UNCERTAINTY PROPAGATION IN GEOTECHNICAL EARTHQUAKE ENGINEERING

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Abstract: This paper describes recent advances in evaluating, quantifying, and propagating various forms of uncertainty in geotechnical earthquake engineering problems. Important developments in the fields of liquefaction engineering, dynamic slope stability, engineering seismology, and lifeline engineering are discussed. The benefits gained through proper treatment of uncertainty include; a well defined measure of the most likely engineering results, a well defined estimate of extreme results, a probability of likelihood ascribed to different realizations, and a mathematical format that lends to performance-based engineering assessment. This paper is by no means comprehensive but highlights some recent studies that contribute to improved probabilistic methodology in the realm of geotechnical earthquake engineering.

1. INTRODUCTION

Uncertainty in its various forms is an unavoidable component of engineering analysis. Geotechnical earthquake engineering is ripe with uncertainty primarily because of the inherent variability of geotechnical materials and the stochastic nature of earthquake ground motions. Capturing and translating uncertainty through any engineering analysis is necessary for resolving the mean or median response with any confidence, and for estimating the dispersion of possible results. Geotechnical earthquake engineering is a pseudo-empirical discipline where theory dictates the trends of the analytical models but data drives the shape, coefficients, and values of the numerical results. A model used in geotechnical earthquake engineering is only as good as its accuracy with respect to the empirical data.

Uncertainty can be conceptually lumped into two groups; the inherent variability of the underlying phenomena, and uncertainty as a function of modeling, measuring, and other engineering machinations that are not part of the phenomena. These are termed aleatory and epistemic uncertainty respectively. These two groups of uncertainty can have a strong influence on the outcome of some engineering analysis and are often not easily separable. Recent advances in probabilistic methods have lead to improved uncertainty analysis in geotechnical earthquake engineering and related fields. This paper describes a select (and by no means comprehensive) group of studies which the author feels demonstrates improvements in how uncertainty is quantified and propagated through the analysis thereby providing a broader understanding of the problem at hand and the desired outcome. The select group of studies is biased towards the author's work but also draws on work from others that strive for the same goal of propagating uncertainty for improved understanding and accuracy.

2. METHODS OF UNCERTAINTY PROPAGATION

The basis for error propagation is founded in the fundamentals of statistics and probability. Statistics is the means of quantifying past occurrences and probability the means of predicting future occurrences. In civil engineering early applications of statistics and probability were well described by Benjamin and Cornell (1970) and Ang and Tang (1975). Quantifying uncertainty can be accomplished through various statistical means if sufficient data exists, or by ascribing a probability distribution based on theory, assumptions, and/or expert solicitation. Quantifying engineering uncertainty using all forms of available information subscribes to the Bayesian philosophy of probability and uncertainty.

Propagating uncertainty involves "pushing" the uncertainty through the model, equation, or analysis to arrive at final results representative of the formulation and the contributing uncertainty. This can be accomplished with exact methods if certain conditions are met (e.g. sum of normally distributed random variables), approximate methods that can often give reasonable results (i.e. first order second moment approximation), and simulation methods (e.g. Monte Carlo simulation). Reliability analysis is a special case of error propagation where the mathematical formulation is defined by a relationship between load and resistance with the goal of characterizing failure. The relative contribution of epistemic and aleatory uncertainty in uncertainty propagation can be complex and there is little agreement as to how best separate the two (Helton 2004). But the methods of propagating uncertainty are well established and can be readily applied to most geotechnical earthquake engineering problems. The following are selected studies that demonstrate useful applications of uncertainty propagation.

2. LIQUEFACTION ENGINEERING

Liquefaction engineering starts with the assessment of the likelihood of liquefaction triggering for a particular soil deposit. Field data drives liquefaction analysis because lab data in most cases fails to capture critical *in situ* soil conditions. There exists uncertainty in the input variables on both the load and the resistance side of the phenomenon. Recent work by Cetin et al. (2004) and Moss et al. (2006) focused effort on quantifying the uncertainty in the earthquake loading, in the form of the cyclic stress ratio (CSR), and the dynamic soil resistance, in the form of corrected resistance values from the SPT and CPT. The results show (Figure 1) the probabilistic relationship between load and resistance that provides a means of making a performance-based engineering decision of the likelihood of liquefaction triggering for a particular soil deposit. This probabilistic relationship is a byproduct of quantifying the uncertainty of each input variable and then propagating that uncertainty through the mathematical interaction of the variables in a reliability format.
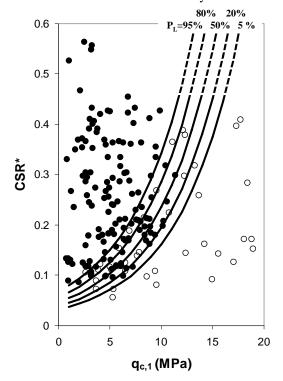

Figure 1 Liquefaction triggering curves for the CPT (from Moss et al. 2006). The x-axis is a normalized cone tip resistance and the y-axis a normalized earthquake load. Probability of liquefaction curves from 5% to 95% are shown.

