A Discotic Disguised as a Smectic: A Hybrid Columnar Bragg Glass

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We show that discotics, lying deep in the columnar phase, can exhibit an x-ray scattering pattern which mimics that of a somewhat unusual smectic liquid crystal. This exotic, new glassy phase of columnar liquid crystals, which we call a "hybrid columnar Bragg glass," can be achieved by confining a columnar liquid crystal in an anisotropic random environment of, e.g., strained aerogel. Long-ranged orientational order in this phase makes *single-domain* x-ray scattering possible, from which a wealth of information could be extracted. We give detailed quantitative predictions for the scattering pattern in addition to exponents characterizing anomalous elasticity of the system.

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Until now, the x-ray scattering pattern given in Fig. 1 would be identified with a system in a somewhat unusual smectic phase with short-ranged translational and long-ranged orientational order *within* the smectic layers, i.e., a smectic composed of *nematic*, rather than *liquid*, layers. The set of on-axis quasisharp Bragg peaks along q_h is a signature of the quasi-long-ranged translational order (i.e., the periodicity of the layering) that is characteristic of the bulk smectic phase. The presence of the other, broadened, peaks and the azimuthal anisotropy about the q_h axis respectively indicate the incipient *short-ranged* translational order and the long-ranged orientational order *within* the smectic layers oriented perpendicular to q_h.

In this Letter we predict the existence of a remarkable new "hybrid columnar Bragg glass" (HCBG) phase [1,2] which, despite differing fundamentally from the smectic phase described above, shares the same qualitative scattering pattern illustrated in Fig. 1. Such mimickry of one phase by a completely different phase is unprecedented.

Columnar phases in pure, bulk (i.e., quenched-disorder-free) liquid crystals are phases that have long-ranged translational order in two directions, and short-ranged translational order (i.e., liquidlike correlations) in the third. I.e., they are regular two-dimensional lattices of one-dimensional liquid columns [Fig. 2(a)]. In this Letter, we show that when such a system is confined in an *anisotropic* quenched random environment, e.g., strained aerogel [3], it becomes translationally disordered, but remains topologically ordered (i.e., free of topological defects such as dislocations). This novel state is the HCBG.

Like the smectic phase, the HCBG has translational order that is quasi-long-ranged in one direction and short ranged in another, as implied by Fig. 1. As illustrated in Fig. 2(b), the columns remain in roughly equidistant rows perpendicular to the stretching direction, but lose long-ranged translational order within each row. Nevertheless, the hexagonal orientational order is preserved, albeit uniaxially distorted due to the stretch. However, there are a number of fundamental differences between the two phases. First, unlike the smectic phase, the exponent $\eta_{\mathbf{G}}$ characterizing the shape of the quasi-long-ranged translational order peaks at $\mathbf{G}$, which are given by $I(\mathbf{q}) \propto |\mathbf{q} - \mathbf{G}|^{-3+\eta_G}$, is independent of temperature. This property would allow the scattering patterns of the two phases to be distinguished through comparison of the line shapes as temperature is varied. Second, the correlations of the quasi-long-ranged order in the HCBG scale isotropically in space, in contrast to the well-known strongly anisotropic scaling of these correlations in the smectic. The third and most crucial difference between the two phases is their topological order, which distinguishes their elasticities but not their scattering. Specifically, the absence of the extra direction of long-ranged translational order, in the smectic phase, is caused by free topological defects, namely, unbound dislocations with Burgers vectors along the smectic planes. Although the HCBG also exhibits translational order that is quasi-long-ranged in one direction and short ranged in another, at long length scales it is distinguished from the smectic by being free of these unbound

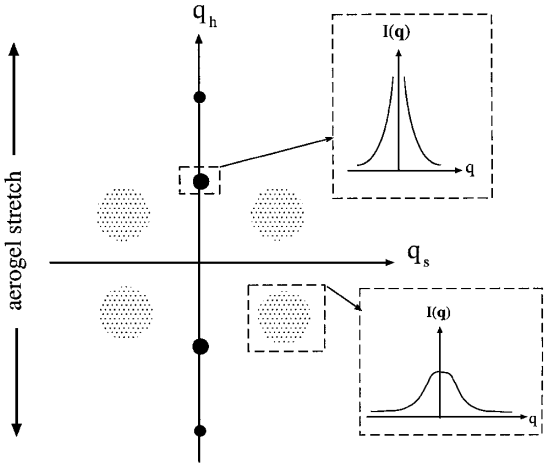

FIG. 1. X-ray scattering pattern in the $\perp$ plane for a class of hybrid columnar Bragg glass.

