PROOF OF CONCEPT FOR PROPELLER PERFORMANCE TESTING IN THE CAL POLY WIND TUNNEL

A Senior Project

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Bachelor of Science in Aerospace Engineering

by

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The goal of this experiment was to execute a proof of concept for a potential propulsion systems laboratory. During the course of this experiment, a data acquisition system was designed, built, and tested in order to collect data from the Cal Poly 3x4 ft draw-down wind tunnel. By operating a brushless outrunner motor and propeller on the sting mount, data was collected for thrust, torque, RPM, current, voltage, and tunnel velocity. By obtaining data for each of these performance metrics, trends were analyzed to confirm the concept and apparatus was functional. The results of the experiment revealed that the concept was successful on a level that could be applied to a teaching laboratory with the implementation of a few key improvements.

**Nomenclature**

\[
\begin{align*}
A & = \text{Propeller disk area (} ft^2 \text{)} \\
C & = \text{Wind tunnel cross-section area (} ft^2 \text{)} \\
C_p & = \text{Power coefficient} \\
C_Q & = \text{Torque coefficient} \\
C_T & = \text{Thrust coefficient} \\
D & = \text{Propeller diameter (} ft \text{)} \\
I & = \text{Current (} A \text{)} \\
J & = \text{Advance ratio} \\
n & = \text{Rotational speed (} rev/s \text{)} \\
P & = \text{Power (} ft \cdot lb/s \text{)} \\
P_s & = \text{Pitot tube static pressure (psfg)} \\
P_T & = \text{Pitot tube total pressure (psfg)} \\
Q & = \text{Torque (} ft \cdot lb \text{)} \\
T & = \text{Thrust (lb)} \\
V_\infty & = \text{Freestream velocity (} ft/s \text{)} \\
V_\infty' & = \text{Corrected freestream velocity (} ft/s \text{)} \\
V & = \text{Volts (} V \text{)} \\
\eta & = \text{Efficiency} \\
\rho & = \text{Density (} slugs/ft^3 \text{)}
\end{align*}
\]
I. Introduction

Propeller performance is not a new field of study, rather one that has been studied extensively due its importance to aircraft performance. Testing and experimentation have been occurring since 400 BC, when Archytas, disciple of Pythagoras, used in inclined plane attached to a cylinder as a propulsive device. However, it wasn’t until 1480 that Leonardo da Vinci sketched a helicopter which utilized a screw and rotating spit for propulsion. By 1798, Robert Fulton was experimenting with four bladed propellers for marine purposes. As 1878 rolled around, William Froude developed the blade element theory which lead up to the revolutionary designs of the Wright brothers.\(^1\) One of the lesser known reasons for the success of the *Wright Flyer* was the fact that Wilbur Wright placed such a large emphasis on wind tunnel testing of propellers.\(^1\) He understood that a propeller is nothing more than a twisted wing oriented in a direction such that the aerodynamic force produced by the propeller was predominately directed along the axis of rotation. The Wrights achieved a propeller efficiency of nearly 70%, an 18% improvement over other current designs. Though we can achieve efficiencies of up to 90% with modern variable-pitch propellers, this was an achievement ahead of its time.\(^1\)

Aircraft performance and propulsion are deeply interwoven in the aircraft design process, as both depend on one another, which is why it is important to have a conceptual understanding of performance of propellers and how their properties affect overall propulsion performance trends. Turboprop engines are very popular choices for propulsion of light to medium sized regional aircraft being designed and produced to this day. Having knowledge of both turboprop performance as well as propeller performance could prove to be a huge benefit to aerospace engineering students as they progress into current markets upon graduation. While the scope of this project will be limited to propellers, and will not deal with turboprop performance, the knowledge gained is still critical to overall comprehension.

The University of Illinois at Urbana-Champaign (UIUC) has conducted wind tunnel testing on over 140 different propellers, with their results well documented.\(^2\) Measurements of thrust and torque coefficients over a range of advance ratios and RPMs were collected and digitized for public reference. This resource provides a set of data for students to refer to after conducting this experiment. While the methods of calibration and correction are different than what will be utilized in this lab, trends and patterns will be available for verification of test data obtained from the Cal Poly wind tunnel.

