

# A Simple Lab Project Integrating Theoretical, Numerical, and Experimental Stress Analysis

## Abstract

Learning is enhanced when students consider problems from different perspectives. Unfortunately, in stress analysis courses, the depth of the mathematical analysis and limited time and resources often restricts the focus to traditional closed-form solutions occasionally supplemented with simple demonstrations. In order to enhance student engagement and understanding, a lab mini-project was developed for teaching Castigliano's method for structural analysis in a stress analysis course.

The mini-project consists of a design evaluation task which is investigated using three different methods: closed-form analysis, finite element analysis, and simple model build and test. The task is to select the better of two alternative support structures for a heavy-duty material-handling conveyor belt. Acceptance criteria in the form of maximum deflection and stress are provided. The closed-form analysis is conducted using Castigliano's method. Beam and shell finite element models are built and analyzed in Abaqus CAE. Structural prototypes are constructed with PASCO Structures System components and tested with simple weights and scales.

The strengths of this combined approach are that the students (a) gain experience with the three different methods of stress/deflection analysis, (b) compare the different methods on a single problem, and (c) check or confirm their own results. By using existing finite element software licenses and available PASCO components, the project took no additional lab time and no additional cost to implement.

Since the first use of this lab project was with in a small class, no direct measures were used to capture improvements in student learning. Instead, Indirect measures (instructor observations, student comments) were used to evaluate the lab's effectiveness. During the build and test phase of the project, students were much more engaged (discussing, changing approach) than in prior years when only the closed-form analysis was included. Student closed-form analysis results were also more complete, with fewer (20% versus 40%) of the student groups forgetting to include parts of the structure in their Castigliano's analysis. On end-of-course evaluation forms, students commented that they enjoyed comparing results between the three methods, and appreciated having something physical to test rather than just performing calculations.

## Introduction

Student conceptual learning can be significantly enhanced by providing opportunities for students to see and feel the effects of their theoretical analysis.<sup>(1)</sup> This means more than just enhanced computer-based presentations. It means hands-on (physical or virtual) projects where the students get to directly test the theories given in textbooks and lectures.

At our university, a quarter-long mechanical engineering senior technical elective course covers intermediate topics in stress analysis, including energy methods (Castigliano's and Rayleigh-Ritz), torsion of non-circular cross-sections, axisymmetric problems, plates, shells, and

introduction to elasticity. Given the time constraints and the depth of the covered topics, this course has typically covered only the analytical methods for solving stress analysis problems. Although feedback from the students has been generally positive, some did lament the fact that they spent a lot of time with calculations and would have liked to ‘see’ the results on something ‘real.’ In partial response to these comments, simple numerical and experimental tasks have been added to the course in recent years. This paper describes one of the resulting labs, which combined analytical, experimental, and numerical work on a single design analysis task.

At different universities, a variety of techniques have been proposed to enhance student learning and engagement in theoretical analysis courses. Some examples are:

- Create integrated analysis/experimental and/or analysis/numerical courses. In many ways this would be optimal, as it shows how well the techniques support each other. However, to give the students time to learn about all three approaches would necessitate a large number of course units, severely restricting the students’ other course options. Alternately, this approach is applied in a survey course, where detailed content is left out.
- Require or suggest analysis, experimental, and numerical courses be covered sequentially. This option means the students see all approaches, but the time lag means many of the benefits would be lost.
- Integrate limited experimental and numerical tasks into an existing analytical course.

This approach has been implemented in a few different ways:

- Incorporate virtual labs so students can “see” the results of their analysis, but do not need to set up or actually perform the experiments. Such techniques are often used at smaller schools where funds and space for experimentation are limited.<sup>(2)</sup><sup>(3)</sup><sup>(4)</sup> If not well implemented, however, this approach may backfire.<sup>(5)</sup>
- Incorporate simple experimental setups that allow students to see the results, but require minimal setup. Although these require some space and setup time, they can often be performed with minimal cost.<sup>(6)</sup><sup>(7)</sup><sup>(8)</sup><sup>(9)</sup>
- Incorporate simple numerical modules that allow students to compare results. Depending on how much the students are expected to learn about the numerical tool, this can be a simple walk-through tutorial or a more complicated analysis where the students build, analyze, and post-process the model.<sup>(10)</sup><sup>(11)</sup>
- Integrate both numerical simulations and simple experimental setups to allow students to analyze, simulate, and experiment with the same system.<sup>(12)</sup><sup>(13)</sup><sup>(14)</sup>

