Temporal Topos and U-Singularities

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Abstract

Several papers and books by C. Isham, C.Isham-A. Doering, F. Van Oystaeyen, A. Mallios-I. Raptis, C. Mulvey, and Guts and Grinkevich, have been published on the methods of categories and sheaves to study quantum gravity. Needless to say, there are well-written treatises on quantum gravity whose methods are non-categorical and non-sheaf theoretic. This paper may be one of the first papers explaining the methods of sheaves with minimally required background that retains experimental applications.

Temporal topos (t-topos) is related to the topos approach to quantum gravity being developed by Prof. Chris Isham of the Oxford-Imperial research group (with its foundations in the work of F. W. Lawvere). However, in spite of strong influence from papers by Isham, our method of t-topos is much more direct in the following sense. Our approach is much closer to the familiar applications of the original algebraic geometric topos where little logic is involved.

The distinguishable aspects of this paper "Temporal Topos and U-Singularity" from other topos theorists' appoaches are the following. For a particle, we consider a presheaf associated with the particle. By definition, a presheaf is a contravarinat functor; however, in the t-topos theory, such a presheaf need not be defined for every object of a t-site over which the topos of presheaves are defined. When such an associated presheaf is not defined (or non-reified), we say that the presheaf (the particle) is in ur-wave state. Therefore, the duality is already embedded in our t-topos theory. We also have the notion of a (micro) decomposition of a presheaf (a particle) to obtain microcosm objects. Another important aspect of our approach is the associated space and time sheaves for a given particle-presheaf. The sheaves associated with space, time, and space-time are treated differently from a particle associated presheaf. Namely, Yoneda Lemma and its embedding are crucial for formulating and capturing the nature of space-time. In this formulation, the space and time sheaves would not exist unless a particle (presheaf) exists. Such a non-locality nature as the EPR type non-locality is also embedded in t-topos. Applications to singularities (a big bang, black holes, and subplanck objects) are formulated in terms of universal mapping properties of direct limit and inverse limit in category theory. Furthermore, the uncertainty principle is formulated through the concept of a micro-morphism in t-site. Our t-topos theoretic approach enables us to formulate a light cone in macrocosm and also in microcosm. However, such a light cone in microcosm has non-reified space-time regions because of the uncertainty principle (a miro-morophsim).

Prologue

Introducing a categorical approach to quantum field theory will avoid divergent expressions, e.g., for the total amplitude of a quantum process. One may also take categorical and sheaf theoretic methods as avoidance of the Dedekind-Cantor continuum approach to physical entities. The Dedekind-Cantor type continuum is one of the sources of infinites in physical theory.

The concept of a sheaf has been effectively used for the foundations of quantum physics and quantum gravity especially among people in the C. Isham school at Imperial College as in [1], [2], [3], Mallios' school as in [4], [5], and Penrose as twistor cohomology of sheaves in [6]. Even though direct connections to our temporal topos method are not known, a few names should be mentioned: Mulvey, Heller, and Sasin. In particular, the noncommutative geometry approach, called virtual topology of F. Van Oystaeyen, seems to be quite relevant to our work (See the treatise *Virtual Topology and Functor Category*, Tayler and Francis Group, 2007).

See [7], [8], [9] for developments and the history of sheaf theory in the theory of holomorphic functions in several complex variables, algebraic analysis, and algebraic geometry.

In this article We will summarize what we have obtained in the series on the fundamentals of the theory of temporal topos following [10], [11], [12]. Our method of temporal topos, referred to as t-topos for short, differs from Isham' s and Mallios' schools, and also from the Russian school directed by A. K. Guts and E. B. Grinkevich. However, we should acknowledge the motivational influence coming especially from the paper [1] by C. Isham. As we have mentioned earlier, compared with other approaches to quantum gravity via sheaves, our method is a more direct and straightforward application of commonly used familiar algebraic geometric (categorical-cohomological) methods. That is, in order to express the changing state of a particle over a time period, the associated presheaf representing the particle is "parameterized" by an object in a t-site. We call such an object in t-site a *generalized time period*. Namely, we introduce such a state controlling parameter as a generalized time period-object in the t-site to keep track of varying states of a particle. (See below for more on t-site.)

