de Vries behavior of the electroclinic effect in the smectic-A* phase near a biaxiality-induced smectic-A*–smectic-C* tricritical point

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Using a generalized Landau theory involving orientational, layering, tilt, and biaxial order parameters we analyze the smectic-A*–smectic-C* (Sm-A*–Sm-C*) transitions, showing that a combination of small orientational order and large layering order leads to Sm-A*–Sm-C* transitions that are either continuous and close to tricriticality or first order. The model predicts that in such systems the increase in birefringence upon entry to the Sm-C* phase will be especially rapid. It also predicts that the change in layer spacing at the Sm-A*–Sm-C* transition will be proportional to the orientational order. These are two hallmarks of Sm-A*–Sm-C* transitions in de Vries materials. We analyze the electroclinic effect in the Sm-A* phase and show that as a result of the zero-field Sm-A*–Sm-C* transition being either continuous and close to tricriticality or first order (i.e., for systems with a combination of weak orientational order and strong layering order), the electroclinic response of the tilt will be unusually strong. Additionally, we investigate the associated electrically induced change in birefringence and layer spacing, demonstrating de Vries behavior for each, i.e., an unusually large increase in birefringence and an unusually small layer contraction. Both the induced changes in birefringence and layer spacing are shown to scale quadratically with the induced tilt angle.

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I. INTRODUCTION

A. Background and motivation

In the last decade there has been significant experimental and theoretical interest in the response of de Vries materials to externally applied electric fields [1]. In the absence of an applied field, de Vries materials exhibit a smectic-A–smectic-C (Sm-A–Sm-C) (or, if chiral, a Sm-A*–Sm-C*) transition with an unusually small change in layer spacing and a significant increase in birefringence upon entry to the Sm-C phase. The increase in birefringence is associated with an increase in orientational order. Some de Vries materials exhibit another unusual feature, namely, a birefringence that varies nonmonotonically with temperature [2,3]. Specifically, the birefringence decreases as the Sm-A*–Sm-C* transition is approached from either the low- or the high-temperature side. de Vries materials generally seem to have unusually small orientational order and follow the phase sequence isotropic Sm-A*–Sm-C*. In several de Vries materials, the Sm-A*–Sm-C* transition seems to occur close to a tricritical point [4–6].

For chiral liquid crystals in general, the application of an electric field to the Sm-A* phase induces a tilt of the average molecular direction, relative to the layer normal, and hence the optical axis. This phenomenon, known as the electroclinic effect, was first predicted using a symmetry-based argument [7] and was then observed experimentally [8]. The electroclinic effect led to the development of electro-optic devices using ferroelectric, i.e., chiral, liquid crystals. However, the quality of these devices has been limited by the formation of chevron defects, which result from a significant layer contraction associated with the electrically induced molecular tilt.

Ferroelectric de Vries materials have generated considerable excitement because in the Sm-A* phase they exhibit an unusual electroclinic effect: a very large reorientation of the optical axis with a very small associated layer contraction. Additionally, there is a very large increase in the birefringence. Aside from being scientifically interesting, such an electroclinic effect makes ferroelectric de Vries materials strong candidates for liquid crystal devices that have large electro-optical response without the associated problem of chevron defects.

There are some details of the electro-optical response in the Sm-A* phase of de Vries materials that merit further discussion. An important characterization of the electroclinic effect is the response curve \( \theta(E) \), where \( \theta \) is the tilt of the optical axis and \( E \) is the strength of the applied electric field. Different types of electroclinic response curves are shown schematically in Fig. 1 and it can be seen that they are generally nonlinear [9,10]. As shown in Fig. 1(a), for systems with a continuous Sm-A*–Sm-C* transition \( \theta(E) \) is also continuous. As is typical for the electroclinic effect, the curvature \( \frac{d^2\theta}{dE^2} \leq 0 \) so the susceptibility \( \chi = \frac{d\theta}{dE} \) is largest at \( E = 0 \). The zero-field susceptibility \( \chi_0 \) diverges as the temperature \( T \) is lowered toward the Sm-A*–Sm-C* transition temperature, \( T_{AC} \). For systems with a first-order Sm-A*–Sm-C* transition the situation is quite different. For temperatures above a critical temperature \( T_c \), the response is continuous but exhibits what has been termed “superlinear growth.” As shown in Fig. 1(b), this corresponds to positive curvature at small fields followed by negative curvature at large fields. It can also be seen that \( \chi \) is largest at the field where the curvature changes sign. As \( T \) is reduced toward \( T_c \) this value of \( \chi \) diverges. For \( T < T_c \) the response becomes discontinuous, as shown in Fig. 1(b), and there is now a jump in the \( \theta \) at \( E_j \).

The value of \( E_j \) decreases continuously to zero as \( T \) is lowered toward \( T_{AC} \). The value of \( \chi_0 \) remains finite as \( T \) is lowered toward \( T_{AC} \).
The response of the birefringence $\Delta n(E)$ in the Sm-$A^*$ phase is also nonlinear and is qualitatively similar to the response of the tilt, $\theta(E)$ [9,10]. For systems with a continuous Sm-$A^*$–Sm-$C^*$ transition, $\Delta n(E)$ is also continuous, while for systems with a first-order Sm-$A^*$–Sm-$C^*$ transition, $\Delta n(E)$ is continuous with superlinear growth for $T>T_c$ and is discontinuous for $T<T_c$, exhibiting a jump at $E_j$. Remarkably, when $\Delta n(E)$ is plotted parametrically against $\theta^2(E)$, the scaling is essentially linear, regardless of the nature (continuous or first order) of the transition [9,10]. Equally remarkable is the fact that for a given system, the slope of the linear scaling varies very little with temperature. This means that for any de Vries material the response of the birefringence is well fitted by $\Delta n(E)=\Delta n(0)+k(T)\theta^2(E)$, where $k(T)$ is a material-dependent parameter that has only a very weak temperature dependence. There is less published data on the response of the layer spacing due to the application of an electric field, other than to show that it decreases with increasing field and is unusually small [11].

To date, there have been two theoretical approaches to modeling the unusual electroclinic effect that is displayed by de Vries materials. The first [6,12–14] is to use a Langevin model (originally proposed by Fukuda in the context of thresholdless antiferroelectricity [15]) in conjunction with the assumption of a “hollow cone” distribution of the molecular directions. For the sake of brevity we refer to this simply as the hollow cone Langevin model. For a hollow cone distribution, the angle $\beta$ between the long axes of the molecules and the layer normal $\hat{N}$ has a preferred value $\theta_\beta$, In the absence of a field the distribution of azimuthal angles, i.e., the projections of the molecular axes onto the layering plane, is uniform so that the average molecular direction, the direction $\hat{n}$, is parallel to $\hat{N}$. One motivation for the use of such a distribution is that it would explain the absence of layer contraction at the Sm-$C^*$ transition because the already tilted molecules need only to align azimuthally in order to reorient $\hat{n}$ away from $\hat{N}$ by an angle $\theta_\beta$. However, it has been pointed out [16] that the hollow cone distribution would have a large negative value of $S_4$ [corresponding to the $P_3(\cos \beta)$ term in an expansion of the distribution in Legendre polynomials], whereas no Sm-$A^*$ materials have been found with negative values of $S_4$ (de Vries materials seem in general to have very small values of $S_4$). The hollow cone Langevin model yields predictions for the electrical response of the director (via the response of the tilt and azimuthal angles) and the birefringence, but not layer spacing, per se. Rather, it is assumed that the response of the layer spacing will be small due to the assumption of a hollow cone distribution. The hollow cone Langevin model cannot describe systems with response curves of the type shown in Fig. 1(b), i.e., systems with first-order transitions. This has motivated the use of a second type of model, namely, that initially presented by Bahr and Heppke in their analysis of a field-induced critical point near the Sm-$A^*$–Sm-$C^*$ transition [17]. While this model provides an accurate description of the response curves, it does not make any predictions regarding the electrical response of the birefringence or layer spacing. Additionally, it does not make any connection to the de Vries behavior of the zero-field Sm-$A^*$–Sm-$C^*$ transition.

B. Summary of results

In this article we present and analyze a model that is a chiral extension of the generalized Landau mean-field theory that was presented in Refs. [18,19]. This model is based on an expansion of the free-energy density in powers of orientational, layering, tilt, and biaxial order parameters. There are chiral couplings of these order parameters to an externally applied field, the effects of which include the electroclinic effect. Our analysis of this chiral model predicts all of the
main experimentally observed features of de Vries materials outlined above: the de Vries behavior (near the zero-field Sm-A′−Sm-C′ transition) of layer spacing d and birefringence Δn, as well as the nonmonotonicity of Δn; proximity of the transition to a tricritical point; the unusually strong electrical response of tilt θ(E) and birefringence Δn(E) in the Sm-A′ phase, along with unusually small layer contraction; the linear scaling of Δn(E) vs θ²(E), regardless of the nature of the zero-field Sm-A′−Sm-C′ transition. Furthermore, all of these features can be accounted for if the system possesses unusually small orientational order and strong layering order, a combination thought prevalent among de Vries materials. These results do not rely on any particular assumptions about the distribution of the molecular directions, other than that the distribution corresponds to small orientational order. Reference [16] presents further details on possible molecular distributions leading to small orientational order but non-negative S₄ value.

1. Zero-field Sm-A′−Sm-C′ transition

Figure 2 shows the Sm-A′−Sm-C′ phase boundary in |φ|²−M space, where |φ| and M are the magnitudes of the layering and orientational order parameters, respectively. They will be defined more rigorously in Sec. II A. It has been observed that the orientational order in de Vries systems has only a very weak temperature dependence. Along with the fact that the nematic phase does not occur for all de Vries materials, this implies [18,19] that the transition to the Sm-C′ phase is driven by an increase in the layering as the temperature decreases. Thus, in the phase diagram of Fig. 2(a), varying the temperature corresponds to a horizontal path. It is important to note that the negative slope of the Sm-A′−Sm-C′ phase boundary implies that the smaller the value of M, the larger the value of |φ| at which the Sm-A′−Sm-C′ transition occurs. This is consistent with the observation [1,16] that de Vries smectics generally have such unusually weak orientational order that their stabilization requires strong layering order, perhaps via microsegregation.