It is the goal of this experiment to conduct a proof of concept of a potential propulsion system laboratory. The goal is to study the effects of advance ratio and propeller pitch on different propeller metrics such as thrust, torque, and power coefficients as well as propulsive efficiency. By building a data matrix for each propeller with a multitude of combinations of fixed RPM and wind tunnel velocity, it should be possible to model a few important trends. These trends should include thrust, torque, and power coefficients vs. advance ratio, as well as propulsive and overall efficiency vs. advance ratio.
II. Analysis

In order to predict the performance of a given propeller, wind tunnel data was collected of thrust, velocity, torque, rpm, current, voltage, and density. The thrust, torque and power of a propeller are typically expressed in coefficient form as shown Eq. (1), (2), and (3),

\[ C_T = \frac{T}{\rho n^2 D^4} \]  
\[ C_Q = \frac{Q}{\rho n^2 D^5} \]  
\[ C_P = \frac{P}{\rho n^3 D^5} \]

where \( T \) is thrust, \( Q \) is torque, \( P \) is power, \( \rho \) is density, \( n \) is the rotational speed, and \( D \) is diameter of the propeller. Eq. (2) and Eq. (3) can be combined to get Eq. (4).

\[ P = 2\pi n Q \]

Eq. (1), (2), and (3) are all functions of advance ratio, \( J \), in Eq. (5),

\[ J = \frac{V_\infty}{nD} \]

where \( V_\infty \) is the wind tunnel’s velocity, measured via a pitot-static tube upstream of the propeller. Using total pressure, \( P_T \), static pressure, \( P_S \), and density, the velocity can be calculated from Eq. (6).

\[ V_\infty = \sqrt{\frac{2(P_T - P_S)}{\rho}} \]

However, propellers in a wind tunnel produce thrust greater than the same propeller in an unrestricted flow.\(^5\) In other words, thrust developed in a wind tunnel would be equal to the thrust expected at a lower velocity in free air. Therefore we add a correction to the free stream velocity\(^5\) in Eq. (7)

\[ \frac{V_\infty'}{V_\infty} = 1 - \frac{\tau \alpha}{2 * \sqrt{1 + 2 * \tau}} \]

where \( \tau \) and \( \alpha \) are

\[ \tau = \frac{T}{\rho A V_\infty^2} \]

\[ \alpha = \frac{A}{C} \]

In Eq. (8) and (9), \( A \) is the propeller disk area and \( C \) is wind tunnel cross-sectional area. Note that \( \alpha \) should not be greater than 0.15.\(^5\) In general, efficiency is defined as the power output of the propeller divided by the power input\(^4\), giving Eq. (10).
\[ \eta = \frac{P_{\text{out}}}{P_{\text{in}}} \]  

(10)

However, power input to the propeller is hard to measure directly, so we use torque derived from Eq. (4) to find the power in and propulsive power as thrust times velocity, resulting in Eq. (11).

\[ \eta_{\text{propeller}} = \frac{TV_{\infty}'}{2\pi n Q} \]  

(11)

Relating Eq. (11) to coefficients and advance ratio results in Eq. (12).

\[ \eta_{\text{propeller}} = \frac{J' C_T}{2\pi C_Q} \]  

(12)

Overall efficiency of the system is power out of the propeller over the electrical power in as shown in Eq. (13),

\[ \eta_{\text{overall}} = \frac{TV_{\infty}}{IV \times 0.738} \]  

(13)

where I is current and V is volts.

III. Apparatus/Hardware

Mount

The goal of the mount system was to maintain an axial orientation while minimizing the drag of the components behind the propeller. To accomplish this, a simple mount system was designed to mate to the sting balance with minimal modification to existing hardware. An adapter was created that mates the electric motor to an existing sting mounting device that was easily modified to mate properly. An exploded view of the system is seen in Fig. 1.

Visible in are the motor, Electronic Speed Control (ESC), ESC plate, tube clamps, adapter, sting tube mount, and key. The key is designed to be silver soldered to the sting tube mount in order to restrict rotational motion of the adapter and motor. The adapter, seen in detail in Fig. 2, is mounted to the rear of the motor with four 3.0mm socket head machine screws. In conjunction with the key, this ensures that the adapter remains in place.
with the adapter, the key also works as a depth limiter to aid in horizontal alignment of the set screw holes with their receiver in the sting tube mount.