Our goals include studying the topos of presheaves (t- topos) defined on a t-site and its applications to quantum gravity. However, in t- topos theory, a presheaf is not always defined on every object in a t-site. When it is defined, a presheaf in t-topos theory satisfies the properties of a contravariant functor. This is one of the issues relevant to the Kochen-Specker theorem in [2] and [3]. The t-topos theory is a background independent theory and also a scale independent theory * (See (*) below.) in the following sense: all the concepts are defined in our theory in terms of presheaves associated with a macro or micro particle together with the associated space, time, and space-time sheaves. For a particle state in the usual sense, we associate a presheaf m so that each particle state of the particle is represented by the reified pair of the presheaf m and an object V (which is called a generalized time period V) in a site S. [At the Second International Conference on Theoretical Physics and Topos, held at Imperial College, London, 2003, (*) C. Isham said (In the definitions in t-topos theory) " --- a particle can be replaced by an elephant."] Such a site as used in t-topos theory is called the t-site.

Recall that a site in general is a category with a Grothendieck topology as defined in [9], [14], [15]. An ur-particle state of the presheaf *m* associated with a particle is expressed as *m(V)* as an object in a product category

$$\prod_{\alpha \in \Delta} C_\alpha . \qquad (0.0)$$

(See [10], [11], [14].) One of the reasons for introducing the product category indexed by a finite set is that for each physical quantity possibly measured, we need a category where such a measurement (interpreted as a morphism in t-topos) can take place. Following the terminology used among topos theorists, the category $\hat{S}$ of presheaves on a site *S* (with a restricted sense as follows) is said to be a temporal topos or simply t-topos. Namely, $\hat{S}$ is the category of contravariant functors from the t-site S to $\prod_{\alpha \in \Delta} C_\alpha$. However, such a t-topos theoretic presheaf is more restricted than the usual definition of a presheaf. That is, $m(V)$ may not be defined for every pair of an object *m* of $\hat{S}$ and an object *V* of *S*.

Definition A presheaf *m*, an object of $\hat{S}$, and a generalized time period *V*, i.e. an object of *S*, are said to be reified (or compatible) when $m(V)$ is defined.

Hence, an object of the t-topos $\hat{S}$ may be more appropriately called an ur-presheaf (or t-presheaf) rather than just a presheaf. Let *m* and *P* be presheaves, i.e., objects of $\hat{S}$. We say that *m* is observed (measured) by *P* over a generalized time period *V* (i.e. an object of the t-site *S*), when there exists a morphism from $m(V)$ to $P(V)$. For a presheaf *m* associated with a particle, there are the space, time, and space-time (pre)sheaves κ_m, τ_m and ω_m associated with *m*. The associated (pre)sheaves with space, time and space-time do not exist without the particle. (See the forthcoming [17] for a complete description of t-topos theory, especially the treatment of space-time sheaf $\omega = (\kappa, \tau)$.)

Also recall that a presheaf *m* is said to be in a *particle ur-state (or ur-particle state)* if there exists an object *V* in *S* such that $m(V)$ is defined. Otherwise, *m* is said to be in a *wave ur-state* (or ur-wave state). For example, when such an object *V* in the t-site cannot be specified between the two as in the case of double slit experiment, *m* is said to be in a wave ur-state. (See [15] for the application of t-topos to a double slit experiment.) Recall also that *m* and *m'* are ur-entangled when presheaves *m* and *m'* are defined always on the same objects of *S*. (See [10], [16] for connections to EPR type non-locality.)

In this paper, for a presheaf *m* representing a particle and for an object *V* in the t-site, a decomposition of *m* and a covering of *V* play major roles in defining a notion of entropy.

For a presheaf *m* in $\hat{S}$, consider a (micro)decomposition of *m* by subpresheaves m_j:

$$m = \prod_{j \in J} m_j , \qquad (0.1)$$

and let

$$\{V \longrightarrow V_k\} \qquad (0.2)$$

be a covering of V by a family of objects in the sense of [9], [13], [14]. We will define the various concepts of entropies as the numbers of defined (reified) pairs of those m_j and V_k. Among all of the decompositions and coverings of m and V as in (0.1) and (0.2), respectively, we have compatible pairs $m_j(V_k)$. We will define a notion of entropy of the state *m(V)* as a number of such compatible pairs in the next section. For a microdecomposition, see [11].