The zero-field model predicts that a Sm-A′−Sm-C′ tricritical point results due to a coupling between biaxiality and tilt. The effect of biaxiality is stronger in systems with small M and large |φ| so that a tricritical point and associated neighboring first-order transition can be accessed by systems with sufficiently small orientational order, M≤M_TC. Here M_TC is the value of the orientational order at which the system exhibits a tricritical Sm-A′−Sm-C′ transition. This is shown in the phase diagram of Fig. 2.

As usual, for systems with continuous Sm-A′−Sm-C′ transitions, the growth of θ upon entry to the Sm-C′ phase scales like θ∝|φ|⁰, where t=1−T/T_{AC} is the reduced temperature and T_{AC} is the Sm-A′−Sm-C′ transition temperature [20]. Away from the tricritical point the scaling is XY like, so β=0.5, and at the tricritical point β=0.25, implying a more rapid growth of θ at tricriticality, as shown in Fig. 2(b). In the Sm-C′ phase, for M>M_TC, there is a crossover in the scaling from XY like to tricritical, as shown in Fig. 2(b). As M is lowered toward M_TC this crossover shrinks to zero. For M>M_TC the transition Sm-A′ to Sm-C′ is first order and there is a discontinuous jump in θ at the transition, also shown in Fig. 2(b).

The behavior of the birefringence near the zero-field Sm-A′−Sm-C′ transition is essentially the same as that for the Sm-A−Sm-C transition. This behavior is best described in terms of the fractional change in birefringence $Δ_n = \frac{Δn_{SM}}{Δn_{SM} + Δn_{AC}}$, where $Δn_{AC}$ is the birefringence in the Sm-A′ phase right at the Sm-A′−Sm-C′ boundary. As discussed in Ref. [18] we find that upon entry to the Sm-C′ phase, for any of the three types of transitions (XY like, tricritical, and first order), $Δ_n$ of

![FIG. 2. (a) The phase diagram in |φ|²−M space near the tricritical point (|φ|_{TC},M_{TC}). The quantity $M$ is a measure of how much orientational order the system possesses and for de Vries materials is effectively athermal. The quantity |φ| is a measure of the strength of the layering. It is a monotonically decreasing function of temperature so that for a given material, decreasing the temperature corresponds to moving horizontally from left to right. The solid line represents the continuous Sm-A′−Sm-C′ boundary while the dashed line represents the first-order Sm-A′−Sm-C′ boundary. These two boundaries meet at the tricritical point (|φ|_{TC},M_{TC}). The dotted line indicates the region in which the behavior crosses over from XY-like to tricritical. The region in which the behavior is XY-like shrinks to zero as the tricritical point is approached. At the tricritical point the slopes of the first-order and continuous Sm-A′−Sm-C′ boundaries are equal but the curvatures are not. Also shown, as double ended arrows, are the three distinct classes of transitions: XY-like, tricritical, and first order. (b) The tilt angle θ as a function of reduced temperature $t=(1−1/T_{AC})$ near the Sm-A′−Sm-C′ transition temperature $T_{AC}$, i.e., for $|φ|≤1$. Upon entry to the Sm-C′ phase the growth of the tilt angle scales like $|φ|^{1/2}$ for a mean-field XY-like transition. For a tricritical transition it scales like $|φ|^{1/4}$ and is thus more rapid. For a first-order transition there is a jump in the tilt angle upon entry to the Sm-C′ phase.
a de Vries type material will grow according to $\Delta_n \propto \theta^2$. While the dependence of $\Delta_n$ on $\theta$ is the same for all three types of transitions, its dependence on temperature is not the same because, as shown in the Fig. 2(b), $\theta$ scales differently with temperature for each type of transition. For an XY-like transition the growth of $\Delta_n$ will be linear, $\propto |t|^{1/2}$, while for a transition at tricriticality it scales like $|t|^{1/2}$ and is thus more rapid. For a first-order transition there will be a jump in the tilt angle and, therefore, an associated jump in $\Delta_n$, although near tricriticality, where the transition is only weakly first order, the jump will be small. Thus, the rapid growth of birefringence observed in de Vries materials can be attributed to the proximity of the system’s Sm-$A^\prime$–Sm-$C^\prime$ transition to a tricritical point, which, as discussed above, can in turn be attributed to unusually small orientational order. Additionally, we predict the possibility of a weakly temperature-dependent birefringence that decreases as the zero-field Sm-$A^\prime$–Sm-$C^\prime$ transition is approached from the Sm-$A^\prime$ phase, which as discussed above, is an unusual feature that has been observed experimentally \[2,3\].

Similarly, the behavior of the layer spacing $d$ near the zero-field Sm-$A^\prime$–Sm-$C^\prime$ transition is essentially the same as that for the Sm-$A$–Sm-$C$ transition, and is best described in terms of the layer contraction $\Delta_d = (d_{AC} - d_c)/d_{AC}$, where $d_{AC}$ and $d_c$ are the layer spacing in the Sm-$A^\prime$ phase (right at the Sm-$A^\prime$–Sm-$C^\prime$ boundary) and in the Sm-$C^\prime$ phase, respectively. We find that for any of the three possible types of transitions, $\Delta_d \propto M_d \theta^2$. Thus, for unusually small orientational order $M_d$, the layer contraction is unusually small, and therefore de Vries like.

2. Electroclinic effect in the Sm-$A^\prime$ phase

With the application of an electric field of strength $E$, we show that our generalized Landau model predicts the following relationship between the induced tilt, $\theta$, and $E$:

$$E = \alpha_{d}(t,M,d) \theta + \beta_{M}(M,d) \theta^2 + \gamma_{M}(M,d) \theta^3.$$  \hspace{1cm} (1)

This relationship is completely analogous to that presented by Bahr-Heppke in the context of a field-induced critical point near the Sm-$A^\prime$–Sm-$C$ transition \[17\]. However, our derivation of Eq. (1) from the more basic level of a generalized Landau theory (in terms of layering and orientational order parameters) allows us to relate the coefficients $\alpha_{d}(t,M,d)$, $\beta_{M}(M,d)$, and $\gamma_{M}(M,d)$ to the orientational order, $M$, and the layer spacing, $d$, in the system. This allows us to do two important things. First, we can determine the nature of the response $\theta(E)$ (i.e., continuous with decreasing slope, superlinear or discontinuous) based on the degree of orientational order $M$ in the system. Second, using Eq. (1) along with the rest of the generalized free energy, we can determine the electrical response of the birefringence (which is proportional to the $M$) and the layer spacing $d$.

The nature of the response depends crucially on the sign of $\beta_{M}(M,d)$. We find that $\beta_{M}(M,d) \propto (M - M_{TC})$ for systems with a continuous Sm-$A^\prime$–Sm-$C^\prime$ transition, $\beta > 0$ and the response is continuous with susceptibility decreasing as $E$ is increased. The response at the continuous Sm-$A^\prime$–Sm-$C^\prime$ transition for small fields scales like $\theta \propto E^{1/8}$. Away from tricriticality ($M > M_{TC}$), $\delta = 3$ while at tricriticality ($M = M_{TC}$), $\delta = 5$ and the response is significantly stronger.

In describing the change in birefringence due to an applied electric field, it is useful to define the fractional change of the birefringence due to the applied electric field, $\Delta_i(E) = \frac{\Delta_i(E) - \Delta_{i0}}{\Delta_{i0}}$ versus the square of the induced tilt, $\theta^2(E)$. For any type of transition [and hence any type of response of $\theta(E)$] we find that the scaling of $\Delta_i(E)$ with $\theta^2(E)$ is linear. The model predicts the possibility of a weakly temperature-dependent slope $\eta(T)$. (b) A plot of the layer contraction due to applied electric field, $\Delta_d(E) = \frac{\Delta_d(E) - \Delta_{d0}}{\Delta_{d0}}$ versus the square of the induced tilt, $\theta^2(E)$. For any type of transition [and hence any type of response of $\theta(E)$] we find that the scaling of $\Delta_d(E)$ with $\theta^2(E)$ is linear. The slope of the scaling is proportional to $M_{E>0}$, the value of the zero-field orientational order, which for de Vries materials is unusually small. Two plots are shown, one for a system with small orientational order $M_{E>0} \ll 1$, for which the contraction will be small, and one for a system with strong orientational order $M_{E>0} = 1$, for which the contraction will be sizable.
fringence, which again is consistent with experiment.

Similarly, the layer spacing \( d(E) \) is affected by the field, and the layer contraction \( \Delta_A(E) = \frac{d(E) - d(0)}{d(0)} \) also scales linearly with \( \theta(E) \) regardless of the nature of the transition (and hence the response), i.e.,

\[
\Delta_A(E) \approx M_{E=0} \theta^2(E),
\]

where \( M_{E=0} \) is the value of the zero-field orientational order, which for de Vries materials is unusually small. Thus, for de Vries materials the contraction of the layers associated with the electroclinic effect will also be unusually small. As with the birefringence, the shape of response curve \( d(E) \) will be nonlinear and discontinuous if \( \theta(E) \) is. However, regardless of the shape, if \( M_{E=0} \) is small, the layer contraction will be too. This is summarized in Fig. 3(b). As discussed above, there is less published data on the response of the layer spacing other than to show that it is small. Further experimental investigation of the response could be in interesting, in order to see if it is consistent with Eq. (3) above.

### C. Outline

The remainder of this article is organized as follows. In Sec. II we review the nonchiral generalized Landau theory. This is done with a view to using it as the basis of our chiral model and we focus in particular on the parts of model that are important for the analysis of the electroclinic effect. Additionally, we review the results for the nonchiral zero-field phase diagram, as it will be argued later that the phase diagram for a chiral system is essentially the same. In Sec. III we generalize the model to reflect the presence of chirality and an external field. The general approach to doing so is to add the relevant chiral terms and field-dependent terms. To strike a balance between making the model realistic and making it manageable, we are selective in what we add to reflect the presence of chirality and a field. The justification behind our selection is discussed in Sec. III. In Sec. IV we analyze the response of the tilt to a field applied to the Sm-A* phase. In Sec. V we analyze the response of the birefringence and layer spacing to a field applied to the Sm-A* phase. We provide a brief recap of our results in Sec. VI. The Appendix includes details of the analysis from Sec. V.

