. The technique was first verified against known radiation patterns, then extended to more complex structures. The pattern produced by a cavity-backed elliptical patch antenna was also computed and compared to patterns constructed using an alternate method. The patterns generated using the proposed method were shown to be in good agreement with the alternate method described at the end of Sec. 4 above, especially for the co-polarization patterns. The patterns for the cross-polarization case also compared well due to the polarization of the aperture fields being normal to the cavity edge. The plots also illustrate that the cavity has a minimal effect on the pattern.

The paper also shows how results obtained for the mounting structure can be used for any radiating element shape and location on the structure in the computation of the radiation pattern. This feature of the technique results in significant improvements in computational
efficiency when the shape or location of the radiating element is varied.

REFERENCES


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