1 Methods of Temporal Topos

We have introduced notions of a microdecomposition and a micromorphism. For example, the concept of a t-topos theoretic light cone is viewed as a light cone with holes where non-reified states occur. This is because the notion of a micromorphism gives the impossibility of factorization between two states corresponding to two generalized time periods. Together with a microdecomposition and a further refinement of a covering in what will follow, we get similar "unreified" pairs of particle-decomposed presheaves and covering-decomposed objects in a t-site. Such a state as unmatched pairs of particle presheaves and objects in the t-site is considered as an ultra-microcosm, and the state is closer to "u-singularity". Even though the method of t-topos is a more kinematical and qualitative theory, the dynamical aspect is embedded in the space and time sheaves. Namely, space-time sheaf $\omega = (\kappa, \tau)$ is associated with a particle. Hence, for example, when the curvature of space-time $\omega_m = (\kappa_m, \tau_m)$ caused by m (representing a particle with mass) is measured, the fundamental composition principle can be used to assign a value. (See what will follow.) Another view of a dynamical aspect of t-topos is the following. When two particles, represented by presheaves m and m' are close enough to influence space-time in the common "region" of two space-time sheaves, then one can associate the two gravitationally interacting particles with the "product space-time" of the associated space-time sheaf induced by m and m'. (See [17] for details.)

Let a presheaf m associated with a particle be observed twice over generalized time periods V and U. Consider the case where m is observed over V first and then over U. That is, time $\tau_m(V)$ precedes time $\tau_m(U)$ in the usual classical linearly ordered sense. Then there exists a morphism g from V to U in the t-site S. Such a morphism g is said to be a *linearly t-ordered morphism*. Note that not every morphism from V to U in S represents such a linear temporal order in the above sense. This is one of the reasons for introducing the concept of a site rather than just a topological space. Recall that an inclusion is the only morphism, if it exists, between two open sets of a topological space.

Suppose that m is measured (or observed) by P over V, then there exists a morphism s_V from *m(V)* to *P(V)* which is the definition of an observation (or measurement). Categorically speaking, this means that s is a natural transformation (a morphism of functors) from m to P. Then we have the following diagram:

$$\begin{array}{ccc} m(V) & \xleftarrow{m(g)} & m(U) \\ \downarrow & \swarrow_{s_V \circ m(g)} & \\ P(V) & & \end{array} \qquad (1.1)$$

where the composition $s_V \circ m(g)$ in the above diagram should be understood as the measurement of *m(U)* by measuring *m(V)* by *P(V)*. Namely, the image of the composite morphism $s_V \circ m(g)$ is the amount of information *P* can obtain on the future state *m(U)* by measuring the state of *m* over *V*. According to the quantum mechanical way of thinking, the phrase that a particle can *"be"* in several different locations at *"the same time"* has been used. However, such expressions need to be examined more carefully since t-topos gives more precise descriptions of such issues. That is, for a particle to be at a place, an object of t-site must be chosen. Then the particle must not be in a wave ur-state since an object in *S* has been specified. Namely, an expression as "An electron moves from point *A* to point *B* taking all available paths simultaneously" assumes the following. If such an electron were observed in addition to the two states corresponding to *A* and *B,* then there would be a non-trivial factorization of $V \xrightarrow{g} U$, i.e., $g = g_2 \circ g_1$ *via* $\{W\}$ in the t-side, corresponding to *A* and *B*. Then in the diagram

$$\begin{array}{ccc} V & \xrightarrow{g} & U \\ & \searrow_{g_1} \quad \nearrow_{g_2} & \\ & W & \end{array} , \qquad (1.2)$$

$V \xrightarrow{g_1} W$ and $W \xrightarrow{g_2} U$ would become non-trivial linearly t-ordered morphisms. In particular, if such a $V \longrightarrow U$ is a micromorphism, then there does not exist such a proper factorization. The number of such paths between *A* and *B* (linearly t-ordered) are precisely equal to the number of non-trivial factorizations by linearly t-ordered morphisms of $V \longrightarrow U$.

For a given state $m(V)$ of *m* over *V*, assume that there exists an object V' in the t-site S so that $\tau_m(V')$ precedes $\tau_m(V)$. Namely, $V' \longrightarrow V$ is linearly t-ordered. Continue the process to obtain a sequence of objects, generalized time periods, of *S*. That is, we get

$$---\rightarrow V'' \longrightarrow V' \longrightarrow V. \qquad (1.3)$$

A definition of a t-topos theoretic light cone is given in [11]. We will give another definition of a light cone using the presheaf associated with a photon.