In all of these cases, the instructor is forced to make decisions about which experiences will provide the best learning opportunities for students. This paper presents one way to integrate closed-form analysis, simple experimentation, and numerical simulation to aid student learning. In contrast to previously published work, the present lab focuses on deflection, rather than stress analysis, and uses simpler components to enable greater deflection visualization.

## Project Overview

The key learning objectives for this lab mini-project are:

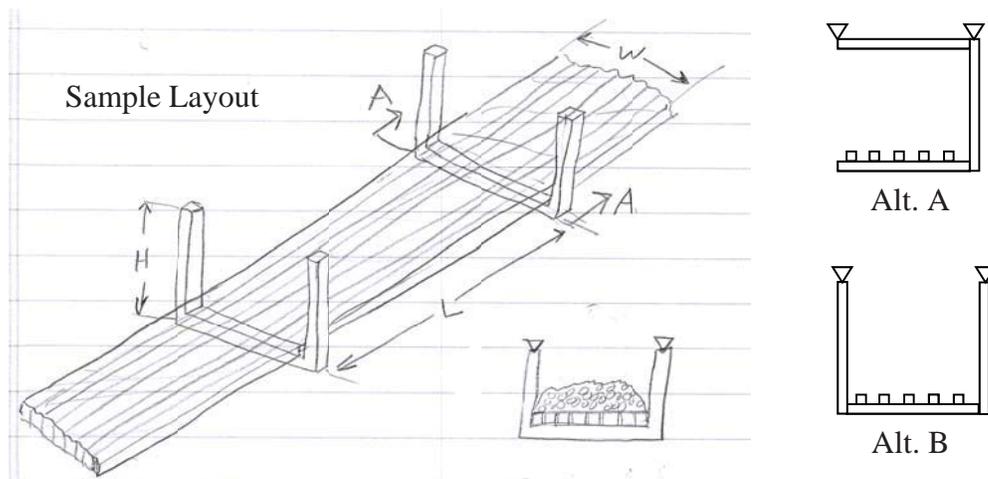
- Develop an accurate usable mathematical model of a physical structure.
- Recognize and use symmetry to simplify a problem.
- Determine strain energy contributions within a structure.
- Explain how local deformations contribute to global deflection of a structure.

- Use Castigliano's method to calculate redundant forces and deflection of a structure.
- Perform simple beam and 2-D solid stress analyses with a commercial FEM code.
- Compare results of analytical, experimental, and numerical analyses of structures.
- Use both stress and deflection predictions to specify the design of a structure.

Two alternative suspended conveyor systems have been designed to transfer processed material in a recycling plant. The vertical support columns will be hollow (1.0-mm gage) 15-mm square tubes, and may be considered attached via pin joints to the ceiling above. The supporting cross-beams (length  $W$ ) are also hollow (1.0-mm gage) 20-mm square tubes. Five parallel hollow (1.0-mm gage) 10-mm square tubes provide support for the conveyor belt between the supports. Dimensions are  $H = 500 \text{ mm}$ ,  $W = 500 \text{ mm}$ ,  $L = 1.0 \text{ m}$ . The material for all parts will be AISI 1010 HR steel ( $E = 207 \text{ GPa}$ ,  $S_y = 180 \text{ MPa}$ ).

As a stress analyst for the conveyor manufacturer, you are responsible for confirming the viability of proposed designs. Your supervisor has just assigned you the task of analyzing these two alternatives. The design criteria, when the conveyor is transporting a uniformly distributed load of  $p = 10\text{-kN/m}^2$ , are:

- 1) No yielding ( $n_d = 1.3$ )
- 2) No deflection greater than 10 mm ( $n_d = 1.2$ )



Analyze both alternative support designs (Alt A and Alt B) and select one for further development.