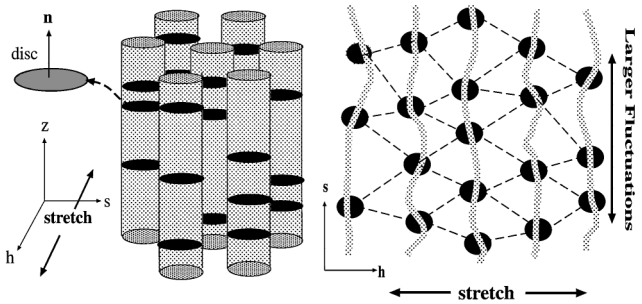

FIG. 2. (a) Alignment of the lattice for a uniaxial stretch. (b) Schematic of the distorted lattice in real space (top view).

dislocations, which would otherwise destroy the columnar phase topology of HCBG. One important experimental consequence of this absence of free dislocations is that the HCBG retains elastic resistance to distortions in the extra direction, albeit, as we discuss below, of a very strange, anomalous sort.

Of course, for *sufficiently* strong disorder free dislocations *will* eventually proliferate. The anisotropy, imposed by the strained aerogel, leads to the interesting possibility that dislocations with their Burgers vectors in the soft direction may unbind before those with their Burgers vectors in the hard direction, leading to the sequence of disorder-driven phase transitions HCBG $\rightarrow$ $m = 1$ smectic Bragg glass [4] $\rightarrow$ nematic elastic glass [5] with increasing aerogel density.

The rest of this Letter gives a more detailed theoretical description of the HCBG phase, including x-ray correlation lengths and universal exponents characterizing the anomalous elasticity. We relegate the technical details to a future publication [6].

Our model for a columnar phase consists of disk-shaped molecules with normals aligned along the $\hat{\mathbf{z}}$ direction. The disks form a hexagonal lattice in the xy ($\perp$) plane and have liquidlike correlations along $\hat{\mathbf{z}}$, as illustrated in Fig. 2(a). We assume, and verify *a posteriori,* that despite considerable distortion, for sufficiently weak quenched disorder, the columnar phase topology is stable, i.e., our discotic liquid crystal remains free of unbound dislocations. Consequently this system can be described within an elastic theory, with a two-component (x and y) lattice site displacement vector $\mathbf{u}(\mathbf{r})$ and the discotic director $\hat{\mathbf{n}}(\mathbf{r})$ (the normal to the disks) as the only important long length-scale degrees of freedom. The disordering tendency of the aerogel is twofold: the strands act both to randomly pin the columnar lattice $[\mathbf{u}(\mathbf{r})]$ and to distort the orientations of the disk normals $[\hat{\mathbf{n}}(\mathbf{r})]$. Our starting Hamiltonian is that of a pure hexagonal discotic in *isotropic* aerogel,

$$H = \int_{\mathbf{r}} \Big[\frac{B_\perp}{2} |\partial_z \mathbf{u} - \boldsymbol{\delta}\mathbf{n}|^2 + \frac{1}{2} \lambda u_{ii}^\perp u_{jj}^\perp + \mu u_{ij}^\perp u_{ij}^\perp + \mathrm{Re} \sum_i V_i(\mathbf{r}) e^{i\mathbf{G}_i \cdot \mathbf{u}(\mathbf{r})} - [\mathbf{g}(\mathbf{r}) \cdot \hat{\mathbf{n}}]^2 \Big] + H_F[\hat{\mathbf{n}}]. \quad (1)$$