Definition 1.1 Let γ be a photon presheaf which is observed over a generalized time period *V*. Then consider all the light cone sequences with respect to the state $\gamma(V)$, or going through *V*

$$---\to \gamma(V_2)\to \gamma(V_1)\to \gamma(V)\to \gamma(V^1)\to \gamma(V^2)---, \qquad (1.4)$$

where

$$---\leftarrow V_2 \leftarrow V_1 \leftarrow V \leftarrow V^1 \leftarrow V^2 \leftarrow --- \qquad (1.5)$$

is an arbitrary sequence of linearly t-ordered objects in *S*. In terms of space-time sheaf, we have

$$---\to \omega(V_2)\to \omega(V_1)\to \omega(V)\to \omega(V^1)\to \omega(V^2)--- \qquad (1.6)$$

associated with γ.

Note that sequence (1.4) emphasizes the states of γ, and sequence (1.6) emphasizes the corresponding space-time.

2 Entropy and Limits

We will define the notion of an entropy for a decomposition as in (0.1) of *m* and for a covering as in (0.2) of *V* of objects in the t-topos $\hat{S}$ and t-site *S*, respectively. Furthermore, we can continue decomposing to get a sequence of decompositions as

$$\prod_{j\in J} m_j \longrightarrow \prod_{k\in K} m_{jk} \longrightarrow ---. \qquad (2.1)$$

Definition 2.1 The t-entropy of the state *m(V)* for a micro-decomposition $\prod_{j\in J} m_j$ and a covering $\{V \longrightarrow V_k\}_{k\in K}$ of *V* is defined by the number of compatible (reifiable) pairs $\{m_j(V_k)\}_{j\in J, k\in K}$.

Definition 2.2 The formal entropy of *m(V)* for the decomposition and the covering is defined by the product of cardinalities of index sets *J* and *K*.

Definition 2.3 The absolute entropy of the state *m(V)* is defined as the maximum number of compatible pairs for all decompositions and coverings of *m* and *V*, respectively.

Note 2.4 Among the compatible pairs in the definitions of entropies, the corresponding generalized time periods need not be linearly t-ordered. Note also that the rest of the pairs between the decomposition and the covering are the collection of non-reified (non-measurable) particle associated presheaves.

Since a covering $\{V_k \longleftarrow V_{k,l}\}_l$ of V_k of a covering $\{V \longleftarrow V_k\}_k$ of *V* is a covering $\{V \longleftarrow V_{k,l}\}_{k,l}$ (See [9, 13, 14].), we get a sequence for V,

$$\{V \longleftarrow V_k\}_k \longrightarrow \{V \longleftarrow V_{k,l}\}_{k,l} \rightarrow ---. \qquad (2.2)$$

Next, we will consider limits of such sequences as in (2.1) and (2.2) and sequences as in (1.5, 1.6).

Definition 2.5 A presheaf m is said to be a *fundamental* presheaf when a decomposition in the sense of the sequence (2.1) becomes stable. That is, further decompositions consist of only several isomorphic objects. Namely, all the components of a decomposition are isomorphic presheaves.

Definition 2.6 An object V of the t-site S is said to be *fundamental* when a covering of V consists of all isomorphic objects to V itself.

Remarks 2.7 (1) A fundamental presheaf should be associated with elementary particles. Such a pair of a fundamental presheaf and a fundamental object of t-site is said to be a *fundamental pair*.

(2) The notion of a direct limit (inverse limit) is defined by a universal mapping property as in [9,13,14]. In this sense, such a notion as a direct (or invese) limit is an ultimate and universal object. Therefore, we propose the following definitions. The direct limit of the sequence (2.1) is said to be an *ur-subplanck decomposition* of m since sequence (2.1) is obtained by decomposing each presheaf in each step. Similarly, the direct limit of sequence (2.2) is said to be an *ur-subplanck covering* of V.