Figure 3 shows the modifications to be made to the sting tube mount. A slot is cut on top of the mount which receives the key. Another slot is cut perpendicular to this which allowed the key to be soldered to the sting tube mount on either side as well as at the rear of the longitudinal slot. These welds were then filed down where necessary to allow the adapter to slide over without interference.
Photos of the interface between the adapter, sting tube and motor can be seen in Fig. 4-7.

In order to assemble the mount system, the adapter must be bolted to the motor before attaching to the sting tube mount. This is done to give the user the ability to access the bolt holes within the adapter. The sting tube mount is then bolted to the sting, and the assembled adapter/motor is then positioned on the sting tube mount and locked in to place with two 4mm set screws. Finally, the ESC is attached to the sting tube mount with a tube clamp similar to that seen in Fig. 1.

Data Acquisition

The data acquisition system that was used for this lab is based on the open source Arduino hardware to control and record RPM as well as record current and voltage into the speed controller. LabView was used to monitor and record sting balance measurements and scanivalve hardware was used to monitor and record pitot tube pressure data to determine wind tunnel velocity. The Arduino was used to control the RPM of the motor through a PID control loop with feedback from an Eagle Tree brushless RPM sensor. This sensor outputs a square wave that follows the commutation frequency of the 3-phase output of the speed controller. This frequency was measured using the interrupt and timing functions built into the Arduino hardware and software and then used to calculate the RPM based
on the number of poles on the motor. The calculated RPM was then used to determine the error in a PID controller implemented on the Arduino, which was used to adjust the throttle percentage in order to hold a target RPM specified over the serial connection. By using the writeMicroseconds() function in the Arduino the PID loop was able to set the throttle percentage with a resolution of 0.1% of the full range of the throttle. The RPM measurements had a significant amount of noise, so a 20 point moving average was used to filter the data before it was used in the PID loop. Without this filter the PID controller was not able to reliably set the RPM of the motor, and large disturbances in throttle were seen. The Arduino also measures and outputs the voltage and current going into the speed controller using an Attopilot voltage and current sensor. This reading is taken by a Texas Instruments 16-bit ADS1115 Analog to Digital Converter (ADC), which is interfaced with the Arduino over an I2C connection. The Arduino and ADC setup can be seen in Fig. 8.

In order to set the motor throttle, the Arduino sent a Pulse Width Modulated (PWM) signal to the ESC, seen in Fig. 9, with a varying pulse width as specified in the writeMicroseconds() command that was the result of the PID control loop. In order to get the full range of throttle inputs and maintain a 0.1% throttle resolution, after each time the speed controller was powered on it would be armed by setting the Throttle to 0% (1000 microseconds) and then to full throttle (2000 microseconds) for 4 seconds to calibrate the throttle range. The throttle was then returned to 0% and the controller was ready to input a target RPM.

The thrust and torque of the prop were measured using the sting balance in the wind tunnel. Each channel of the sting balance is measured by a Wheatstone bridge configuration of strain gauges as seen in Fig. 10. An excitation voltage of 5 volts was applied to each of the six channels and then the differential output was amplified before...
being input into a National Instruments DAQ system. The output of the DAQ was then viewed and recorded using LabView.

IV. Procedure

This section is written with the presumption that the other issues involving the sting (described later) become resolved. It is written as the test should be run with no special directions to counteract any would-be flaws in the data acquisition system.

1. Calibration
   1.1. Secure calibration bars on sting
   1.2. Open LabView program "propeller_lab_v1.vi" and start the program. Set the file name and path to the desired location. The file name should correspond to the weight used in the calibration test.
   1.3. Ensure that nothing is connected to the calibration bars and take data for a zero force intercept point. One minute of data is sufficient to average out noise. Stop the recording and ensure that the data file is there.
   1.4. Stop the vi (red stop sign) and start it again to name a new file to save data to.
   1.5. Using relevant file names, calibrate axial force using the standard weights and bucket placed on the pulley wheel.
   1.6. Calibrate roll moment with the weight bucket hung from the cross bar, taking care to note the moment arm from the centerline of the sting arm.
   1.7. Load the axial and roll data in to MATLAB for each weight or moment applied, and average the voltages for the corresponding outputs.
   1.8. Fit a line to the data using the known weights and moments. The final calibration curves should accept an input of voltage and output the corresponding force or moment.
   1.9. Save the slope and intercept of the calibration curves in matrices to be used with the "polyval" command later. Save these matrices to a .mat file for easy access.