Cetin et al. (2004) and Moss et al. (2006) capture and propagate the uncertainty from the back-analysis of a liquefaction case history database using FORM (first order reliability method), SORM (second order reliability method), and MC (Monte Carlo) simulations. Work by Fang et al. (2006) focuses on including the uncertainty in the forward-analysis of liquefaction triggering using FORM.

The work by Fang et al. assesses the uncertainty from the site specific aspects of loading and resistance, providing a probabilistic forward-analysis that combines with the uncertainty from the back-analyzed case histories to give a comprehensive probabilistic approach.

The probability of liquefaction at a site is conditional on the probability of a particular level of ground shaking being exceeded. Kramer and Mayfield (2007) present a study that integrates the probability of ground shaking with the probability of liquefaction which results in an annualized return period of liquefaction. The return period of liquefaction provides a more consistent measure of liquefaction potential when comparing sites from different seismo-tectonic regions. This work lends to more uniform liquefaction hazard maps on a regional and national scale.

One issue that is often overlooked is the uncertainty that creeps into an analysis when converting between *in situ* index tests (e.g. CPT to SPT). Moss and Hollenback (2009) discuss this issue with respect to post-liquefaction effective stress normalized undrained strength (s_u/σ_{vo}'), commonly called liquefied residual strength or mobilized shear strength ratio. When converting from one index test to another, the measurement uncertainty from the index test combine with the statistical uncertainty between the correlated index tests to produce a compounded uncertainty. This can result in ambiguous and inaccurate estimates of the median converted value as well as large dispersion. Even in the simple process of converting from one index test value to another, the propagation of uncertainty should be performed to evaluate the impact of this uncertainty on the final results.

3. DYNAMIC SLOPE STABILITY

Dynamic slope stability is a concern for natural and man-made slopes. The work by Bray and Travasarou (2007) provides an improved simplified slope displacement model that captures the uncertainty associated with the seismic loading and the dynamic slope resistance. The bulk of the uncertainty in dynamic slope stability is due to the inherent variability of the input ground motion. Bray and Travasarou (2007) statistically account for this variability by evaluating almost 700 ground motions and corresponding displacements using a nonlinear fully coupled stick-slip sliding block model. The resulting slope displacement model can be used in a fully probabilistic manner for predicting the distribution of anticipated slope displacements in a hazard analysis.

A novel aspects of this research is the use of mixed random variables (discrete and continuous) to separate slopes that exhibit small displacements that would be of no engineering concern from slopes that exhibit displacements that warrant engineering attention. Figure 2 shows the distribution of the mixed random variable used to model this dual-mode displacement response. A mixed random variable is useful for removing the bias of slopes that indicate negligible engineering displacements.
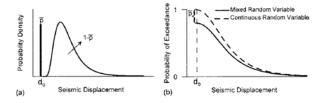
Figure 2 Mixed random variable used for seismic slope displacement analysis; (a) probability density function and (b) probability of exceedance for mixed and continuous random variable (from Bray and Travasarou, 2007).

4. ENGINEERING SEISMOLOGY

Geotechnical earthquake engineering projects rely on engineering seismology models to define the loading for design. Many sources of uncertainty contribute to the overall uncertainty for a particular measure of seismic loading. Probabilistic seismic hazard analysis (PSHA) provides a means of combining the most readily quantifiable sources of uncertainty into a comprehensive probabilistic measure of seismic loading at a particular site. The dispersion of PSHA results however can be ill defined and there is much debate as to the scale and magnitude of the true dispersion. This is particularly important for long return period projects such as nuclear power plants where the project life is often in the 10,000 year range and uncertainty drives the upper bound values (Bommer et al. 2004). Several recent studies have delved into the source and impact of uncertainty in seismic loading and are discussed here.