### II. MODEL AND RESULTS FOR A NONCHIRAL SYSTEM

In constructing the free-energy density for a chiral smectic we follow the usual strategy of starting with a nonchiral free-energy density and then adding the terms that reflect the breaking of the chiral symmetry and the presence of a field. In this section we discuss the nonchiral model and results.

#### A. Free-energy density for a nonchiral system

The nonchiral free-energy density includes orientational, tilt (azimuthal), biaxial, and layering order parameters. The complex layering order parameter \( \psi \) is defined via the density \( \rho = \rho_0 + \text{Re}(\psi e^{i\mathbf{q} \cdot \mathbf{r})} \) with \( \rho_0 \) constant and \( \mathbf{q} \) the layering wave vector, the arbitrary direction of which is taken to be \( \hat{z} \).

The remaining order parameters are embodied in the usual second-rank tensor orientational order parameter \( \mathbf{Q} \), which is most conveniently expressed as

\[
Q_{ij} = M \left[ -\cos(\alpha) + \sqrt{3} \sin(\alpha) \right] e_1 e_{1j} + \left[ -\cos(\alpha) - \sqrt{3} \sin(\alpha) \right] e_2 e_{2j} + 2 \cos(\alpha) e_3 e_{3j},
\]

where \( e_1 = \hat{c} + \sqrt{1-c^2} \hat{z} \) is the average direction of the molecules’ long axes, i.e., the director. Here, in either the Sm-A or Sm-C phase, \( \hat{z} \) is normal to the plane of the layers. The projection, \( \hat{c} \), of the director onto the layers is the order parameter for the Sm-C phase. The other two principal axes of \( \mathbf{Q} \) are given by \( e_1 = \hat{c} \times \hat{\mathbf{z}} \) and \( e_2 = \sqrt{1-c^2} \hat{c} - c \hat{\mathbf{z}} \). These unit eigenvectors are shown in Fig. 4. The amount of orientational order is given by \( M \approx \sqrt{\text{Tr}(Q^2)} \), which is proportional to the birefringence. The degree of biaxiality is described by the parameter \( \alpha \). The Sm-A phase is uniaxial with \( \alpha = 0 \), and the Sm-C phase is biaxial with \( \alpha \neq 0 \).

The nonchiral generalized free energy was presented previously [18] as a sum, \( f = f_Q + f_\psi + f_{Q\psi} \) of orientational (\( f_Q \)), layering (\( f_\psi \)), and coupling (\( f_{Q\psi} \)) terms. The orientational term consists of terms \( \propto \text{Tr}(Q^n) \), with integer \( n > 1 \). The layering term consists of terms \( \propto \text{Tr}(Q^{2n}) \) with integers \( n \geq 0 \) and \( m > 0 \). The coupling term \( f_{Q\psi} \) is only of real scalar combinations of \( \mathbf{Q} \), \( \mathbf{Q} \), and \( \psi \), e.g., \( q_{ij} Q_{ij} \). The analysis tractable, the coefficients of these coupling terms (and were (will be) assumed to be small. Minimization with respect to the biaxiality \( \alpha \) yielded the nonchiral free-energy density \( f = f_M + f_\psi + f_{M\psi} + f_c \). The pieces \( f_M \) and \( f_\psi \) only involve the orientational and layering order parameter \( M \) and \( \psi \), respectively, and are given by

\[
f_M = \frac{1}{2} M^2 - \frac{1}{4} M^3 + \frac{1}{4} \alpha M^4,
\]

and
\[ f_\psi = \frac{1}{2} r_\psi |\psi|^2 + \frac{1}{4} u_\psi |\psi|^4 + \frac{1}{2} K (q_\psi^2 - q_\psi^0)^2 |\psi|^2. \] (6)

The coefficients \( w, u_\psi, r_\psi, \) and \( K \) are positive. As usual in Landau theory, the parameters \( r_\psi \) and \( r_t \) are monotonically increasing functions of temperature and control the “bare” orientational and layering order parameters, \( M_0 \) and \( \phi_0 \), respectively. By “bare” we mean the values the order parameters would take on in the absence of the coupling term \( f_\psi \) and an externally applied field. Similarly, the constant \( q_0 \) is the bare value of the layering wave vector. From Eqs. (5) and (6) above, we immediately find \( M_0(r_\psi) = (w + \sqrt{w^2 - 4uwr_\psi})/2u_\psi \) and \( |\phi_0| = \sqrt{r_\psi}/u_t \). As discussed in Sec. I, de Vries behavior occurs in materials where the layering and orientational order parameters are the primary and secondary order parameters, respectively. This would imply a virtually athermal \( r_\psi \) (and thus, an athermal \( M_0 \)) so that for a given material \( M_0 \) can be thought of as a fixed quantity. This means that the temperature variation in orientational order \( M \) is effectively due to its coupling to the temperature dependent layering, i.e., via \( f_M \) and \( f_c \). The term \( f_M/\phi \) is given by

\[ f_M = q_\psi^2 |\psi|^2 \left[ -a(q^2) + b |\psi|^2 + c M + d |\psi|^2 M \right], \] (7)

where \( a(q^2) = a_0 + a_1 (q^2 - q_\psi^0) \). The coefficients \( a_0, a_1, b, c, \) and \( d \) are positive and, as discussed above, are treated perturbatively throughout [21]. For notational simplicity, we suppress the explicit \( q \) dependence of \( a, \) i.e., we use \( a = a(q^2) \). The coupling term \( f_c \) involves the tilt (azimuthal) order parameter \( c \) and is given by

\[ f_c = \frac{1}{2} r_c c^2 + \frac{1}{4} u_c c^4 + \frac{1}{6} v_c c^6. \] (8)

The coefficients \( r_c, u_c, \) and \( v_c \) are given by \( r_c = 3aq^2 |\psi|^2 \), \( u_c = 9 c q |\psi|^2 M \), \( u_c := 9 c q |\psi|^2 M \), and \( v_c := 3 s q^3 |\psi|^2 M^3 \), with \( s \) another coupling coefficient that is treated perturbatively throughout. The parameter \( \tau = \frac{\sqrt{\bar{\phi}^3 (\bar{\phi}^2 + 2h |\phi|^2)} M^3}{M} \) controls the zero-field transition. The proximity of the zero-field transition to tricriticality is measured by the tricritical proximity parameter \( \mu \) which will be discussed below.

B. Zero-field Sm-A−Sm-C transitions

1. Continuous Sm-A−Sm-C transition

At the continuous Sm-A−Sm-C transition the dimensionless parameter \( \tau \) and thus \( r_c \) changes sign. For materials, such as de Vries smectics, with orientational order \( M \) that is weakly temperature dependent, this transition occurs due to the layering order \( |\phi| \) increasing as temperature decreases. Using the above expression for \( \tau \), the continuous transition temperature \( T_0 \) is defined via \( |\phi_0(T_0)|^2 = a_0 = -g + 2h q_\psi^0 M_0^3 / b \). Figure 5 shows the continuous Sm-A−Sm-C boundary as a straight line in \( |\phi|^2 \)−\( M \) space. At this point we make a notational distinction. In referring to the Sm-A−Sm-C transition temperature generally (i.e., without distinguishing between continuous or first order) we use \( T_{AC} \). When referring specifically to either a continuous or a first-order transition we use \( T_0 \) and \( T_{1st} \), respectively. It is useful to work with a reduced temperature \( t = T / T_0 - 1 \) which, near the continuous transition, can be related to \( \tau \) via \( \tau = 1 - \frac{|\phi_0(T)|^2}{|\phi_0(T_0)|^2} \approx pt \). Here we have Taylor expanded \( |\phi_0(T)|^2 \) near \( T = T_0 \). The dimensionless parameter \( \mu = -\frac{a_0}{|\phi_0(T)|^2} \) controls the nature of the continuous Sm-A−Sm-C transition. The dimensionless tricritical point \( \mu \) incorporates the renormalization of the \( c^2 \) term due to the coupling between biaxiality \( \alpha \) and tilt \( c \) (in the absence of such a coupling \( \mu = 1 \)) and depends on the amount of orientational and layering order. It is given by

\[ \mu(T) = 1 - \frac{g}{2h q^3} \left( \frac{w M(T)}{g q^3 |\phi(T)|^2} - 1 \right)^{-1}, \] (9)

where the temperature dependence of \( \mu \) is a consequence of the temperature dependence of both \( \psi \) and \( M \). For de Vries materials, in which the orientational order \( M \) varies very little with temperature in the Sm-A phase, the temperature dependence of \( \mu \) in the Sm-A phase is due primarily to the temperature variation in the layering order \( |\phi| \). Figure 5 shows the location of \( \mu = 0 \) in \( |\phi|^2 \)−\( M \) space. The nature of the transition is determined by the sign of \( \mu_{AC} = \mu(T_{AC}) \), the value of \( \mu \) at the zero-field Sm-A−Sm-C transition. For \( \mu_{AC} > 0 \) (for small and large values of \( |\phi|^2 \) and \( M \) respectively) the transition is continuous while for \( \mu_{AC} < 0 \) (for large and small values of \( |\phi|^2 \) and \( M \), respectively) the transition is first order. When \( \mu_{AC} = 0 \) the quartic term vanishes and the transition is tricritical. As shown in Fig. 5 the asso-
cated tricritical point \(|\psi_{TC}|^2, M_{TC}\) is located where the continuous Sm-A–Sm-C boundary meets the locus of \(\mu=0\). For de Vries materials with a virtually athermal \(M\) the sign of \(\mu_{AC}\) is determined by the size of the system’s orientational order. For a transition close to tricriticality, \(\mu_{AC}\) is most conveniently expressed as

\[
\mu_{AC} = \frac{m}{\left(\frac{M-M_{TC}}{M_{TC}}\right)^2},
\]

where \(m=1+\frac{2\eta_0}{\beta}\) is a dimensionless constant. To lowest order in the coupling parameters, \(M_{TC}=\frac{ma\sigma^2}{3\eta}\). The corresponding value of layering order at the tricritical point is \(|\psi_{TC}| = \sqrt{a/b}\). In previous models \([22]\) of the Sm-A–Sm-C transition the parameter analogous to \(\mu\) has been assumed to be independent of temperature. In our model, as discussed above, \(\mu\) will vary with temperature via the temperature dependence of \(|\psi(T)|\). For the time being we will use a constant \(\mu\) approximation, \(\mu(T) \approx \mu_{AC}\), valid near the Sm-A–Sm-C transition. In Sec. IV C we discuss in further detail the temperature dependence of \(\mu\) and some of the related consequences for the electroclinic response.