2. Test Procedure
   2.1. Mount steel sting tube to sting.
   2.2. Mount the E-Flite Power 15 motor to the aluminum motor adapter.
   2.3. Secure the propeller to the motor shaft.
   2.4. Slide the aluminum motor adapter over the sting tube until it stops. Tighten the set screws to secure the adapter on the sting tube.
   2.5. Turn on main power supply, power up the Arduino and open the serial connection on your computer. Ensure that the PID control sketch is uploaded by verifying that the serial is outputting voltage and current to the screen. These numbers should be close to zero at this point.
   2.6. Clear the area inside the wind tunnel and close any access ports that may be open. Make sure that there are no tools or debris loose in the tunnel.
   2.7. Plug in the speed control power cable to the power output on the main power supply. The voltage on the serial monitor should read over 12 volts momentarily, and the motor will begin calibrating. The serial monitor will inform you when it is ready to begin accepting RPM set points.
   2.8. Prepare the LabView and scanivalve programs to record data, naming files appropriately for the conditions to be measured.
2.9. Turn on the wind tunnel using the directions posted and set the frequency to 10.2 Hz for a speed of approximately 25 ft/s. Actual speed will vary.

2.10. Set the motor to 2500 RPM and wait for the measured RPM to reach steady state. This lower RPM will increase with increasing tunnel speed.

2.11. Begin recording data on the scanivalve and LabView. Data collection should be between 20 seconds and 1 minute for each RPM value.

2.12. Increase the RPM in increments of 500 and repeat data collection, making sure to separate files to avoid recording transients. The electronic speed controller will reach its maximum throttle setting around 9500-11000 RPM. This is the maximum RPM that will be recorded.

2.13. After the last RPM, make sure to bring the motor down to 0 RPM and allow the tunnel to run for 2-3 minutes to let the motor cool.

2.14. Repeat steps 2.8 through 2.13 for each tunnel speed. Frequencies of 10.2 Hz, 12.3 Hz, 16.5 Hz, 20.8 Hz, 25 Hz, 29.2 Hz, 33.5 Hz, 37.7 Hz, and 41.9 Hz should give approximate speeds of 25, 30, 40, 50,…,100 ft/s respectively.

2.15. After the final test, copy and paste the data from the Arduino serial monitor to a text file in Notepad. Save this data to be imported in MATLAB. Be sure to eliminate the calibration output from the beginning of the file. This is up to and including the line, “Ready!”

2.16. Disconnect the power from the test motor and turn off the wind tunnel motor.

V. Results and Discussion

In order to validate our test we compared results to the UIUC propeller database. This database has $C_T$, $C_P$, and efficiency over advance ratios for nearly 140 propellers. Figures 11-16 display the UIUC data for the 11x5.5 and the 11x7 propellers that were tested.

![Fig. 11 - UIUC Data: $C_T$ vs. $J$, 11x5.5 Propeller](image1)

![Fig. 12 - UIUC Data: $C_T$ vs. $J$, 11x7 Propeller](image2)
After analyzing data, many issues were discovered with the magnitude of values related to thrust and torque. The results of the experiment are plotted in Fig. 17-26 on the following pages.
Fig. 17 - Thrust Coefficient, APC 11x5.5 Prop

Fig. 18 - Thrust Coefficient, APC 11x7 Prop
Fig. 19 - Torque Coefficient, APC 11x5.5 Prop

Fig. 20 - Torque Coefficient, APC 11x7 Prop
Fig. 21 - Power Coefficient, APC 11x5.5 Prop

Fig. 22 - Power Coefficient, APC 11x7 Prop
Fig. 23 - Propeller Efficiency, APC 11x5.5 Prop

Fig. 24 - Propeller Efficiency, APC 11x7 Prop
Fig. 25 - Overall Efficiency, APC 11x5.5 Prop

Fig. 26 - Overall Efficiency, APC 11x7 Prop
When compared to the UIUC database, our values for thrust and torque, and thus $C_T$ and $C_P$, are much larger. This caused both the propeller and overall efficiencies to be larger than one, and corrected velocity to be less than zero in some cases. Because of this, the plots above are not using corrected velocity. Some possible explanations for these discrepancies will be explored in the next section.