where $u_{ij}^\perp = \frac{1}{2}(\partial_i^\perp u_j + \partial_j^\perp u_i - \partial_k u_i \partial_k u_j)$ is the rotationally invariant symmetric strain tensor, $\delta\mathbf{n}(\mathbf{r}) \equiv \hat{\mathbf{n}}(\mathbf{r}) - \hat{\mathbf{z}}$, the $B_\perp$ term reflects the tendency of the molecular director (disk normal) $\hat{\mathbf{n}}(\mathbf{r})$ to lie along the local tangent $\hat{\mathbf{t}} \approx \hat{\mathbf{z}} + \partial_z \mathbf{u}$ to the liquidlike columns, $H_F[\hat{\mathbf{n}}]$ is the Frank free energy of the molecular directors, and $V_i(\mathbf{r})$ is a complex random pinning potential that couples to lattice site fluctuations along the reciprocal lattice basis vector, $\mathbf{G}_i$. At long length scales its correlations can be accurately represented as zero mean with *short-ranged* Gaussian statistics: $\overline{V_i(\mathbf{r}) V_j^*(\mathbf{r}')} = \tilde{\Delta}_V \delta_{ij}^\perp \delta^d(\mathbf{r} - \mathbf{r}')$ [5], where throughout this paper $\overline{x}$ denotes a quenched average over the disorder of the quantity x, while $\langle x \rangle$ denotes a thermal average. The last term describes the tendency of the disk normals $\hat{\mathbf{n}}(\mathbf{r})$ to align along the random local aerogel strand directed along $\mathbf{g}(\mathbf{r})$. This "random tilt" disorder is described by *short-ranged isotropic* correlations $\overline{g_i(\mathbf{r}) g_j(\mathbf{r}')} = 1/2\sqrt{\Delta}\delta_{ij}\delta^d(\mathbf{r} - \mathbf{r}')$ [5]. Δ_V and Δ are phenomenological parameters which, in the simplest microscopic model, are proportional to the aerogel density, ρ_A. As for smectics [5], only these two types of disorder have important long distance effects.

A detailed analysis [5,6] has shown that this system exhibits a columnar Bragg glass phase with only short-ranged translational order. However, this changes if the aerogel is anisotropic. Aerogel anisotropy could be realized, e.g., by applying a strain to the strands. For heterotropic alignment between the disk normals and strands (assumed throughout), a uniaxial *compression* will lead to a phase in the same universality class as the Bragg glass phase of an Abrikosov flux lattice [1,2], with quasi-long-ranged translational order in both directions of the $\perp$ plane [6]. The more interesting HCBG phase with quasi-long-ranged order in only one $\perp$ direction can be obtained by applying a uniaxial *stretch* to the strands. For homeotropic alignment of strands and disk normals, the two phases reverse with respect to stretch and compression leaving all of our other predictions unchanged.

Uniaxial stretch to the aerogel strands (along $\hat{\mathbf{e}}_h$) causes the disk normals to align $\perp$ to this axis of stretch (Fig. 2). This can be accounted for by the addition of a term $\int_{\mathbf{r}} \gamma(\hat{\mathbf{e}}_h \cdot \hat{\mathbf{n}})^2$ to the Hamiltonian, where γ is a phenomenological parameter, which we expect to be a monotonically increasing function of ρ_A and strain. The uniaxial stretch also breaks both the hexagonal symmetry of the lattice and its rotation invariance. Thus, we are forced to consider both a more general lattice structure and an elastic Hamiltonian that is not invariant under rotations of the lattice. We can take into account both of these effects by replacing the harmonic pieces of the hexagonal elastic energy $\int_{\mathbf{r}} (\frac{1}{2}\lambda u_{ii}^\perp u_{jj}^\perp + \mu u_{ij}^\perp u_{ij}^\perp)$ with the more general harmonic elastic energy $\frac{1}{2}\int_{\mathbf{r}} C_{ijkl} \partial_i^\perp u_j \partial_k^\perp u_l$, where the elastic constant tensor C_{ijkl} is *not* symmetric under interchange of its first two or second two indices, due to the lack of in-plane rotation invariance just discussed. The terms cubic and quartic in $\mathbf{u}$ in (1) must also

be so generalized, of course. Because of the $B_\perp$ term in Eq. (1), fluctuations of $\hat{\mathbf{n}}$ from the local column tangent $\hat{\mathbf{t}}$ are small, i.e., $\delta\mathbf{n} \approx \partial_z \mathbf{u}$.