(3) As such a decomposition in (2.1):

$$\prod_{j \in J} m_j \longrightarrow \prod_{k \in K} m_{jk} \longrightarrow ---$$

approaches the direct limit of this sequence, more fundamental presheaves appear. Then a fundamental presheaves m_α associated with short-lived particles cause severe curvatures of space-time in microcosm. Note that being short-lived means the smallness of the assigned value via *FUNC* of the corresponding time sheaf $\tau_{m_\alpha}(V_{\underset{\rightarrow}{\lim}})$ evaluated at the corresponding $V_{\underset{\rightarrow}{\lim}}$ after sufficient refinements of the covering with more fundamental objects of S:

$$\{V \longleftarrow V_k\}_k \longrightarrow \{V \longleftarrow V_{k,l}\}_{k,l} \rightarrow --- \;)$$

as in (2.2). Note also that the entropy of such a condition is also small, because the number of reified fundamental pairs of presheaves and objects of S decreases, i.e., there are more isomorphic objects in $\hat{S}$ and S, respectively. One might associate this t-topos theoretic interpretation of the ultra-microscopic state of short-lived fundamental presheaves $\{m_\alpha\}_\alpha$ and fundamental objects $\{V_{\underset{\rightarrow}{\lim}}\}$ of the t-site (i.e., generalized time periods) with the foam-like condition. For the connections to other singularities, see Remark 3.1(2) in the next section and the Epilogue, where we introduce the notions of ur-big bangs of the 0^{th} stage and $(-1)^{st}$ stage, respectively.

3 U-Singularities

Let be m_Ω a fundamental presheaf of $\hat{S}$. Assume that m_Ω was observed at a generalized time period V of S. Then we can consider such a situation as we have considered earlier. Namely, for this given state of m_Ω over V, i.e., $m_\Omega(V)$, assume that there exists an object V' in the t-site S so that $\tau_{m_\Omega}(V')$ may precede $\tau_{m_\Omega}(V)$. Namely, $V' \longrightarrow V$ is linearly t-ordered. Continue this process successively to obtain a sequence of objects, i.e., generalized time periods of S. That is, as we evaluate at the (contravariant) fundamental presheaf m_Ω, we get the following sequence

$$---\leftarrow m_\Omega(V'')\longleftarrow m_\Omega(V')\longleftarrow m_\Omega(V). \qquad (3.1)$$

The direct limit $m_\Omega(V_{\underset{\leftarrow}{\lim}}) \overset{def}{=} \underset{\longrightarrow}{\lim}(---\leftarrow m_\Omega(V'')\longleftarrow m_\Omega(V')\longleftarrow m_\Omega(V))$ of (3.1) is said to be the *inverse u-singularity* of the state $m_\Omega(V)$. On the other hand, in Remark (3), $\{m_\alpha(V_{\underset{\rightarrow}{\lim}})\}$ is said to be the *direct u-singularity* of the state $m(V)$. By the very definition of a direct limit, the corresponding space-time $\omega_{m_\Omega}(V_{\underset{\leftarrow}{\lim}})$, and in particular $\tau_{m_\Omega}(V_{\underset{\leftarrow}{\lim}})$, can not be preceded by any usual classical time. We are assuming the existence of a particle hich has survived from the earliest universe. As a candidate for such a particle represented by m_Ω, we can consider cosmic background radiation.

At the inverse and direct u-singularity states, only isomorphic fundamental presheaves and fundamental generalized time periods (appearing in the direct limit of a covering sequence as in (2.2)) are available to be reified.

Remarks 3.1 (1) One may like to associate the "Ancestor's Rule" with the further decompositions of a given state in (2.1) to obtain (or to reach) fundamental presheaves in the following sense. Each individual has 2^n ancestors in the n generations back. After a certain number of generations, individuals have only several common fundamental ancestors.

(2) The inverse u-singularity and the direct u-singularity correspond to a big bang type singularity and to a microcosm quantum fluctuation, respectively. Notice that the definition of the inverse u-singularity is defined as the direct limit of the sequence (3.1) induced by the linearly t-ordered sequence of generalized time periods (i.e., objects of the t-site). Namely, it is possible to have reified pairs of fundamental presheaves and fundamental generalized time periods *without* having linearly t-ordered fundamental objects *beyond* the inverse u-singularity. That is,

beyond the inverse u-singularity, there is no distinction of the time notion of "before and after" in the linearly t-ordered sense and hence also in the classical sense. One might phrase such a state as: possible reified pairs simply exist without the past-future notion. Then note also that space sheaf κ and time sheaf τ can not be distinguished since there is no particular role difference without a linearly t-ordered concept. Further beyond such a state as no past-future, our t-topos approach does not tell whether there can exist such a situation as no reified pairs of fundamental presheaves and the t-site objects at all or not.