**ISSUES**

One of the biggest issues which was encountered during testing involved the consistency of data being acquired. Partially through the experiment it was observed that efficiency values being calculated were above one, which was a red flag that an issue had emerged. After checking for any unit conversion errors, it was determined that the calibration curve which was previously generated was no longer accurate. Further investigation revealed that every test that was run experienced the same sort of inconsistency and variation of the calibration intercept point over time. Not only this, but anytime the sting balance was touched at all, the zero point would shift. In order to see how much of an effect this phenomenon was having on our data, we conducted a test over 16 hours outputting static axial force voltages, converted to force using the original calibration curve, which can be found in the appendix. The plot of this data is shown in Fig. 27.

![Figure 27 - Float of static axial force over time](image_url)

**Fig. 27 - Float of static axial force over time**

One of the possible solutions to this problem is to account for the floating zero point by running a static calibration prior to each individual test run in the wind tunnel, but this zero point has clearly been shown to fluctuate over time. There is also evidence that large fluctuations occur over a short period of time on the order of a single test duration, as seen in Fig. 28.
With this in mind, and the fact that over the 16 hour test axial force varied by over 25 ounces, there is potential that all data which was collected has been affected by this issue. This leads to concerns that experiments performed prior to this could have also been affected by tainted data. An attempt was made to determine the source of this variation in static axial readings. The sting outputs were connected directly to the Arduino and static axial voltage outputs were monitored without any amplification for just over 39 minutes. It was found that the output was relatively constant; after filtering the data with a moving average and applying a first order curve fit, the best fit slope of the data was minimal. This indicated two possible sources of the error, both of which could account for the variation seen previously. The first possibility was that the amplifier which was already in place was malfunctioning and distorting the data as it came from the sting balance. Upon inspecting the existing amplifier, a significant lack of shielding was found along with a lack of a consistent ground to the main power source. This could allow charges to build up and dissipate over time in the circuitry, causing the time based fluctuations seen in Fig 27. The other reason which could explain why the data collected directly with the Arduino looked constant was because the voltages were so small that there was a lack of resolution capability to see any distortion in the data. It would be possible to test if this was the case by obtaining a different amplifier and properly shielding the circuitry downstream of the sting to see how the amplified signals behave when bypassing the existing amplifier and its hardware.

Another potential reason for the error found in test data could be the result of excess noise in the system. Though it is true that any error from noise was minimal when compared to the error from the floating zero phenomenon, it is worth taking into account. One source of this unwanted noise could be attributed to Electromagnetic Interference (EMI) from different components of the test system. EMI comes from any apparatus which has an electric current flux, such as a component operating with AC current. This means that the large power supply which the wind tunnel fan uses likely generates quite a bit of interference. The rest of the wiring and the componentry used in the acquisition of sting data could have all contributed some amount of EMI in the form of noise, which showed up in the analysis of collected data.
In addition to floating zero point uncertainty and EMI, vibrations present in the system could have contributed to error seen in our analysis as well. During operation, the propeller and electric motor combination could have potentially contributed to the excess noise seen in the system through vibrations induced by the spinning propeller. As RPM increased it would be expected that noise in the system due to vibration increased as well. Vibrations resulting from aeroelastic effects could have caused some vibration in addition to the propeller and must be accounted for. The test apparatus was constructed with a plate on the side, placed in the propwash. By being downstream of the propeller, it was possible that vibrations were induced on this plate causing noise to be added to the system.

A final source of error which could have played a role in skewing test data is the fact that the existing amplifier does not have an ON/OFF switch but rather plugs into an electrical outlet, and thus stays on most of the time. Overuse by constantly being in a state of operation could have caused degradation of the components to the point that the amplifier no longer outputs useable data. If this was the case, the fix would simply be to replace the amplifier, preferably with a model capable of shutting off with the push of a button.

In order to test the hypothesis that the float of the sting outputs could be the cause of the discrepancies between the UIUC data and the data collected in this experiment, a brief sensitivities test was performed. The effect of an offset corresponding to the magnitude of the variation of the axial force intercept seen in Fig. 27 was examined. After subtracting this offset from the data for the 11x5.5 propeller, we obtained the propeller efficiency curves shown in Fig. 29. The efficiencies are now below one, and half of the RPM curves are starting to show reasonable trends. The additional error and false trends could be due to offsets in torque or varying offset during the test. This gives evidence that the calculations are correct.