We have analyzed [6] this generalization of the model [Eq. (1)] by using renormalization group (RG) methods [5]. One of the most surprising conclusions of this analysis is that, at long length scales, fluctuations $u_h \equiv \hat{\mathbf{e}}_h \cdot \mathbf{u}$ along the direction of stretch decouple from those $u_s \equiv \hat{\mathbf{e}}_s \cdot \mathbf{u}$ orthogonal to this direction, where we have denoted the axis of stretch, $\hat{\mathbf{e}}_h$, as "*hard*" (h) and called the other $\perp$ axis, orthogonal to $\hat{\mathbf{e}}_h$ "*soft*" (s), i.e., $\mathbf{r}_\perp = (r_h, r_s)$. That is, all couplings between u_s and u_h that are present in the full elastic tensor C_{ijkl} are effectively irrelevant, leaving C_{ijkl} in the form $C_{ijkl} = B_{ss}\delta_{is}\delta_{js}\delta_{ks}\delta_{ls} + B_{sh}\delta_{is}\delta_{jh}\delta_{ks}\delta_{lh} + B_{hs}\delta_{ih}\delta_{js}\delta_{kh}\delta_{ls} + B_{hh}\delta_{ih}\delta_{jh}\delta_{kh}\delta_{lh}$.

The total Hamiltonian for the system can therefore be expressed as a sum of *decoupled* Hamiltonians for u_h and u_s: $H_{\rm tot}[u_h, u_s] = H_{XY}[u_h] + H_{m=1}[u_s]$, with

$$H_{XY} = \frac{1}{2}\int_{\mathbf{r}}\Big[\gamma|\partial_z u_h|^2 + B_{sh}|\partial_s u_h|^2 + B_{hh}|\partial_h u_h|^2 + \mathcal{R}e\sum_i V_i(\mathbf{r})e^{i(\mathbf{G}_i\cdot\hat{\mathbf{e}}_h)u_h(\mathbf{r})}\Big], \tag{2a}$$

$$H_{m=1} = \int_{\mathbf{r}}\Big\{\frac{K}{2}(\partial_z^2 u_s)^2 + B_{ss}\Big[\partial_s u_s - \frac{1}{2}(\partial_z u_s)^2\Big]^2 + B_{hs}(\partial_h u_s)^2 - g_z(\mathbf{g}\cdot\hat{\mathbf{e}}_s)(\partial_z u_s)\Big\}. \tag{2b}$$

These two parts of $H_{\rm tot}$, which describe the fluctuations of the hard and soft phonon fields u_h and u_s, are not new. Hamiltonians of precisely this form have been previously used to describe the random field XY model [2] and the "$m = 1$ smectic Bragg glass" [4], respectively, and have been studied extensively [1]. However, a columnar phase confined in anisotropic aerogel, whose Hamiltonian, $H_{\rm tot}$, is a combination, or *hybrid*, of the two, is entirely novel.

The u_h fluctuations of our system are the same as those of a random field XY model (with anisotropic stiffness) and are given by $\overline{\langle(u_h(\mathbf{r}) - u_h(\mathbf{0}))^2\rangle} = C(d)\ln r/G_{0h}^2$, where G_{0h} is the lattice spacing of the projection of the discotic reciprocal lattice onto the hard axis. They diverge logarithmically as a function of distance, implying that the translational order along the hard direction is quasi-long-ranged. While these elastic distortions are reminiscent of the famous Landau-Peierls $\ln r$ fluctuations of *bulk* smectics, they differ crucially in two ways. Firstly, they are *disorder*, rather than *thermally*, driven with $C(d)$ *universal* [$C(3) \approx 1.1$] and the logarithm persisting in all $2 < d < 4$. Secondly, they are *isotropic* in their scaling. In contrast, in a bulk smectic the layer fluctuations within the layers scale differently from those along the normal to the layers.