**Figure 1: Lab Mini-Project Problem Statement**

The objectives are accomplished by having the students work in teams of two to complete a design analysis problem by choosing between two alternatives. With specific stress and deflection criteria, the students are asked to analyze the two alternatives and choose one for further development (Figure 1). The project description goes on to suggest that each team:

1. Develop and test simple models of each structure using the PASCO Structure System,
2. Complete a theoretical (Castigliano's) closed-form analysis of the structures, and
3. Perform finite element analyses of the two structures.

These steps are described in the following sections.

In the original course format, the analysis of this problem would take two lab periods (six hours of focused time over two weeks), and a separate tutorial was used to give the students experience with finite element analysis. In the revised format, the first three lab periods (nine hours over three weeks) were used to perform the experiment, analysis, and FEA on this single problem. The course generally has 30 students, although when this lab was introduced, there were only 12 enrolled. Because of the small number of students, only qualitative results of the change were captured.

### Part 1: The Experiment

The experimental part of the mini-project takes place during the first 90 minutes of the first lab of the course. In order to give the students the opportunity to see and feel the effects of loading on a structure, they are asked to construct models of both supporting structure designs using simple components.

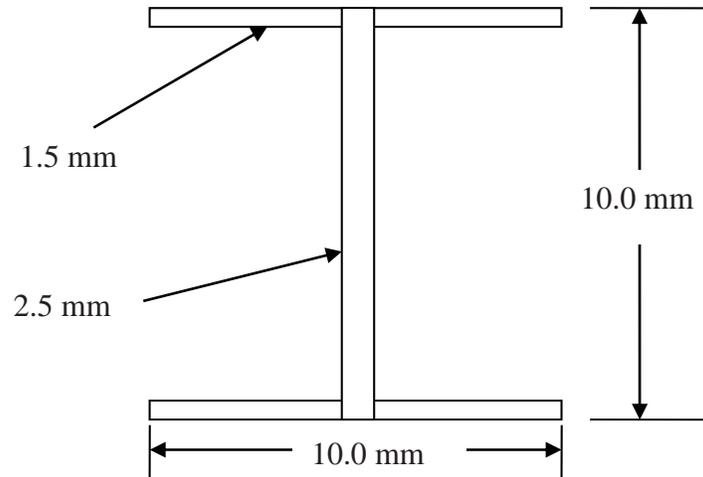
PASCO Structure System elements (Figure 2) were chosen for this purpose since they:

- Are readily available, low-cost structural elements
- Have a predictive, repeatable performance under load
- Are easy to analyze
- Are easily deformable, allowing hand measurements of deflection under load.

PASCO beam members have a constant cross section (Figure 3) and are made of POLYLAC PA-757 ABS with a flexural modulus of 380 ksi.



**Figure 2: PASCO Structures System elements**<sup>(15)</sup>



**Figure 3: PASCO beam cross-section**

While the teams set up their structures, the instructor circulates, asking questions to guide the students, probe their level of understanding, and encourage them to think about alternatives:

- Given the somewhat rigid nature of the joints, does beam length affect how closely this model will represent the final design? [longer beams are more representative as these reduce the influence of the joints in the overall deflection]
- Do you need to model the whole structure or just a portion? [Only one support frame is required to simulate the response. However, two connected by a short length of conveyor is more stable during testing]
- Can you match the exact loading on the structure? If not, how will this affect your results? [Typically, point loads are used for simplified testing. In this case, deformation shape will be similar, but peak deflection is increased. Analysis needs to compensate.]
- How do your boundary conditions affect your results? [If the top connections are held, rather than allowed to rotate, the deflection changes.]
- Look closely at how the structure deforms. Can you see where strain is occurring? [With Alt A, all three members bend. This is visible during testing, and is one of the strengths of using the PASCO Structure System parts!]
- Can you see how deformation in one area contributes to the total deflection at the load? [Discussing how bending of one member leads to rotation of a joint, leading to more deflection lower, really helps students understand why all members must be included in deflection analysis]

Once their structures are completed, students use a number of different free weight plates with attached hooks to simulate the structural loading. Under load, they observe how the structure deforms and measure the maximum deflection (at point of load application). Students are encouraged to check for permanent set (due to slippage of the joints) after the loading, and re-test after tightening the joints.