Moss (2009) and Moss and Der Kiureghian (2006) evaluate the influence of parameter uncertainty on the variance of ground motion prediction equations. The focus of this work is primarily evaluating the influence of measurement uncertainty of V_{S30} (thirty meter shear wave velocity) on the resulting overall uncertainty of a ground motion prediction equation (defined by the standard deviation in natural log units). The measurement uncertainty in V_{S30} is quantified using existing blind and comparative studies in Moss (2008). This uncertainty is then propagated through the ground motion prediction equations using Bayesian regression as well as the approximate methods of Monte Carlo simulation and FOSM (first order second moment). Figure 3 shows a 10% reduction that can be achieved by evaluating the influence of V_{S30} measurement uncertainty on the overall uncertainty in a ground motion prediction equation.

Wang and Takada (2007) present an excellent paper using Bayesian updating to, in most cases, reduce the uncertainty of a site specific ground motion prediction by using new data near the site. This paper presents the mathematical methodology, closed-form solution using conjugate priors, and data-based example showing the utility. By updating a regional ground motion prediction equation using recent site specific recordings the authors achieved an appreciable reduction in the standard deviation thereby affording a better defined median ground motion for design purposes.

Atkinson (2006) presented an interesting study delving into the impact of site effects and travel path on the overall uncertainty of ground motion prediction equations. The argument made is that regression of a large database of ground motions from diverse regions that are questionably grouped together results in an artificially large dispersion. To control for site and travel path effects Atkinson looked at the dispersion of a single site that experienced multiple earthquakes. The results, based on the limited data set for this site, indicate that site effects alone contribute 10% of the uncertainty as measured by the standard deviation, and that travel path and site effects together can contribute 40% to the uncertainty. Controlling for site and travel path effects is not necessarily feasible in a predictive analysis but this study raises questions about a weak correlation between V_{S30} and the complexity of site effects, and binning strong motion data from diverse regions to define the dispersion. A large database of diverse motions will provide a stable median value and well defined trends, but dispersion may be more accurately estimated on a site specific basis if there exist enough recordings at the site, or on a region specific basis where the data is carefully binned to reflect the travel path and site effects consistent with the site of interest.
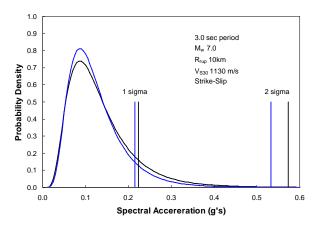

Figure 3 The influence of V_{S30} measurement uncertainty on ground motion prediction equation is most pronounced at the longer periods. Here the Chiou and Youngs (2008) ground motion prediction equation is used as the basis to demonstrate a 10% reduction in one standard deviation for the 3.0 second period spectral ordinate when V_{S30} uncertainty is properly accounted for within the regression procedure (from Moss 2009).

5. LEVEE/LIFELINE ENGINEERING

Engineering of lifelines presents interesting problems that are unique to spatially distributed man-made structures. With respect to geotechnical engineering there has been progress following the large consequences of the levee failures in New Orleans from Hurricane Katrina. Levee risk has been an ongoing concern in the Netherlands since catastrophic failures there in the 1950's. Probabilistic

methods are the obvious choice for assessing the failure potential of levees and levee systems however implementing uncertainty propagation for levee analysis and design has proven difficult. Static stability methods pertain to geotechnical earthquake engineering because static analysis is the starting point for dynamic analysis of levee systems in seismic parts of the world like Japan, China, and the western US.

The US Army Corp of Engineers commissioned as study for revising the methodology of levee analysis using risk and reliability concepts. Wolff (1994) developed approximate methods for levee analysis that are compatible with existing Corp deterministic methods, however these methods have not been adopted to date. Corp guidelines for seismic dam analysis (HQUSACE 1997) are more advanced than that for seismic levee analysis and provide a template for engineering issues that relate to both types of structures.

The current forefront of probabilistic levee analysis is driven by the levee systems in the California Bay Delta. In the Delta there are over 1000 km of waterways hemmed by poorly engineered and poorly maintenanced levees that protect land largely below sea level. To compound the problem there is a high likelihood of seismic activity in the vicinity that can load these levees dynamically. At risk are major transportation corridors, power transmission lines, high dollar agricultural fields, residential housing areas, metropolitan areas, and a water transmission system that delivers fresh water to over 20 million users in Central and Southern California. The author's research is focused on improving methods for quantifying risk and propagating uncertainty for the Delta levee system (Moss and Eller 2007). In this ongoing research, that is as yet unpublished, a number of novel concepts have been implemented in the realm of levee risk analysis. Some of these are listed below.