A commonly used \([22]\) measure of how close the continuous Sm-A–Sm-C (or Sm-A’–Sm-C’) transition is to tricriticality is the magnitude of the reduced temperature, \(\bar{t}_c\), when \(|r_c| = u_c^2/v_c\). Using this condition, it is straightforward to show that

\[
t_c = -\kappa_1 \left(\frac{M-M_{TC}}{M_{TC}}\right)^2,
\]

where the dimensionless constant \(\kappa_1 = \frac{a^4\eta_0}{3\eta^2}\). In the Sm-C phase, well within the corresponding temperature window \(T_s < T < T_0\), where \(T_s = T_0(1-|t_c|)\), the quartic term \(\kappa\bar{c}^4\) is important, and the behavior is \(XY\) like. Sufficiently far outside this window, i.e., \(T < T_s\), it can be neglected, and the behavior of the system is tricritical. Figure 5 shows the corresponding crossover region in \(|\psi|^2-M\) space, in which the system’s behavior goes from being \(XY\) to tricritical. The reduced temperature \(t_c\) can be obtained \([22]\) from measurements of the specific heat at the continuous Sm-A–Sm-C (or Sm-A’–Sm-C’) transition. Work has been done to relate the size of the reduced temperature window \(|t_c|\) to system parameters, e.g., the width of the Sm-A phase \([23]\). However, to the best of our knowledge, no one has yet investigated a possible relationship between the size of the reduced temperature window and the size of the orientational order \(M\). The above expression for \(t_c\) provides a prediction for such a relationship \([24]\).

3. First-order Sm-A–Sm-C transition

It was shown \([18]\) that when the tricritical proximity parameter \(\mu_{AC}<0\), i.e., \(M<M_{TC}\), a first-order Sm-A–Sm-C transition occurs at a value of \(t\) given by \(t_{1,A} = \frac{2}{M_{TC}}|t_c| > 0\). As discussed in Ref. \([18]\) the size of the latent heat at the first-order Sm-A–Sm-C transition is proportional to \(\mu_{AC}\) and thus, calorimetric studies can measure the proximity of the first-order transition to the tricritical point. It is important to keep in mind that the first-order Sm-A–Sm-C will occur at \(t>0\) and thus \(T_{1,M} > T_0\). Correspondingly, the value of layering order \(|\psi|\) at the first-order Sm-A–Sm-C boundary is smaller than would be necessary for a continuous Sm-A–Sm-C transition. Figure 5 shows an extrapolation of the continuous Sm-A–Sm-C boundary in \(|\psi|^2-M\) space for \(M>M_{TC}\). The difference between the layering at the extrapolated boundary and the first-order Sm-A–Sm-C boundary is proportional to \((M-M_{TC})^2\).

C. Roles of orientational order and layering order in de Vries behavior and the nature of the Sm-A–Sm-C transition

As shown in Ref. \([18]\) de Vries behavior, i.e., an unusually small change in the layer spacing at the Sm-A–Sm-C transition, can be explained by unusually small orientational order and coupling parameters. The de Vries behavior, i.e., unusually rapid change, of the birefringence at the Sm-A–Sm-C transition can be explained by proximity of the transition to a tricritical point. It has been experimentally observed \([4–6]\) that several materials exhibiting de Vries behavior also have a Sm-A–Sm-C transition that is close to a tricritical point. Our model implies that de Vries behavior and proximity of the Sm-A–Sm-C transition to tricriticality can be connected by unusually small orientational order. Indeed, it has been observed that de Vries materials do have unusually small orientational order. Consequently it has been argued \([1, 16]\) that stabilization of materials with such small orientational order must be provided by unusually strong layering order, perhaps via microsegregation. The phase diagram in \(|\psi|^2-M\) space, shown in Fig. 2, is consistent with such an argument; the negative slopes of both the continuous and first-order phase boundaries mean that systems with smaller orientational order require larger layering order to make the transition from the Sm-A phase to the Sm-C phase.

To the best of our knowledge, no direct measurement of the layering order in de Vries materials has been published. We believe such measurements would be valuable in understanding the role that layering order plays in driving the Sm-A–Sm-C (or Sm-A’–Sm-C’) transition, as well as the nature of the transition (i.e., continuous, tricritical, or first order) and how de Vries like the system is. While direct measurements of the layering have not been reported, there is published data \([25]\) on the width of the Sm-A phase in a homologous series of hexyl lactates (nHL) exhibiting Sm-A’–Sm-C’ transitions that range from conventional to de Vries like. It is found that the temperature width of the Sm-A’ phase window increases as the system becomes more de Vries like. Making the conventional assumption that the layering order at the Sm-A’–Sm-C’ transition is a monotonically increasing function of the temperature width of the Sm-A’ phase, this data is consistent with our model. However, one must be careful in making this assumption for systems (e.g., de Vries materials) that have first-order isotropic (Iso–Sm-A (or Sm-A’)) transitions where the layering does not necessarily grow continuously from zero. For example, it could be possible that the layering order at the Iso–Sm-A (or Sm-A’) transition is larger in systems with smaller orientational order. Thus, the layering at the Iso–Sm-A (or Sm-A’) transition may already be large enough so that it is not nec-
essary to have a wider temperature window for the Sm-A (or Sm-A’*) phase [26]. This is another reason that a systematic experimental investigation of the layering and orientational order in these systems would be valuable.

III. INCORPORATING THE EFFECTS OF CHIRALITY AND EXTERNAL FIELDS TO THE FREE-ENERGY DENSITY

Having set up the nonchiral zero-field free energy we next add terms to reflect the presence of chirality and an externally applied field. The most important such term is the one which models the electroclinic interaction of the molecules with the applied electric field \( \mathbf{E} \). To lowest order in the orientational and layering order parameters it is

\[
f_{EC} = e e_{ij} q_{jl} |\psi|^2 E_j Q_{il} = e q^2 M |\psi|^2 \hat{Z} \cdot (\mathbf{E} \times \mathbf{c}),
\]

(12)

where \( e_{ij} \) is the Levi-Cevita symbol and the Einstein summation convention is implied. In coupling the electric field directly to the tilt \( \mathbf{c} \), instead of via the electrostatic polarization \( \mathbf{P} \), we are making the standard assumption that \( \mathbf{P} \propto \hat{Z} \times \mathbf{c} \) [27]. The coefficient \( e \) depends on the strength of the electrostatic coupling between the field and the molecules. This in turn depends on amount of chirality in the system and for a racemic mixture \( e = 0 \). Here we take \( e > 0 \); switching the handedness, e.g., left to right, of the molecules simply switches the sign of \( e \). In making the approximation in Eq. (12) above, we include only the lowest-order contribution of tilt \( \mathbf{c} \) from the orientational order tensor \( \mathbf{Q} \) and we neglect the biaxial part of \( \mathbf{Q} \). It can be shown that close to the tricritical point the coupling between the field and biaxiality is negligible.

In order to make the model manageable, we also omit other contributions, each of which lead only to secondary less important effects. The first of these is the nonchiral coupling of the system to the electric field which would contribute terms such as \( \mathbf{E}_j Q_{ij} \). All such terms scale like \( E^2 \) and in the limit of small field can be shown to be much smaller than the electroclinic term in Eq. (12) above, which scales linearly with \( E \).

We also assume a spatially uniform tilt \( \mathbf{c} \) and thus ignore a second group of contributions involving spatial variations in \( \mathbf{c} \), including manifestly chiral terms that depend on the sign of \( \nabla \times \mathbf{c} \). We have analyzed the difference that such terms make to our model. One zero-field effect of these terms is to shift the location of the Sm-A’–Sm-C’ phase boundary, by renormalizing the coefficients in the free-energy expression, Eq. (8), for \( f_c \). In particular, increasing the chirality of the system lowers the quadratic coefficient, \( r_c \), the effect of which is to increase the Sm-A’–Sm-C’ transition temperature. Increasing the chirality also lowers the value of the quartic coefficient \( u_c \), thus driving a continuous transition toward tricriticality or a first-order transition away from tricriticality. The behavior of the layer spacing and birefringence are also somewhat affected via the renormalization of these coefficients. However, in the limit (which we assume throughout) of small orientational order and small couplings between layering and orientational order parameters, the renormalization of these coefficients is negligible.

Thus, the zero-field behavior of the chiral system should essentially be the same as described for the nonchiral system.

The absence of terms involving spatial variations in \( \mathbf{c} \) also precludes the possibility of a superstructure involving a spatial modulation of \( \mathbf{c} \), which in the zero-field Sm-C’ phase would be helical. In the past [17] the assumption of a spatially uniform tilt has been justified by consideration of electric field strength above that necessary to unwind a helical superstructure. However, it is not obvious that a helical superstructure would form when the tilt is electrically induced (as opposed to spontaneously developing at the zero-field Sm-A’–Sm-C’ transition.) For example, it has been shown [28] that in a two-dimensional Sm-A’ film, the electroclinic effect can lead to a spatially uniform tilt at small and large fields and to a modulated tilt for fields of intermediate strength. To the best of our knowledge the situation for three-dimensional Sm-A’ systems has yet to be analyzed, although we plan to do so in the near future. It should be pointed out that one proposed explanation [29] for the strong electroclinic effect in de Vries materials is that the Sm-A’ phase is actually a Sm-C’ phase that is made up of an ordered array of disclination lines and walls, and thus assumes a strong spatial modulation of the tilt in the Sm-A’ phase. We do not explore that possibility here.

In summary, because we are interested primarily in the electroclinic effect and do not wish to overburden the model with less important secondary effects, the only extra term we add to our nonchiral model is that given in Eq. (12).