In $H_{m=1}$, the combination of relevant anharmonic terms and large disorder-induced u_s fluctuations leads to strong anomalous elasticity [5,6]. By anomalous elasticity we mean that the full, *anharmonic* theory with *constant* K, B_{ss}, and Δ can, at small wave vector $k \ll \xi_{\rm NL}^{-1}$ (where $\xi_{\rm NL}$ is a nonuniversal length determined by material parameters, e.g., aerogel density), be effectively replaced by a *harmonic* theory with wave vector dependent K, B_{ss}, and Δ, given by

$$K(\mathbf{k}) = K k_z^{-\eta_K} f_K(k_h/k_z^{\zeta_h}, k_s/k_z^{\zeta_s}), \tag{3a}$$

$$B_{ss}(\mathbf{k}) = B_{ss} k_z^{\eta_B} f_B(k_h/k_z^{\zeta_h}, k_s/k_z^{\zeta_s}), \tag{3b}$$

$$\Delta(\mathbf{k}) = \Delta k_z^{-\eta_\Delta} f_\Delta(k_h/k_z^{\zeta_h}, k_s/k_z^{\zeta_s}). \tag{3c}$$

B_{hs} is not significantly renormalized, that is, $B_{hs}(\mathbf{k}) = B_{hs}$, independent of wave vector. Here the anisotropy exponents $\zeta_s \equiv 2 - (\eta_B + \eta_K)/2$ and $\zeta_h \equiv 2 - \eta_K/2$. The exponents, evaluated using the RG and a high precision ϵ-expansion were found to be $\eta_K = 0.50$, $\eta_B = 0.26$, and $\eta_\Delta = 0.13$ [4,6]. We also predict that the anomalous exponents will obey the following *exact* scaling relation in $d = 3$:

$$1 + \eta_\Delta = \eta_B/2 + 2\eta_K. \tag{4}$$

The translational order of the system along the soft direction is short ranged and is characterized by the algebraic and anisotropic divergence of u_s correlations,

$$\overline{\langle(\delta u_s(\mathbf{r}))^2\rangle} = \begin{cases} (\frac{K}{B_{ss}})(\frac{r_z}{\xi_z})^{\chi_z}, & r_z \gg r_{s,h}, \\ (\frac{K}{B_{ss}})[\frac{r_s}{\xi_z^2}(\frac{K}{B_{ss}})^{1/2}]^{\chi_s}, & r_s \gg r_{h,z}, \\ (\frac{K}{B_{ss}})[\frac{r_h}{\xi_z^2}(\frac{K}{B_{sh}})^{1/2}]^{\chi_h}, & r_h \gg r_{s,z}, \end{cases} \tag{5}$$

where we have defined $\delta u_s(\mathbf{r}) \equiv u_s(\mathbf{r}) - u_s(\mathbf{0})$, $\chi_z \equiv 1 - \eta_K + \eta_B/2 + \eta_\Delta = \eta_B + \eta_K$, $\chi_{s,h} \equiv \chi_z/\zeta_{s,h}$, and $\xi_z \equiv K^2 B_{sh}^{1/2}/(\Delta B_{ss}^{1/2})$. The exact scaling relation (4) could be experimentally tested by using the more general expressions for χ_z, χ_s, χ_h in terms of all three exponents, and verifying that η_B, η_K, and η_Δ obey the relation (4). Our ϵ-expansion results for the η's imply $\chi_z = 0.76$, $\chi_s = 0.47$, and $\chi_h = 0.43$. The fluctuations given in Eq. (5), like those along the hard direction, are disorder, rather than thermally driven.