When measurements are done, the students move to the second part of the project – the closed-form (Castigliano's) analysis.

## Part 2: The Closed-Form Analysis

The closed-form analysis is the one activity traditionally completed in a stress analysis course. For this lab, it still constitutes the most time-consuming part. Students begin working on this during the remaining 90 minutes of the first lab, continue working on it as homework, and spend a second lab period completing it. Although they are required to set up the problems and list all the equations by hand, they are encouraged to make use of computer equation solvers to check their results.

Student teams must complete four distinct analyses for this portion of the project: Alt A & Alt B with PASCO Structures dimensions and properties, and Alt A & Alt B with the final dimensions and properties. However, since only the dimensions and properties change, this approach encourages them to perform their analyses in symbolic form (an important analysis technique), and then substitute values at the end. In lecture, Castigliano's method is developed and used to analyze simple straight and curved beams, then adapted to solving indeterminate structures. It is these approaches that the students practice during the closed-form analysis part of this lab.

The major tasks of the analysis are:

1. Identify symmetry (Alt B) to simplify analysis [analyze just the left or right half, starting from the top attachment].
2. Add a dummy load ( $Q$ ) at the location of maximum deflection [the lower left corner in Alt A and the middle of the lower span in Alt B].
3. Use statics to determine the vertical reactions.
4. Identify one reaction as redundant ( $R$ ) in Alt B [horizontal force on either side].
5. Develop expressions for internal bending moments in all parts of the structure, as a function of the applied loads, dummy load, and redundant reaction load (Alt B):

ALT A:

$$M_{upr} = \left(\frac{pWL}{2} + Q\right)x, \quad M_{upr} = \left(\frac{pWL}{2} + Q\right)W, \quad M_{upr} = \left(\frac{pLx}{2} + Q\right)x$$

ALT B:

$$M_{vert} = Ry, \quad M_{lwr} = RH - \left(\frac{pWL}{2} + Q\right)x + \frac{pLx^2}{2}$$

6. Develop expressions for strain energy in all parts of the structure as a function of the applied loads, dummy load, and redundant reaction load (neglect shear and axial contributions since bending predominates).
7. For Alt B, use Castigliano's theorem (differentiate the strain energy with respect to the redundant reaction load) to determine the unknown reaction (set deflection equal to zero):

$$0 = \frac{RH^3}{3I_{vert}} + \frac{RH^2W}{2I_{lwr}} - \frac{pLHW^3}{16I_{lwr}} + \frac{pLHW^3}{24I_{lwr}}$$

8. Calculate stress in all members of the support structure (neglect shear, but include axial):

$$\sigma = \frac{Mc}{I} + \frac{P}{A}$$

9. Use Castigliano's theorem (differentiate the strain energy with respect to the dummy load) to determine the maximum deflection of the support structure:

ALT A:

$$\delta = \frac{1}{2EI_{upr}} \int_0^w pWLx^2 dx + \frac{1}{2EI_{vert}} \int_0^H pW^3 L dx + \frac{1}{2EI_{lwr}} \int_0^w pLx^3 dx$$

ALT B:

$$\delta = \frac{2}{EI_{lwr}} \int_0^{w/2} \left( RHx - \frac{1}{2} pWLx^2 + \frac{1}{2} pLx^3 \right) dx$$

The students complete these tasks first with the PASCO Structures properties and dimensions, and compare results to their testing. After they have confidence in the results, they repeat the analysis using the actual design properties and dimensions. Next, they move on to the numerical simulations.