Spatial variability of soil properties is a reality for long linear engineered structures such as levees. To properly account for the influence of spatial variability on stability analysis a general relative variogram, GRV (Issaks and Srivastava 1989), is calculated for each levee reach using evenly spaced *in situ* data. The GRV determines the length of a levee reach by defining the distance needed to achieve a minimum statistical correlation, thereby ensuring the maximum statistical independence between levee reaches. The GRV is also compatible with, and includes in this study, point estimates of measurement uncertainty represented by the squared coefficient of variation (Figure 4).

The GRV of the foundation soils for a reach are constrained by the geomorphology and depositional environment of the soil, and the GRV of the levees are constrained by the composition material, construction methods, and level of maintenance. Spatial variability in other studies, if is accounted for at all, is treated as a fixed pseudo-probabilistic value with an ambiguous mathematical basis. It has been found that probability of failure calculations are highly sensitive to the reach length and a robustly defined reach length will provide a quantitative basis for eliminating this sensitivity.
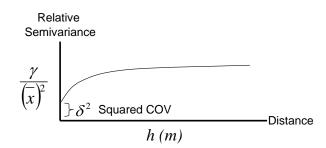

Figure 4 Conceptual diagram of exponential curve of a general relative variogram, GRV. The x-axis is the lag or distance. The y-axis is the semivariance divided by the squared mean of the spatial sample. The nugget or intercept value is the squared coefficient of variation, a point estimate of measurement uncertainty. The reach length where statistical correlation is minimized is at the sill or exponential plateau. Semivariance is calculated from SPT, CPT, and V_S measurements.

In following the lead of the Dutch (van Manen and Brinkhuis 2005) the calculation of risk, usually the product of the probability of failure and the consequences of failure in that order, is performed in reverse order setting the consequences first and then working through the failures modes that could result in the consequences. This is a subtle change but provides a consequences driven analysis that identifies all the possible failure modes that could result in a particular consequence. This results in a much more efficient means of computing the risk and focuses effort on the highest consequence scenarios.

The failure of a levee, regardless of the particular failure mode, is controlled by the weakest levee section along the length of the levee and the lowest resistance values within that levee section. In system reliability terms this is a series system with failure occurring within the weakest component. In observing past levee failures (e.g. 17th Street Canal failure as discussed in Seed et al., 2008) it has been found that the weakest levee section is often controlled by extreme low values in resistance, yet static and dynamic levee stability analyses are often performed using mean or median values of resistance. It is more appropriate to apply extreme value statistics to the resistance to better define the most probable location of the failure and the dispersion of the low values. A Type III smallest or Weibull distribution is used in this research to better define the lowest values in a particular weak levee section. The continuity or spatial extent of these extreme low values will be mapped by extrapolating the median trends of the levee or foundation GRV to the extreme values. As in the case of the variograms, the *in situ* data is used to define the statistics of the extreme values both in their point distributions and spatial continuity.

Time is a factor not just for the load variables (i.e. seasonal water loads or stochastic seismic loads) but also for resistance variables. Soil strength changes occur due to various geotechnical processes, and overall levee degradation occurs due to biological or maintenance factors.

The distributions of the resistance variables will be subjected to time alterations based on degradation/aggradation models of the altering effects. The probability of failure is calculated per levee reach throughout the Delta at a fixed point in time, then recalculated for each time increment progressively to capture the time rate effects of the altered resistance distributions.

This research on the risk analysis of the Bay Delta uses FORM (first order reliability method) and MC (Monte Carlo) simulations in a structural reliability formulation to calculate the component failure probabilities for each levee reach. The reverse risk modeling is performed using an event tree format with the consequences driving the analysis. A very important component of this work is the final calibration using existing failure case history data that provides a rough estimate of the time rate of failure and the general spatial distribution of failures. This bounds and provides a check on the reasonableness of the results. Calibration is performed ostensibly on the static (non-seismic) failure modes. Being that the static and dynamic resistance values are integrally linked, this then provides some means of bounding the dynamic failure modes.

6. CONCLUSIONS

A review of some recent research pertaining to liquefaction engineering, dynamic slope stability, engineering seismology, and levee/lifeline engineering has been presented in this paper. The thread that ties all this research together is the common goal of propagating uncertainty through the respective problems to determine how this uncertainty influences the results. Probabilistic methods for propagating uncertainty are coming of age and geotechnical earthquake engineering is the ideal field for these methods due of the large amount of uncertainty that exists in both the loading and resistance aspects of these problems. This review has been biased towards the author's work but effort was made to include important progress made by other researchers with similar interests. By quantifying and propagating uncertainty, as demonstrated in these types of earthquake engineering problems, more reliable engineering analysis and design can be accomplished.

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