Despite this lack of translational order, our detailed calculations [6] indicate that dislocation loops remain bound for weak disorder, and therefore the low temperature phase replacing the columnar phase must be distinct from the smectic and hexatic, separated from them by a thermodynamically sharp dislocation unbinding phase transition.

The stability of this exotic glass phase is contingent upon our assumption of long-ranged orientational order. We validate this assumption by calculating $\overline{\langle|\mathbf{n}(\mathbf{r}) - \mathbf{n}(\mathbf{0})|^2\rangle} = \overline{\langle|\partial_z \mathbf{u}(\mathbf{r}) - \partial_z \mathbf{u}(\mathbf{0})|^2\rangle}$ and showing that it does *not* diverge as $\mathbf{r} \to \infty$ [6]. Although equilibration into the ground state might be slow and therefore require field alignment, this orientational order would allow experimentalists to investigate single-domain samples of HCBG. The anisotropic scaling information which is usually lost in a powder averaged x-ray scattering experiment would be retained in a single-domain experiment, allowing detailed tests of our predictions for η_K, η_B, and η_Δ.

The scattering pattern in the $\perp$ plane, obtained from a single-domain sample, would consist of a set of spots rather than the set of rings that one would expect from a powder sample. This pattern depends crucially on the relative orientations within the $\perp$ plane of the reciprocal lattice and the axis of stretch, $\hat{\mathbf{e}}_h$, which could vary from discotic to discotic since it depends on the microscopic interactions between the disks and strands. The intensity of a Bragg spot at a reciprocal lattice vector $\mathbf{G}$ is

$$I(\mathbf{G}) \propto \int_{\mathbf{r}} \exp\{-\overline{\langle[\mathbf{G}\cdot(\mathbf{u}(\mathbf{r})-\mathbf{u}(\mathbf{0}))]^2\rangle}/2\} \quad (6a)$$

$$\propto \int_{\mathbf{r}} \exp\{-[G_h^2\overline{\langle(u_h(\mathbf{r})-u_h(\mathbf{0}))^2\rangle} + G_s^2\overline{\langle(u_s(\mathbf{r})-u_s(\mathbf{0}))^2\rangle}]/2\}. \quad (6b)$$

Unless $G_s = 0$, the algebraically diverging u_s fluctuations dominate the logarithmically diverging u_h fluctuations and the integrand is stretched-exponentially damped, leading to an anisotropically broadened Bragg peak. If, however, $G_s = 0$, then the exponential becomes $r^{-0.55n^2}$, where $n = G_h/G_{0h}$, with G_{0h} being the magnitude of the *smallest* $\mathbf{G}$ lying *on* the hard axis, and for $n < 3$ the integral diverges as $r \to \infty$, leading to quasisharp peaks for those n's. We therefore predict two classes of hybrid columnar Bragg glasses. The first, which we call a *commensurate* HCBG, has some reciprocal lattice vectors that lie *along* the hard axis, and will exhibit a scattering pattern with peaks lying on the hard axis, with the first two quasisharp. In the second, *incommensurate* HCBG, class, all of the peaks lie off the hard axis, and are anisotropically broadened by the contribution from the u_s fluctuations. The smecticlike scattering pattern (Fig. 1), with a quasisharp peak on the first ring, will therefore be observed only for commensurate HCBG's.

The dependence of the anisotropically broadened peak widths on the bare elastic constants is the same for both classes of HCBG. Setting the $u-u$ correlation functions [given in Eq. (5)] equal to G_s^{-2}, and solving for $(r_z = \xi_z^X)^{-1}$, $(r_s = \xi_s^X)^{-1}$, and $(r_h = \xi_h^X)^{-1}$ gives the width of the peak at $\mathbf{G}$ along the z, s, and h directions:

$$(\xi_z^X)^{-1} = \xi_z^{-1}(G_s^2 K/B_{ss})^{\chi_z^{-1}}, \quad (7a)$$

$$(\xi_s^X)^{-1} = \xi_z^{-2}(K/B_{ss})^{1/2}(G_s^2 K/B_{ss})^{\chi_s^{-1}}, \quad (7b)$$

$$(\xi_h^X)^{-1} = \xi_z^{-2}(K/B_{sh})^{1/2}(G_s^2 K/B_{ss})^{\chi_h^{-1}}. \quad (7c)$$