Epilogue

Our basic approach toward quantum behavior of a particle (elementary or not) is to capture an ur-particle state as a reified pair of the associated presheaf m and an object V of t-site. Generally speaking, for a macrocosm presheaf, there are more decompositions, and for a macrocosm covering, there are further refinement coverings. The t-topos theory is a scale independent theory not only because all the concepts are defined independently of the scales, but also in the following sense. For example, for a given morphism in the t-site, the shorter the sequence of the factorization of the morphism is, the more microscopic the morphism. A similar statement can be asserted for a presheaf.

When presheaf m does not have an object to be reified, m is said to be in an ur-wave state. This ur-wave state includes the case of the double slit experiment because such a choice of one object of the t-site can not be determined. When applied to the notion of a light cone of a particle in a microcosm, such a t-topos theoretic microscopic light cone is a light cone with missing states where the associated particle presheaves do not have objects from t-site to be reified. One of the missing elements in our approach to t-topos is the aspect of dynamics. However, in the t-topos theory, there is a notion for such relativistic dynamics in terms of the space and time presheaves depending upon a particle (locally defined). For a full-scale description of the space-time sheaf as a measuring device of a particle, see the forthcoming [17]. However, further study is needed to develop the t-topos theory to treat more applications. The development of t-topos methods is still at the early stage. The t-topos aspect of the time delay effect, for example near a black hole, is yet to be formulated. In the near future, our plan is to investigate the t-topos theoretic interpretations of Hawking radiation and quantum tunneling. See our forthcoming papers, e.g., [17]. Our theory may belong to a hidden variable approach (with direct experimental applications) as indicated in [19].

A similarity between a back hole type singularity and a big bang type singularity is the concept of u-singularity, i.e., the categorical notion of a limit (inverse or direct). Namely, for a compatible pair of a presheaf and an object (generalized time period) of the t-site, a quantum fluctuation type singularity is described as limits of microdecompositions of the given presheaf and of micro coverings of the object of the t-site. Meanwhile, a big bang type singularity is given as a limit of a linearly t-ordered sequence for such a compatible object of the t-site with an arbitrary fundamental presheaf. In Remark 3.1, we have considered that fundamental pairs may exist, but no linearly t-ordered time, and furthermore, the totally incompatible (non-reified) state of fundamental presheaves and fundamental objects of the t-site. We may call such states the *ur-big bang* of the 0^{th} *stage* and the *ur-big bang* of $(-1)^{st}$ *stage*, respectively. The ur-big bang of stage $(-1)^{st}$ is the "unmatched melting pot" of presheaves of t-topos and t-site objects without any compatible pairs. Notice that there are similarities between the singularity type of a big bang and ultra microcosm in the sense of the direct u-singularity in Remark 2.7. The difference is also clear as well. The inverse u-singularity is induced by a linearly t-ordered sequence, but the direct u-singularity is induced by the coverings. However, at the level of ur-big bang state of the 0^{th} stage and the subplanck covering level, there is a similarity since the both cases are consisting of fundamental pairs without the usual space-time notion. Note also that some results from particle physics may tell us how many fundamental objects are in t-topos and t-site and in particular at the big bang. In order to make the t-topos theory into a quantitative theory, we may be able to use the so called the fundamental composition principle as in [2] and [3] for V defined for an operator in a Hilbert space H corresponding to a physical quantity. Namely, the following diagram consisting of the vertical morphism of Hilbert space H induced by a function from $\mathbb{R}$ of real numbers to $\mathbb{R}$

$$\begin{array}{ccc} H & \longrightarrow & \mathbb{R} \\ \downarrow & & \downarrow \\ H & \longrightarrow & \mathbb{R} \end{array}$$

is commutative. See [2], [3], [10] for details. For the mathematical foundations for t-topos theory, see the forthcoming [17]. In this paper, sheaf cohomology per se does not appear. However, sheaf cohomology via coverings is crucial for Penrose's work as mentioned in Prologue. The papers [4] and [5] by Mallios and Rptis, De Rham cohomology, i.e., the hypercohomolgy with coefficient in the complex of sheaves of differential forms, plays an important role. The methods of sheaf cohomology also appears in [20]. Namely, in order to obtain the Veneziano amplitude, Volovich's p-adic string theory requires the computation zeta function obtained from the 1_{st} p-adic cohomology group of Fermat curve over a finite field in characteristic p. (See the references in [20].) More general treatments of cohomologies can be found in [9] and [14]. As for cohomologies of sheaves for physics, see the forthcoming [17].

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