IV. RESPONSE OF TILT

In this section we explore the response of the tilt order parameter \( \mathbf{c} \) to an externally applied electric field \( \mathbf{E} \). Of particular interest is the response near the tricritical point shown in Fig. 2. As shown in Fig. 4 we take the field to point in the \( y \) direction so that the free energy is minimized by a tilt in the \( x \) direction, i.e., \( \mathbf{c} = \mathbf{c}_x \mathbf{\hat{x}} \) and \( f_{EC} = -b q^2 |\psi|^2 E_c \). The magnitude \( c \) of the tilt induced by the applied field can be determined using the tilt portion of the free energy, \( f_c + f_{EC} \). Minimizing this free energy with respect to the tilt \( c \) one obtains the following relationship between \( c \) and \( E \):

\[
E = \alpha_c c + \beta_c c^3 + \gamma_c c^5,
\]

(13)

where the electroclinic coefficients \( \alpha_c, \beta_c \), and \( \gamma_c \) are given by

\[
\alpha_c = \frac{3 \alpha pt}{e},
\]

(14)

\[
\beta_c = \frac{9 \mu h q^2 M}{e},
\]

(15)

\[
\gamma_c = \frac{81 s q^4 M^2}{4 e^4},
\]

(16)

where the reader is reminded that the tricritical parameter \( \mu \) generally depends on temperature via its dependence on orientational \( (M) \) and layering order \( (|\psi|) \), given in Eq. (9). As
discussed in Sec. II B 2 the orientational order of the Sm-\(A^*\) phase in de Vries materials varies very little with temperature so the temperature dependence of \(\mu\) in the Sm-\(A^*\) phase is due primarily to the temperature variation in the layering order \(|\psi|\). The relationship [Eq. (13)] between \(E\) and \(c\) is analogous [30] to that derived by Bahr and Heppke in their analysis of a field-induced critical point near the Sm-\(A^*\)–Sm-\(C\) transition [17]. There are, however, a couple of distinctions that should be pointed out. The first is motivation. In Ref. [17] the primary motivation was to establish the existence of and to analyze a line of first-order Sm-\(A^*\)–Sm-\(C\) transitions in the temperature-field plane that terminates at a critical point. Our motivation is to model and explain the unusually large electroclinic response of de Vries materials. It will be shown that this can be done by analyzing Eq. (13) in a similar manner to Ref. [17].

A second related distinction is that, as a result of starting with a generalized Landau theory in terms of orientational and layering order parameters, we can relate our coefficients \(\alpha_c, \beta_c, \gamma_c\) to the strengths of orientational order (and hence birefringence) and layering order, as well as the layer spacing (via \(q\)) in the system. Of particular interest is the origin of a negative quartic coefficient \(\beta_c < 0\) in Eq. (13) above, which is necessary for a field-induced first-order Sm-\(A^*\)–Sm-\(C\) transition. In Ref. [17], this was assumed (justifiably) on the basis of the existence of a zero-field first-order Sm-\(A^*\)–Sm-\(C\) transition. Here, a negative quartic coefficient can be explained as resulting from sufficiently weak orientational order, which, as discussed in Sec. II C necessitates strong layering order, in order to stabilize the system. Thus, our generalized Landau theory shows that an unusually strong electrical response of the tilt can be explained as resulting from a combination of weak orientational order and strong layering order, which makes the quartic coefficient \(\beta_c\) either positive and small (corresponding to a continuous zero-field Sm-\(A^*\)–Sm-\(C\) transition that is near a tricritical point) or negative (corresponding to a first-order zero-field Sm-\(A^*\)–Sm-\(C\) transition). A related distinction between this analysis and that of Bahr and Heppke is that our quartic coefficient, \(\beta_c\), depends on temperature via the temperature dependence of \(\mu\). We will next analyze the electroclinic response implied by Eq. (13).

A. Electroclinic response near the continuous zero-field Sm-\(A^*\)–Sm-\(C\) transition

We begin our analysis by approximating the tricritical proximity parameter \(\mu\) as being temperature independent, i.e., \(\mu(T) \approx \mu_{AC}\), which is valid sufficiently close to the Sm-\(A^*\)–Sm-\(C\) transition. The effect of \(\mu\)'s temperature dependence will be discussed in Sec. IV C. For \(\mu_{AC} > 0\), corresponding to a continuous zero-field Sm-\(A^*\)–Sm-\(C\) transition, \(\beta_c > 0\). For such systems, the response of the tilt \(c\) to an applied field \(E\) is continuous. Additionally, as shown in Fig. 6, the susceptibility \(\chi_c = \frac{\partial c}{\partial E}\) gets smaller with increasing field. Its largest value, at \(E = 0\), is \(\chi_c(0) = \alpha_c(0)^{-1}\), which diverges as the system approaches the continuous zero-field Sm-\(A^*\)–Sm-\(C\) transition at \(\alpha_c(T_c) = 0\), a standard result for continuous transitions. The response at the Sm-\(A^*\)–Sm-\(C\) transition for small fields is \(c \approx E^{\delta/\delta_t}\), with \(\delta = 3\) away from tricriticality and \(\delta = 5\) at the tricritical point.

It is interesting to consider how the response \(c\) at fixed reduced temperature \((t > 0)\) and field \((E > 0)\) is affected by lowering \(\mu_{AC}\) toward zero, i.e., driving the continuous transition to tricriticality. It is straightforward to show that

\[
\frac{\partial c}{\partial \mu_{AC}} = -\frac{9hq^2M^3c}{e} < 0, \tag{17}
\]

which, as expected, shows that the response, at fixed \(E\) and \(t\), should be larger for systems with smaller \(\mu_{AC}\), i.e., systems in which the orientational order is small \((M \approx M_{TC})\). This is shown graphically in Fig. 6(b) and is reminiscent of an experimentally obtained comparison [1,31] of electroclinic responses for a homologous series of hexyl lactates \((nHL)\), with each response being measured at the same reduced temperature. The response is observed to be larger for small \(n\) values. The compounds have zero-field continuous Sm-\(A^*\)–Sm-\(C\) transitions that range from conventional to de Vries like [25]. We speculate that if one were to measure the proximity of each compound’s transition to a tricritical point, one would find that 9HL’s and 12HL’s transitions are closest and furthest respectively, i.e., \(0 < \mu_{AC_{9HL}} < \mu_{AC_{12HL}}\).

B. Electroclinic response near the first-order zero-field Sm-\(A^*\)–Sm-\(C\) transition

Next we consider the response when the tricritical proximity parameter \(\mu_{AC} < 0\) (and thus \(\beta_c < 0\)) corresponding to a first-order zero-field Sm-\(A^*\)–Sm-\(C\) transition. As shown in Fig. 7, for large reduced temperatures \(t\) the response is continuous and will show a positive curvature \((\frac{d^2c}{dE^2} > 0)\) at small fields followed by a negative curvature \((\frac{d^2c}{dE^2} < 0)\) at large fields. The positive curvature has been referred to in the literature (e.g., in Ref. [12]) as “superlinear growth.” For sufficiently small temperatures the response curve is \(S\) shaped (i.e., has a portion with negative slope) and there is a jump in the tilt as the electric field is increased from zero. Thus, an unusually strong discontinuous electroclinic effect will be exhibited by systems with sufficiently small orientational order \(M < M_{TC}\).

We define \(t_c\) as the value of reduced temperature below which the response curve, \(c(E)\), exhibits a negative slope, and hence a discontinuity in the response. As shown in Fig. 7, at \(t = t_c\), the curve has divergent slope and curvature at \(E_c\) and \(c_c\). Thus, the values \(t_c, E_c,\) and \(c_c\) are specified by \(\frac{dE}{dc} t_c = 0\) and \(E_c = E(t_c, c_c)\). At the level of the mean-field theory presented here and elsewhere [17], the associated critical point \((t_c, E_c, c_c)\) is analogous to the liquid-vapor critical point. However, it has been pointed out [32] that when fluctuations are included the universality class of this critical point is distinct from that of the liquid-vapor critical point. It is straightforward [17] to calculate the critical values \((t_c, E_c, c_c)\). We rederive these values, primarily with a view to presenting them in terms of the degree of orientational order in the system. We also provide extra details that might be useful in experimentally investigating whether the strong response of de Vries materials is indeed a
result the proximity of the Sm-A–Sm-C* tricritical point. The value of the critical reduced temperature is found to be $t_c = \frac{12}{5} t_{1st}$, where $t_{1st}$ is defined in Sec. III. Keeping in mind the fact that the first-order transition occurs at $t_{1st} > 0$, one can find the temperature difference between $T_{1st}$ and $T_c$:

$$\frac{T_c - T_{1st}}{T_{1st}} \approx \frac{21}{80} \kappa \left( \frac{M_{TC} - M}{M_{TC}} \right)^2,$$

(18)

where the dimensionless constant $\kappa = \frac{4\kappa_0}{3\mu}$ was defined earlier in Sec. II and the approximation applies close to tricriticality where $M \approx M_{TC}$. The value of $c_c$ is found to be

$$c_c = \sqrt{\frac{7}{5}} c_{1st} \approx \left( \frac{M_{TC} - M}{M_{TC}} \right)^{1/2},$$

(19)

where $c_{1st}$ is the size of the jump in the tilt order parameter at the zero-field transition, which is found [18] to be $c_{1st} = \frac{2}{\sqrt{5}} T_{1st}$. The above equation implies that the ratio $\frac{c_c}{c_{1st}} = \sqrt{\frac{7}{5}}$ should hold for any system, a prediction that should be straightforward to test experimentally. Lastly, we find

$$E_c = \frac{8}{15} \alpha_c(t_c) c_c \approx \left( \frac{M_{TC} - M}{M_{TC}} \right)^{5/2},$$

(20)

where $\alpha_c(t)$ is given by Eq. (14). Using the fact that $\alpha_c(T) = \chi_c(T)^{-1}$, we define the following combination:

$$\lambda_c = \frac{\chi_0(T_c) E_c}{c_c},$$

(21)

where $\chi_0(T_c)$ is the value of the zero-field susceptibility at $T=T_c$. Together, Eqs. (20) and (21) predict that $\lambda_c = 8/15$ for every material that has a first-order Sm-A–Sm-C* transition. It would be interesting to investigate experimentally whether this is accurate for de Vries materials with first-order Sm-A–Sm-C* transitions. If so, it would indicate that the mean-field theory described here is suitable to describe the strong electro-optic response of de Vries type materials.