![Fig. 29 - Sensitivity analysis of float variation on propeller efficiency](image-url)
VI. Conclusion

It is unfortunate that the data did not turn out as well as desired, and the expected trends were not seen. After examining the possible reasons for the poor data obtained during this test, key changes that can be made were formulated and are suggested below. In addition, other potential uses are given.

Future Improvements

During the tests, it was observed that current increased with rpm. This led to an increase in voltage loss at the load as seen in Fig. 30. To improve the consistency of the test, voltage regulation at the load could be implemented. This function is built into the power supply and only requires two wires to be run from the power supply’s regulator to the load input. This would cause the voltage at the load to be constant with changing current.

In order to get a more stable sting calibration point it would be necessary to bypass the amplifier hardware that is currently connected to the sting. The Arduino setup that has been implemented has enough 16 bit ADC’s to accomplish this and is set up to provide the required 5 volt excitation voltage, although the output voltage of the sting is much smaller than the full scale voltage of the ADC input which has a minimum setting of 0.256 V. Because of this, the resolution when monitoring the sting is roughly 4 oz., which is not small enough for our purposes. By using instrumentation amplifiers to increase the signal voltage going into the ADC we could get the resolution needed to take useful data from the Arduino, although this may also amplify the noise. Ideally amplifiers would be used that have good common mode rejection ratios, which would be able to filter out noise due to EMI being transmitted through the wiring.

Using the Arduino we were able to take all 6 channels of sting data at a sample rate of nearly 20 Hz. This would be sufficient for our measurement needs, although for other tests higher data rates may be necessary. In order to increase the sample rate, different ADCs could be used; preferably these would have an SPI interface instead of I2C, as the Arduino is able to handle higher data rates over SPI. Another way to increase the data rate would be to use the Arduino Due as the base for the DAQ. The Due utilizes an 84 MHz 32 bit ARM processor which is much faster than the 16 MHz 8 bit processor in use on the Arduino Nano.
Potential Uses

The propeller test apparatus, when fully functional, would offer a wide range of alternate uses, from air breathing propulsion labs to supporting Masters theses. It would allow the newly formed Cal Poly UAS design class the ability to experimentally test their chosen propellers and motors. The UAS class could model battery and propulsion system responses to changing demands in a simulated flight condition. Ducted fan performance could be tested with a new mount. The propeller test apparatus could open doors for many future projects needed at Cal Poly if the current errors could be mended.

Acknowledgments

We would like to thank Professor Dan Wait for his support, encouragement and technical expertise. Without his patience and willingness to share knowledge, the journey would have been much more arduous. In the end, the emotional support that he provided helped keep our chins up as we ran in to many unforeseen problems.

Additionally, we would like to thank Cody Thompson for his mechanical and manufacturing expertise, without which the sting tube would have taken much longer to obtain. Dr. Rob McDonald has also earned our gratitude by lending us a power supply to power the motor as well as pointing us in the right direction for the analysis and wind tunnel velocity corrections. We would also like to thank Dr. Jin Tso for allowing us to make use of the Cal Poly 3'x4' Wind Tunnel as well as Brian Barker for assisting in setting up the pitot tube data acquisition system. Adam Chase and Brad Schab also helped review the initial concept for the test apparatus and motor/prop matching. Finally, we owe a great deal of thanks to JWD Machine, Inc. for donating 5 hours of CNC machine and machinist time to manufacture our aluminum motor adapter.

References


4McDonald, R 2012, Propeller Theory, lecture notes distributed in AERO 425 at California Polytechnic State University, San Luis Obispo in Fall 2012.

Appendix

Calibration Curve

The calibration curves for thrust and torque were calculated through a careful process of applying known forces and moments, respectively, and measuring the average voltage produced by the sensors. These voltages and forces were then plotted and a first order best fit line was applied to obtain the following calibration equations:

\[
Q = 674.0718(V_Q) + 498.2726 \quad (14)
\]

\[
T = 358.2344(V_T) + 1468.6 \quad (15)
\]

Note that both the thrust and torque voltages are measured negative on the order of -4 and -0.8 respectively. The positive thrust direction was considered opposite \( V_\infty \), while the positive torque direction followed the right hand rule in the same direction.