The temperature dependence of $\xi_{z,s,h}^X$ could be used to determine the exponents η_K, η_B and η_Δ since the *bulk* $K(T)$, $B_{ss}(T)$, and $B_{sh}(T)$ in Eqs. (7a)–(7c) have T dependences that can be extracted from data on bulk materials. A more direct way to observe the anomalous elasticity would be a measurement of the $u-u$ correlation function $I(\mathbf{q}) \propto \overline{\langle|u_s(\delta\mathbf{q})|^2\rangle}$, which can be obtained [6] for *large* $\mathbf{q}$ (i.e., $\mathbf{q}$'s with at least one component bigger than the corresponding inverse x-ray correlation length quoted above) by looking at an intermediate regime in the "tails" of the broad x-ray scattering peaks. In those tails, [i.e., for $\mathbf{q} = \mathbf{G} + \delta\mathbf{q}$ with $\xi_{\rm NL}^{-1} \gg |\delta q_\alpha| \gg (\xi_\alpha^X)^{-1}$ for at least one Cartesian direction $\alpha = (h, s, z)$ [7]], it can be shown that

$$I(\mathbf{q}) \propto \frac{\Delta(\delta\mathbf{q})q_z^2}{[B_{ss}(\delta\mathbf{q})\delta q_s^2 + K(\delta\mathbf{q})\delta q_z^4 + B_{hs}\delta q_h^2]^2}. \quad (8)$$

Hence, the $\mathbf{q}$ dependence of $B_{ss}(\mathbf{q})$, $K(\mathbf{q})$, and $\Delta(\mathbf{q})$ in Eqs. (3a) and (3b) could be tested directly by a fit of scattering data to these tails.

A related experimental approach, which has the advantage of *not* being restricted to wave vectors larger than the inverse x-ray correlation lengths, but can, rather, explore arbitrarily small $\mathbf{q}$'s, is light scattering, which measures director fluctuations. These can be related to the $u-u$ correlations via our condition $\delta\mathbf{n} \approx \partial_z\mathbf{u}$. This yields

$$\overline{\langle|\delta n_s(\mathbf{q})|^2\rangle} = \frac{\Delta(\mathbf{q})q_z^4}{[B_{ss}(\mathbf{q})q_s^2 + K(\mathbf{q})q_z^4 + B_{hs}q_h^2]^2}, \quad (9a)$$

$$\overline{\langle|\delta n_h(\mathbf{q})|^2\rangle} = \begin{cases} [C(3)/2G_{0h}^2](q_z^2/q^3), & \text{commensurate} \\ k_B T q_z^2 G(\mathbf{q}) + \Delta_h(\mathbf{q})q_z^4 G(\mathbf{q})^2, & \text{incommensurate} \end{cases}, \quad (9b)$$

where $\Delta_h(\mathbf{q})$ in Eq. (9b) is a renormalized $\mathbf{q}$-dependent tilt disorder variance obeying the scaling law $\Delta_h(\mathbf{q}) = \Delta_h q_z^{-(\eta_\Delta+\eta_B)} f_{\Delta_h}(q_h/q_z^{\zeta_h}, q_s/q_z^{\zeta_s})$ and $G(\mathbf{q}) = 1/(B_{sh}q_s^2 + \gamma q_z^2 + B_{hh}q_h^2)$. The commensurate and incommensurate cases differ because in the commensurate case there *is* a random field acting on u_h, while in the incommensurate case there is no random field, leaving the random tilt as the dominant disorder.

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[7] We have shown in [6] that the ratio $\xi_{\rm NL}/\xi_\alpha^X \to 0$ as the aerogel density goes to zero. Hence, one could make the width of this window in δq_α as large as one likes simply by making the aerogel density sufficiently small.