C. Effects of the temperature dependence of the tricritical proximity parameter $\mu$ on the electroclinic response near the Sm-A–Sm-C* transition

In previous models [22] of the Sm-A–Sm-C (and Sm-A–Sm-C*) transitions the parameter analogous to $\mu$ has been assumed to be independent of temperature. In our model $\mu(T)$, given by Eq. (9), will vary with temperature via the temperature dependence of $|\phi(T)|$ and to a lesser degree $M(T)$. From Eq. (9) it can be seen that $\mu(T)$ decreases if $M(T)$ and $|\phi(T)|$ decrease and increase, respectively. We have argued here and elsewhere [18,19] that in de Vries materials the system is driven toward the Sm-C* phase as the layering $|\phi|$ increases with decreasing temperature. Additionally, the nonmonotonicity of $M(T)$, which is both pre-
The Sm-A"–Sm-C" transition is approached from above. Since we are primarily interested in the evolution of the shape of the curves we do not specify units for $E$. The curves are now $S$ shaped which implies a discontinuous response. A set of curves for fixed $\alpha_e=0.01125$ (and thus, fixed reduced temperature $t>0$), $\gamma_e=0.4$ and different values of $\beta_e=\mu_{AC}/0$. Different values of $\beta_e$ imply varying degrees of proximity of the first-order Sm-A"–Sm-C" transition to a tricritical point. The values of $\beta_e$ are given in terms of $\beta_{ec}=-\left(1/3\right)^{1/2}$, the value of $\beta_e$ below which the curves become $S$ shaped. (i) $\beta_e=0$, (ii) $\beta_e=\beta_{ec}$, (iii) $\beta_e=1.3\beta_{ec}$. Making $\beta_e$ (and thus, $\mu_{AC}$) more negative would increase the temperature window $T_c-T_{1st}$ so that the discontinuous response occurs further away from the Sm-A"–Sm-C" transition.

As discussed in the preceding two sections, decreasing $\mu(T)$ leads to a strengthening of the electrical response of the tilt. Thus, we speculate that the electroclinic response in de Vries materials is further strengthened by the thermal behavior of the layering and orientational order. It would be interesting to extract the temperature dependence of $\mu(T)$ (perhaps through fitting the response curves at different temperatures) to see if it does have a temperature dependence and, if so, whether it decreases as the Sm-A"–Sm-C" transition is approached from above. There may also be an observable feature associated with the temperature dependence of $\mu(T)$ and the nonmonotonicity of $M(T)$. It has been predicted [2,3] in de Vries materials, implies that $M(T)$ decreases as the Sm-A"–Sm-C" transition is approached from above. Thus, each of these effects causes $\mu(T)$ to decrease toward $\mu_{AC}$ as the Sm-A"–Sm-C" transition is approached from above.

Figure 7. (Color online) Response curves $c(E)$ for systems with $\beta_e<0$, i.e., systems with first-order Sm-A"–Sm-C" transitions. The curves show the electrically induced tilt $c$ due to the application, in the Sm-A" phase, of a field of magnitude $E$. The curves were obtained using Eq. (13). Since we are primarily interested in the evolution of the shape of the curves we do not specify units for $E$ (i.e., we use arbitrary units, a.u.). (a) A set of curves for fixed $\gamma_e=0.4$ and $\beta_e=-0.1$ (and thus, fixed $\mu_{AC}/0$) and different values of $\alpha_e$. The values of $\alpha_e$ are most usefully expressed in terms of $\alpha_{ec}=\frac{E}{3\gamma}=0.00125$, which is the value of $\alpha_e$ at the critical reduced temperature $t_c$. Below this value the curves become $S$ shaped. Since $\alpha \approx t$, the ratio $t/t_c$ is the same as $\alpha/\alpha_{ec}$, and we label the curves according to the value of $t$ in terms of $t_c$: (i) $t=t_{ec}$, (ii) $t=t_{ec}$, (iii) $t=0.65t_{ec}$, and (iv) $t=t_{ec}=0.5t_{ec}$. For $t>t_c$, the response is continuous but “superlinear,” with $\frac{d^2c}{dt^2}>0$ at small $E$. At $t=t_c$, the response has divergent susceptibility (i.e., slope) and curvature at $E$, and $c$, indicated with a dot. The curves are now $S$ shaped which implies a discontinuous response. (b) A set of curves for fixed $\alpha_e=0.01125$ (and thus, fixed reduced temperature $t>0$), $\gamma_e=0.4$ and different values of $\beta_e=\mu_{AC}/0$. Different values of $\beta_e$ imply varying degrees of proximity of the first-order Sm-A"–Sm-C" transition to a tricritical point. The values of $\beta_e$ are given in terms of $\beta_{ec}=-\left(1/3\right)^{1/2}$, the value of $\beta_e$ below which the curves become $S$ shaped. (i) $\beta_e=0$, (ii) $\beta_e=\beta_{ec}$, (iii) $\beta_e=1.3\beta_{ec}$. Making $\beta_e$ (and thus, $\mu_{AC}$) more negative would increase the temperature window $T_c-T_{1st}$ so that the discontinuous response occurs further away from the Sm-A"–Sm-C" transition.

V. RESPONSE OF THE BIREFRINGENCE AND LAYER SPACING TO AN ELECTRIC FIELD APPLIED TO THE Sm-A" PHASE

Having analyzed how the tilt order parameter, $c$, will respond to an electric field $E$ being applied to the Sm-A" phase, we now investigate how the birefringence, $\Delta n$, and layer spacing, $d$, are simultaneously affected. We do this with a view to providing insight into the response of birefringence and layer spacing for de Vries materials in particular. First we summarize the main experimental observations [1]. In de Vries materials the response of the tilt is unusually strong, which as discussed above, can be explained by an unusually small orientational order which leads to a Sm-A"–Sm-C" transition that is either continuous and close to tricriticality or first order. The response of the birefringence (which is proportional to the orientational order in the system) is also unusually strong. However, the contraction, i.e., fractional change $\Delta d$, of the layer spacing $d$ associated with the tilt is unusually small. The combination of a large response in the tilt and birefringence and a small contraction of the layer

$M(T)$ begins to decrease. If this were so, there may be an associated anomaly in the electroclinic response as $T$ is lowered through $T_{max}$.
spacings is technologically desirable. The unusually small contraction of the layers eliminates buckling of the layers and the associated chevron defects which lead to unwanted striping in ferroelectric liquid crystal displays.

Another noteworthy experimental observation is the scaling of the birefringence response with tilt response. The tilt \( c(E) \) and the birefringence \( \Delta n(E) \) each scale nonlinearly with applied field \( E \), and the shape of the nonlinear curves change significantly as temperature is varied. However, a parametric plot of \( \Delta n(E) \) vs \( c^2(E) \) is very close to being linear. Remarkably, this linear scaling seems to hold regardless of the nature (i.e., continuous, tricritical, or first order) of the transition [9,10]. Additionally, the slope of this linear scaling varies very little with temperature. There does not seem to be any published parametric plots of \( \Delta n(E) \) as a function of \( c^2(E) \). As discussed in more detail below, we predict that while \( \Delta n(E) \) will scale nonlinearly with applied field it will scale linearly with \( c^2(E) \). The slope of this linear scaling is proportional to the orientational order and will thus be unusually small in systems with unusually small orientational order. Unlike the birefringence we predict that the slope of the absolute change in layer spacing \( \Delta(M) \) [as opposed to fractional change \( \Delta n(E)/n \)] vs \( c^2(E) \) will not be weakly temperature dependent.

In what follows we first investigate the response of the birefringence to an applied electric field. The general methods described here are also applied to investigating the response of the layer spacing.

A. Response of the birefringence to an electric field applied to the Sm-\( A^+ \) phase

In analyzing the response of the birefringence we use the fact that birefringence is proportional to the orientational order in the system and find the change in orientational order due to an applied field. We define the zero-field orientational order as \( M_{E=0} \). It is important to note that this differs from \( M_0 \), given after Eq. (6), which is defined as the zero-field orientational order in the absence of coupling between orientational order and layering. As discussed in the analysis of the nonchiral zero-field model [18], the effect of the coupling of the orientational order to layering order is to increase the orientational order above its zero coupling value \( M_0 \). Here, with our chiral model, we are focusing on the additional effect on orientational order due to the application of an electric field. Thus, we use the notation \( M_{E=0} \) to represent the zero-field orientational order, which includes the increase due to the zero-field coupling of orientational order to layering. This means that \( M_{E=0} > M_0 \). As was shown in Ref. [19] \( M_{E=0} \) is a nonmonotonic function of temperature. As temperature is lowered toward \( T_{AC} \), \( M_{E=0} \) decreases (albeit weakly), a feature which, while unusual, has nonetheless been observed experimentally [2,3]. Upon entry to the Sm-C phase, \( M_{E=0} \) increases with decreasing temperature. For continuous transitions the rate of increase is larger the closer the transition is to tricriticality and for first-order transitions the increase is larger the further transition is from tricriticality.