Error Analysis

Error propagation begins with the most basic elements of the calculations performed. These include thrust and torque voltage outputs from the sting, room pressure and temperature measurements, and gauge pressure values from the pitot tube (total and static pressure). The purpose of this error analysis is to determine the level of precision that our values have, and not their accuracy. Precision represents repeatability on the individual data points while accuracy represents the nearness of those data points to the true value. In order to calculate precision, we assume that there is no error on conversion of the thrust and torque voltages, so that

\[
\frac{\sigma_{\text{axial voltage}}}{\text{Axial Voltage}} = \frac{\sigma_T}{|T|} \quad (16)
\]

\[
\frac{\sigma_{\text{torque voltage}}}{\text{Torque Voltage}} = \frac{\sigma_Q}{|Q|} \quad (17)
\]

It is also assumed that error on propeller diameter is zero, because it is given as a specification from the manufacturer.

Error percentages were calculated for thrust, torque, current, voltage and RPM for all combinations of propeller pitch, RPM, and freestream tunnel velocity. The maximum percent error among these combinations was examined for each of these five values. Finally, the maximum of the five individual values was used to determine the case to propagate error on. These values were presented in Table 1.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Maximum Error</th>
<th>$V_\infty$ (ft/s)</th>
<th>Target RPM</th>
<th>Prop</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>0.38%</td>
<td>23.21</td>
<td>2500</td>
<td>11x7</td>
</tr>
<tr>
<td>$I$</td>
<td>3.09%</td>
<td>99.02</td>
<td>8500</td>
<td>11x7</td>
</tr>
<tr>
<td>$V$</td>
<td>0.11%</td>
<td>39.52</td>
<td>10000</td>
<td>11x5.5</td>
</tr>
<tr>
<td>$T$</td>
<td>2.89%</td>
<td>79.21</td>
<td>10000</td>
<td>11x5.5</td>
</tr>
<tr>
<td>$Q$</td>
<td>3.04%</td>
<td>24.06</td>
<td>6500</td>
<td>11x7</td>
</tr>
</tbody>
</table>

As visible in Table 1, the maximum error of these variables is on current, which occurs for the case with $V_\infty = 99.02 \text{ ft/s}, n = 8500 \text{ rpm}$ on the 11x7 propeller. From this point forth, all error calculations will be performed using error values and average values from this case.

For the case highlighted in Table 1, the temperature was measured at $T = 62^\circ F$ and pressure at $P = 29.76 \text{ inHg}$, leading to a density of $0.0023 \text{ slugs/ft}^3$. Additional values for this case are outlined in Table 2.

| Variable | Standard Deviation | Magnitude | Error ($\sigma_x/|x|$) |
|----------|-------------------|-----------|------------------------|
| $n$      | 12.0228 rpm       | 8499.6 rpm| 0.0014                 |
| $I$      | 0.0324 A          | 1.0471    | 0.0309                 |
| $V$      | 0.00042 V         | 14.6804 V | 0.000029               |
| $T_{voltage}$ | 0.0189 V     | 3.9311 V  | 0.0048                 |
| $Q_{voltage}$ | 0.0191 V     | 0.7613 V  | 0.0250                 |
| $P_T$    | 0.1743 psfg       | 0.8134 psfg| Not used               |
| $P_S$    | 0.1823 psfg       | 12.3171 psfg| Not used               |

These values will be used wherever applicable in the analysis that follows.
Density
Continuing calculation of the fundamental values, we find the error on density as follows:

\[ \rho = \frac{p}{RT} \]  
\[ (18) \]

\[ \frac{\sigma_{\rho}}{\rho} = \sqrt{\left(\frac{\Delta p}{p}\right)^2 + \left(\frac{\Delta T}{T}\right)^2} \]
\[ = \sqrt{\left(\frac{0.01}{29.76}\right)^2 + \left(\frac{1}{62 + 460.67}\right)^2} \]
\[ = 0.0019 = 0.19\% \]  
\[ (19) \]

Freestream Velocity
The error on freestream velocity is as follows:

\[ V_\infty = \sqrt{\frac{2(P_T - P_S)}{\rho}} \]  
\[ (20) \]

\[ \sigma_{V_\infty} = \sqrt{\left(\frac{\partial V_\infty}{\partial P_T}\right)^2 (\delta P_T)^2 + \left(\frac{\partial V_\infty}{\partial P_S}\right)^2 (\delta P_S)^2 + \left(\frac{\partial V_\infty}{\partial \rho}\right)^2 (\delta \rho)^2} \]  
\[ (21) \]

where

\[ \frac{\partial V_\infty}{\partial P_T} = \frac{1}{2} \left(\frac{2(P_T - P_S)}{\rho}\right)^{-\frac{1}{2}} \]
\[ = \frac{1}{V_\infty \rho} \]  
\[ (22) \]