We define \( \Delta M_E \) as the fractional change in the orientational order due to the application of an electric field, i.e., \( M=M_{E=0}(1+\Delta M_E) \). The response \( \Delta M_E \) is obtained by minimizing the free energy with respect to \( \Delta M_E \). This is made tractable by assuming that \( \Delta M_E \) is small and expanding the free energy to quadratic order in \( \Delta M_E \). Details of the analysis are given in the Appendix. We find that within the Sm-\( A^+ \) phase, for small \( t \) and \( \mu_{AC} \), i.e., close to a Sm-\( A^+ \)-Sm-\( C^- \) transition which is close to tricriticality, the fractional change in orientational order is given by

\[
\Delta M_E = \frac{3m}{2\gamma_M} q_{E=0} |\psi(T)| [1 - O\left(\frac{c^2(E)}{c_M^2}\right)] c^2(E),
\]

where \( m = 1 + \frac{2q_{E=0}^2}{\gamma_M} \) is a dimensionless constant and \( \gamma_M = \frac{d^2J_M}{dM^2}\big|_{M=M_0} \), where \( f_M \) is given in Eq. (5). (The zero-field layering wave vector, \( q_{E=0} \), is distinct from the bare \( q_0 \), in that it includes the effects of the zero-field coupling between orientational and layering orders. The dimensionless parameter \( c_M \) can be thought of as the value of \( c \) where the scaling of \( \Delta M_E \) with \( c \) crosses over from being quadratic to quartic. We define \( c_M \) in the Appendix and show it to be \( O(1) \), which makes the quartic contribution negligible in our theory, where it is assumed that \( c \ll 1 \). It should also be pointed that the largest experimentally measured values of \( c \), obtained for large fields, are on the order of 0.5 [corresponding to \( c_{\max} = \sin(\theta_{\max}) \), \( \theta_{\max} = 30^\circ \)]. Thus, at all but the largest values of \( c \) the scaling of \( \Delta M_E \) with \( c \) is quadratic, which is consistent with experiment. Most importantly, the above result, Eq. (22), is valid for both continuous and discontinuous \( c \) response curves. Of course, the linear scaling of \( \Delta M_E \) with \( c^2(E) \), implied by Eq. (22), means that if there is a strong or discontinuous response of tilt \( c \) to applied field \( E \), there will be a correspondingly strong response of \( M \), and hence birefringence, to applied field. This is also consistent with experiment.

Having shown that the change in orientational order (and hence birefringence) scales linearly with \( c^2(E) \), we next consider the slope of this scaling, in particular its temperature dependence which, as discussed above, is experimentally observed to be weak. In most published work (e.g., Refs. [6,12]) that has analyzed the change in birefringence as a function of tilt, it is the absolute change rather than the fractional change that has been analyzed. Here we are interested in the absolute change in orientational order \( \Delta(M) = M - M_{E=0} \), which is given by

\[
\Delta M = M_{E=0}(T) [q_{E=0}(T)|\psi(T)|^2] c^2(E),
\]

where we have used \( \Delta M = M_{E=0} \Delta M_E \) and in going from Eqs. (22) and (23) we have kept only the leading-order temperature dependence (which we now display explicitly) of the \( c^2(E) \) prefactor. Thus, the temperature dependence of the slope of \( \Delta(M) \) vs \( c^2(E) \) is determined by the temperature-dependent combination \( \sigma(T) = M_{E=0}(T) q_{E=0}(T)|\psi(T)|^2 \). Since \( M_{E=0}(T), q_{E=0}(T) \), and \( |\psi(T)|^2 \) each remain finite within the Sm-\( A^+ \) phase, both \( \sigma(T) \) and the slope will also remain finite. In particular there will be no dramatic change in the slope as the Sm-\( A^+ \)-Sm-\( C^- \) transition is approached from above. Given that the temperature dependence of \( M_{E=0}(T) \) is weak, any change in the slope should be due to a change in the combination \( q_{E=0}(T)|\psi(T)|^2 \). We have already argued that
|φ(T)| increases monotonically as the Sm-A′–Sm-C′ transition is approached from above. It is generally observed experimentally that as the Sm-A′–Sm-C′ transition is approached from above, there is a monotonic dilation of the layer spacing, which corresponds to a monotonic decrease in qE=0(T). Thus, we speculate that the temperature changes in qE=0(T) and |φ(T)|² offset each other which leads to only a weak temperature dependence of the slope of the birefringence vs c²(E).

B. Response of layer spacing to an electric field applied to the Sm-A′ phase

To analyze the change in layer spacing due to the application of a field, we first obtain the change in the wave vector q=2π/d. As with the orientational order we define qE=0 to be the zero-field wave vector. This is distinct from q0, the zero-field wave vector in the absence of coupling between orientational order and layering. Since the wave vector only appears as q², it is convenient to define a fractional change Δq in q² due to the application of an electric field, i.e., q²=qE=0(1+ΔqE). In finding ΔqE, we follow the same method as described in Sec. V A and relegate the details to the Appendix. Within the Sm-A′ phase, close to tricriticality, i.e., for small μAC, we find

\[
\Delta q_{E} = \frac{3|a_1|}{2K} M_{E=0} \left[ 1 + O\left(\frac{c^2(E)}{c_0^2}\right) \right] c^2(E),
\]

where, as in Ref. [18], a layer contraction (as opposed to dilation) requires a₁ to be negative. As with cM, the dimensionless parameter c_0 can be thought of as the value of c where the scaling of ΔqE with c crosses over from being quadratic to quartic. We also define c_q in the Appendix, showing it to be O(1), which for the same reasons as outlined above, allows us to neglect the quartic contribution.

Using the above equation and the relationship between layer spacing (d) and wave vector (q=2π/d), we next seek the change in the layer spacing. This contraction is equivalent to the fractional change in the layer spacing Δ\(d_\) = (dE=0−d)/dE=0, where dE=0 is the zero-field value of the layer spacing in the Sm-A′ phase. We find that the contraction is given by

\[
\Delta d = \frac{3|a_1|}{4K} M_{E=0} c^2(E).
\]

Since c(E) is a nonlinear function of E (and is not \(\propto \sqrt{E}\)) the above equation implies that the contraction Δd(E) will also be a nonlinear function of E, and if c(E) is discontinuous, then ΔE will also be discontinuous. However, the above equation predicts that, like the birefringence, Δd(E) will scale linearly with c²(E), regardless of the nature of the transition. Thus, for small tilt angle θ, which implies c\(\approx\)θ, the fractional change in layer spacing scales like θ². In addition, our theory predicts that this fractional contraction is also proportional to the size of the orientational order M_{E=0}. Thus, de Vries systems which have unusually small orientational order will, under the application of an electric field, exhibit an unusually small layer contraction, as shown in Fig. 3(b).

Since \(M_{E=0}(T)\) is, as discussed in Sec. V A, only weakly temperature dependent, the slope of the Δd(E) vs c²(E) should also be weakly temperature dependent. However, the slope of the absolute change in layer spacing \(d_E=0−d\) = Δd(E) vs c²(E) should not be weakly temperature dependent. This is because \(Δd(E) = d(T)Δd(E)\) and \(d(T)\) has been shown experimentally to exhibit a noticeable monotonic increase as the Sm-A′–Sm-C′ transition is approached from above. Thus, we expect that as temperature is lowered there should be a noticeable increase in the slope of Δd(E) vs c²(E).

VI. SUMMARY

In summary, we have analyzed a generalized Landau theory for chiral smectics, one that tracks orientational, layering, tilt, and biaxial order parameters as well as layer spacing. A combination of small orientational order and large layering order leads to Sm-A′–Sm-C′ transitions that are either continuous and close to tricriticality or first order. The model predicts that the change in layer spacing at the zero-field transition will be proportional to the orientational order. It also predicts that in systems having zero-field transitions that are continuous and close to tricriticality or first order, the increase in birefringence upon entry to the Sm-C′ phase will be especially rapid. Thus, both the small change in layer spacing and the rapid increase in birefringence can be attributed to the system possessing a combination of small orientational order and large layering order. This is consistent with the observation that de Vries materials usually possess unusually small orientational order, which in turn means that strong layering order is required for stabilization.

The model also predicts that as a result of the zero-field Sm-A′–Sm-C′ transition being either continuous and close to tricriticality or first order, the electroclinic response of the tilt will be unusually strong. In the case of a system that has a zero-field first-order Sm-A′–Sm-C′ transition, the electroclinic response tilt will exhibit a jump. Thus, as with the zero-field features of de Vries materials, our model indicates that the strong electrical response is a result of a combination of small orientational order and strong layering order.

The equation governing the response of the tilt is completely analogous to that derived by Bahr and Heppke to describe a field-induced critical point near a Sm-A′–Sm-C′ transition [17]. However, our derivation of the response equation from a more basic generalized Landau theory allows us to incorporate the effects of the layering and orientational orders, which we can in turn relate to the strength and nature of the tilt response. In addition, it also allows us to derive the electroclinic response of the orientational order (and thus, birefringence) and the layer spacing. We find that the change in birefringence scales quadratically with the electrically induced tilt. This means that an unusually strong tilt response implies an unusually strong response of the birefringence, as is the case in de Vries materials. The quadratic scaling is also consistent with experiment. Similarly, we find that the electrically induced change in layer spacing also scales quadratically with tilt, although the scaling is also proportional to orientational order.
Thus, the theory predicts that a system with small orientational order and strong layering order will exhibit a combination of strong electro-optic response (in both reorientation of the optical axis and change in birefringence) and small layer change. Such a combination is technologically desirable for ferroelectric liquid crystal (FLC) based liquid crystal devices.

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APPENDIX: FIELD-INDUCED CORRECTIONS TO THE ORIENTATIONAL ORDER AND TO THE LAYERING WAVE VECTOR

In this appendix we provide further details of the method by which we find the fractional changes $\Delta f_c$ and $\Delta q$, to the orientational order and to the layering wave vector, respectively, due to the application of an electric field in the Sm-A* phase. This is done near a Sm-A*-Sm-C* transition (continuous or first order) that is close to tricriticality.

1. Correction to the zero-field orientational order

As discussed in Sec. V A we are interested in finding the correction to the zero-field value of the orientational order $M_{E=0}$. This zero-field value already includes the increase due to the zero-field coupling of orientational order to layering.

In the zero-field Sm-A* phase the tilt is zero and the zero-field value $M_{E=0}$ was found [18] by analyzing the part of the free energy that does not include tilt, i.e., $f_{c0}=f_{E=0}+f_M$. Specifically, we Taylor expanded $f_{c0}(M)$ about $M_0$, the value of the orientational order in the absence of coupling to layering, i.e., the value that minimizes $f_M$. This gave

$$f_{c0}=f_{c0}(M_0)+f_M'(M_0)(M-E_0-M_0)+\frac{1}{2}f_M''(M_0)(E_0-M_0)^2,$$  

where $f_M'(M_0)=df_M'(M_0)/dM$ and $f_M''(M_0)=df_M''(M_0)/dM$. We have neglected the term $\alpha f_E'(M_0)$ which contributes terms higher order in coupling compared to $f_M'(M_0)$. Minimization of the above $f_{c0}$ then gave $M_{E=0}=M_0$, $f_M'(M_0)$.