And similarly,

\[ \frac{\partial V_\infty}{\partial P_S} = -\frac{1}{V_\infty \rho} \]  
\[ (23) \]

\[ \frac{\partial V_\infty}{\partial \rho} = \frac{P_T - P_S}{V_\infty \rho^2} \]  
\[ (24) \]

Thus by evaluating plugging in these partials we come to

\[ \sigma_{V_\infty} = \sqrt{(4.3045)^2(0.1743)^2 + (-4.3045)^2(0.1823)^2 + (-0.00021)^2(0.0019)^2} \]
\[ = 1.0901 \text{ ft/s} \]  
\[ (25) \]

And finally, the error on \( V_\infty \) is:

\[ \frac{\sigma_{V_\infty}}{V_\infty} = \frac{1.0901}{99.02} = 0.0110 = 1.10\% \]  
\[ (26) \]
Thrust Coefficient

\[ C_T = \frac{T}{\rho n^2 D^5} \]  
\[ \frac{\sigma_{C_T}}{C_T} = \sqrt{\left(\frac{\sigma_T}{T}\right)^2 + \left(\frac{\sigma_p}{p}\right)^2 + \left(\frac{2\sigma_n}{n}\right)^2 + \left(\frac{5\sigma_D}{D}\right)^2} \]  
\[ \frac{\sigma_{C_T}}{C_T} = 0.0048 = 0.48\% \]  

Torque Coefficient

\[ C_Q = \frac{Q}{\rho n^2 D^4} \]  
\[ \frac{\sigma_{C_Q}}{C_Q} = \sqrt{\left(\frac{\sigma_Q}{Q}\right)^2 + \left(\frac{\sigma_p}{p}\right)^2 + \left(\frac{2\sigma_n}{n}\right)^2 + \left(\frac{4\sigma_D}{D}\right)^2} \]  
\[ \frac{\sigma_{C_Q}}{C_Q} = 0.0250 = 2.5\% \]  

Power Coefficient

\[ C_P = 2\pi C_Q \]  
\[ \frac{\sigma_{C_P}}{C_P} = 2\pi \left(\frac{\sigma_{C_Q}}{C_Q}\right) = 0.1587 = 15.87\% \]  

Advance Ratio

\[ J = \frac{V_{\infty}}{nD} \]  
\[ \frac{\sigma_J}{J} = \sqrt{\left(\frac{\sigma_{V_{\infty}}}{V_{\infty}}\right)^2 + \left(\frac{\sigma_n}{n}\right)^2 + \left(\frac{\sigma_D}{D}\right)^2} \]  
\[ \frac{\sigma_J}{J} = 0.0111 = 1.11\% \]  

Propeller Efficiency
\[ \eta_{prop} = \frac{TV_{\infty}}{2\pi n Q} \]  

(38)

\[ \frac{\sigma_{\eta_{prop}}}{\eta_{prop}} = \frac{1}{2\pi} \sqrt{\left( \frac{\sigma_T}{T} \right)^2 + \left( \frac{\sigma_{V_{\infty}}}{V_{\infty}} \right)^2 + \left( \frac{\sigma_n}{n} \right)^2 + \left( \frac{\sigma_Q}{Q} \right)^2} \]  

(39)

\[ \frac{\sigma_{\eta_{prop}}}{\eta_{prop}} = 0.0044 = 0.44\% \]  

(40)

**Overall Efficiency**

\[ \eta_{overall} = \frac{TV_{\infty}}{IV \left( \frac{550}{745.7} \right)} \]  

(41)

\[ \frac{\sigma_{\eta_{overall}}}{\eta_{overall}} = \left( \frac{745.7}{550} \right) \sqrt{\left( \frac{\sigma_T}{T} \right)^2 + \left( \frac{\sigma_{V_{\infty}}}{V_{\infty}} \right)^2 + \left( \frac{\sigma_{I}}{I} \right)^2 + \left( \frac{\sigma_{V}}{V} \right)^2} \]  

(42)

\[ \frac{\sigma_{\eta_{overall}}}{\eta_{overall}} = 0.0450 = 4.50\% \]  

(43)