When a field is applied to the Sm-A* phase, a tilt is induced and the tilt-dependent part of the free energy becomes nonzero. Thus, to find the correction to $M_{E=0}$, we Taylor expand the full free energy $f=f_{c0}+f_c+f_{EC}$ about $M_{E=0}$. Doing so gives

$$f\approx f_{c0}(M_{E=0})+f_c'(M_{E=0})M_{E=0}+f_{EC}'(M_{E=0})M_{E=0} \Delta M_E,$$

where $\Delta M_E(M_{E=0})=f_M'(M_{E=0})$, $f_{c0}'(M_{E=0})=df_{c0}/dM$, $f_{EC}'(M_{E=0})=df_{EC}/dM$, and $f_{c0}''(M_{E=0})=df_{c0}/dM^2$. As above, we neglect the term $\alpha f_E'(M_{E=0})$ which contributes terms higher order in coupling compared to $f_M'(M_{E=0})$. Minimization of $f$ now gives

$$\Delta M_E(E)=-\frac{[f_c'(M_{E=0})+f_{EC}'(M_{E=0})]}{f_{c0}''(M_{E=0})}.$$

Keeping only terms to lowest order in coupling coefficients, $f_{c0}''(M_{E=0})=f_M''(M_0)=\gamma_M$. The dependence of $\Delta M_E(E)$ on $E$ enters via the dependence of $(f_c+f_{EC})/E$ on $E$ and $c(E)$. Since we seek to relate the correction $\Delta M_{E}(E)$ to $c(E)$, it is useful to express $(f_c+f_{EC})$ in terms of $c(E)$ and not $E$ explicitly. This can be achieved using Eq. (13) for $E$ in terms of $c$, giving

$$f_c+f_{EC}=-\frac{1}{2}r_c c^2(E)-\frac{1}{4}q_c c^4(E)-\frac{1}{6}v_c c^6(E).$$

To obtain $\Delta M_E(E)$, as given in Eq. (A2), we must differentiate $f_c+f_{EC}$ with respect to $M$ which enters via the coefficients $r_c(M,q^2)$, $u_c(M,q^2)$, and $v_c(M,q^2)$. These coefficients were introduced after Eq. (8), but it is convenient to present them again,

$$r_c(M,q^2)=3a(q^2)q^2|\psi|^2/M\tau(M,q^2),$$

$$u_c(M,q^2)=9\mu(M,q^2)hq^2|\psi|^2M^2,$$

$$v_c(M,q^2)=\frac{81}{4}sq^2|\psi|^2M^3,$$

where $a(q^2)$, $\tau(M,q^2)$, and $\mu(M,q^2)$ are given by

$$a(q^2)=a_0+a_1(q^2-q_0^2),$$

$$\tau(M,q^2)=1-\frac{b|\psi|^2+\alpha g+2hq^2q^2}{a(q^2)},$$

$$\mu(M,q^2)=1-\frac{g}{2hq^2}\left(\frac{wM}{gq^2|\psi|^2}-1\right)^{-1}.$$

Differeniating Eq. (A4) with respect to $M$, inserting the result into Eq. (A3) and keeping terms to lowest order in couplings, $t$ and $\mu_Ac$, i.e., close to a Sm-A* -Sm-C* transition which is close to tricriticality, we find

$$\Delta M_E(E)=-\frac{3m}{2\gamma_M}gq^2|\psi|^2\left[1-\left(\frac{c(E)}{c_M}\right)^2+\left(\frac{c(E)}{c_M}\right)^4\right]c(E),$$

where $m=1+\frac{2hc}{g}$ is a dimensionless constant, and

$$c_M=\left(\frac{2g}{3hq^2}\right)^{1/2},$$

$$c_M=\left(\frac{4mg}{27M_{E=0}q^4}\right)^{1/4}. \quad \text{(A8)}$$

If $g$ is of the same order or larger than $hq^2$ then $c_M$ is $O(1)$.

This is not unreasonable since $g$ and $h$ are both coupling constants, which we take to be small and of the same order. Similarly,
for $M=M_{TC}$, which is of the same order as the coupling constants, $c_{MM}\gg 1$. Thus, for the small $c$ values assumed for our theory and observed experimentally, the $(\frac{c_{MM}}{c_{MM}})^2$ and $(\frac{c_{MM}}{c_{MM}})^4$ contributions are small and the scaling of $\Delta M(E)$ with $c(E)$ is quadratic. Note that in going from Eqs. (A7) and (24) we omit the $\delta(E)$ term and replace $M$ with $M_{E=0}$.

2. Correction to the zero-field wave vector

In this part of the appendix we present details of our analysis of the fractional change, $\Delta_{q_E}(E)=\frac{q_{E=E_0}}{q_{E=0}}-1$, in $q^2$. As with the orientational order, we are seeking the correction to the zero-field value $q_{E=0}^2$ which already includes the correction due to the zero-field coupling of orientational order to layering. The method we use to obtain $\Delta_{q_E}(E)$ is completely analogous to that used above to find $\Delta_{M}(E)$. Taylor expanding the free energy $f$ about $q_{E=0}^2$ and minimizing with respect to $\Delta_{q_E}(E)$, we find

$$\Delta_{q_E}(E) = -\frac{f'_E(q_{E=0}^2) + f''_E(q_{E=0}^2)q_{E=0}^2}{f''_E(q_{E=0}^2)},$$

(A9)

where $f'_E(q^2)=\frac{df_E}{dq^2}$, $f''_E(q^2)=\frac{d^2f_E}{dq^2}$. Keeping only terms to lowest order in coupling coefficients, $f''_E(q_{E=0}^2)\approx f''_E(q_0^2)=K|\phi|^2$. We again use Eq. (A4) for $f'_E$ but now we are interested in the $q^2$ dependence of the coefficients $r_n(M, q^2)$, $u_n(M, q^2)$, and $u_t(M, q^2)$, which are given by Eqs. (A5) and (A6). Differentiating Eq. (A4) with respect to $q^2$, inserting the result into Eq. (A4) and keeping terms to lowest order in couplings, $t$ and $\mu_{AC}$, i.e., close to a Sm-A′−Sm-C′ transition which is close to tricriticality, we find

$$\Delta_{q_E}(E) = \frac{3|a_1|}{2K}M \left[ 1 + \left( \frac{c(E)}{c_E} \right)^2 - \left( \frac{c(E)}{c_{M1}} \right)^4 \right] c^2(E),$$

(A10)

where $m=1+\frac{3m_2}{g}$ is a dimensionless constant, and

$$c_E = \left( \frac{|a_1|}{3gq_{E=0}M} \right)^{1/2},$$
$$c_{M1} = \left( \frac{4|a_1|}{27M^2s^4} \right)^{1/4}.$$ (A11)

For $M=M_{TC}$, which is of the same order as the small coupling constants, both $c_{M}$ and $c_{M1}$ are $\gg 1$. Thus, as with the correction to orientational order, for the small $c$ values assumed for our theory and observed experimentally, the $(\frac{c_{MM}}{c_{MM}})^2$ and $(\frac{c_{MM}}{c_{MM}})^4$ contributions are small and scaling of $\Delta_{q_E}(E)$ with $c(E)$ is quadratic. Note that in going from Eqs. (A10) and (24) we omit the $\delta(E)$ term and replace $q$ with $q_{E=0}$.

[9] For experimental data showing the tilt response $\theta(E)$ and birefringence response $\Delta n(E)$ in a system with a continuous Sm-A′−Sm-C′ transition, see, for example, Ref. [6].
[10] For experimental data showing the tilt response $\theta(E)$ and birefringence response $\Delta n(E)$ in a system with a first-order Sm-A′−Sm-C′ transition, see, for example, Ref. [12].
[20] It should be noted that here, and throughout this article, exponents are calculated within mean-field theory, and do not include the effects of fluctuations. For example, it is known that when fluctuation effects are included in analysis of the three-dimensional $XY$ transition, $\theta$ scales like $(1-\frac{\beta}{T_c})^8$, with $\beta=0.35$, whereas in mean-field theory $\beta=0.5$. The use of mean-field theory is justified by the fact that virtually all continuous AC transitions are observed to be mean field like.
[21] We deviate slightly from the notation used in [18], dropping the subscript 0 on the $a, b, g, h, \mu$ and $s$ coefficients. We also use $r_n$ and $r_s$ instead of $t_n$ and $t_s$, for the coefficients of $M^2$ and $|\phi|^2$. 

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respectively. This is done to avoid confusion with the reduced temperature $t$.


[24] Of course, different materials will also have different values of $\kappa$. It is not clear how strongly these parameters will vary from material to material.


[26] In a separate analysis, which we do not present here, we have found that the layering order at the Iso-Sm-A transition is indeed larger in systems with smaller orientational order. However, we have also found that for a given decrease in orientational order, the increase in layering required at the Sm-A–Sm-C transition was larger than the increase in layering at the Iso-Sm-A transition. This would imply that a wider temperature window for the Sm-A phase will result as the orientational order is reduced.

[27] Assuming $P \approx z \times \epsilon$ reflects the fact that the polarization $P$ is a secondary order parameter, with the tilt $\epsilon$ being the primary order parameter. However, the relationship between $P$ and $\epsilon$ may be more complicated than the linear one assumed here. See S. Dumrongrattana, C. C. Huang, G. Nounesis, S. C. Lien, and J. M. Viner, Phys. Rev. A 34, 5010 (1986); T. Carlsson, B. Zeks, A. Levstik, C. Filipic, I. Levstik, and R. Blicci, ibid. 36, 1484 (1987). With a view to making our analysis manageable, we avoid introducing another order parameter $P$ and assume the simplest linear relationship.


[30] It should be noted that in the Landau theory of [17], there was a coupling between tilt and polarization. This coupling leads to an innocuous renormalization of the continuous Sm-A$^+$/Sm-C$^-$ transition temperature.