### Part 3: The Numerical Analysis

The third, and final, lab period on this mini-project is devoted to numerical analysis of the same two design alternatives previously investigated experimentally and analytically. This replaces a Finite Element Analysis (FEA) tutorial lab that was previously used in the course. FEA is introduced not only as another tool to analyze a structure, but also as an application of energy methods. In lecture, it is shown how the same strain energy used in Castigliano's method is part of the total potential energy of a structure. Minimizing that potential energy leads to the finite element method. Before coming to lab to use the commercial code, students complete Rayleigh-Ritz and simple FEA problems by hand for homework.

During their junior-level design course, students complete an introductory tutorial in the finite element method (FEM) using ABAQUS CAE. The finite element analysis senior elective course also uses ABAQUS as the main analysis tool. As a result, ABAQUS was an ideal choice for this stress analysis lab.

Before analyzing the two alternatives, students first complete another tutorial on stress analysis in ABAQUS by looking at stresses in a redundant beam structure that was previously analyzed in closed form as a homework assignment. A copy of this tutorial is available on the ABAQUS site ([http://www.simulia.com/academics/tutorial\\_pdfs/Tutorials/Cant\\_you\\_see\\_the\\_stress.pdf](http://www.simulia.com/academics/tutorial_pdfs/Tutorials/Cant_you_see_the_stress.pdf)). Following the tutorial, students then construct and analyze beam and 2-D solid models of the two final design alternatives previously analyzed by hand. (Note: Although shell elements are needed to properly model the closed box sections, the skills to develop and analyze models with these elements are best left to a dedicated finite element course.)

The details of building the model will not be discussed here, as they are specific to the commercial code used, and are given in detail on the ABAQUS site referenced above. What is

important is that the students learn to go through the each step of the process by completing two detailed tutorials (one in their junior-level design course, and another in this lab) and then apply what they have learned to analyzing a new structure. Although this assignment could be given as homework, since so many things can go wrong during the development and analysis of a new FEA model, it is best if the students have immediate access to assistance while working on it.

After performing the numerical analysis, the students have now successfully studied the same two structures using three different approaches – experimentation, hand calculations, and numerical simulation. To complete the experience, each team writes a report where they compare and contrast the results of the three methods and discuss the strengths and weaknesses of each. In addition, they reflect on their own feelings about each approach and how they can be used together to offset weaknesses of each individual one.

## Results & Discussion

Although the goal of this project was to improve student learning through greater engagement and direct comparison of alternative methods, direct measures of improvements have not been gathered. Rather, indirect measures, primarily student comments on final evaluation forms and instructor observations during and following the lab, were used.

One of the main observed benefits of combining the three different techniques on a single problem is an increase in student engagement. In contrast to the analysis-only assignment conducted in prior years (where students predict deflection using closed-form analysis of a problem), students in the combined lab were much more active, had more in-depth discussions, and experienced more ‘ah-ha’ moments. Student comments immediately after the lab and, later, in the final course evaluations identified this lab as one of the highlights of the course. Although no quantitative analysis of the improvements was performed, lab analysis results were significantly better (20% of the teams had omission errors versus 40% in prior years), and exam results showed a high level of understanding of the process of predicting structural deflection. In the future, more quantitative evaluation is planned.

When building and testing the two alternatives with PASCO Structures components, the students quickly confirm what their intuition told them: Deflection of the cantilever section (Alt A) is significantly greater than that of the simply supported section (Alt B). However, more importantly, they use the experimental results to observe how the local deformations lead to global deflection. The Alt A design is ideal in this respect, as the students can see the bending of the top beam and discuss how the slope at the end of that beam influences the angle of the lower horizontal member. Similarly, bending of the vertical member can be directly observed and related to the lower member.

Experimentation also enables the students to quickly vary the loading and boundary conditions to see how these affect the local and total deflection of the structure. Many students initially assume that the pin joints at the top of the structure are equivalent to fixed boundary conditions since no loads act on the upper horizontal member. However, when they clamp the two upper attachments and observe the deflection decrease, they have a better understanding of how structures work.

Working through a closed-form analysis of the structures they just built and tested allows the students to build on what they have seen. They can remember to include all parts of the structure in the strain energy calculation since they have seen strain in all parts of the structure. Prior to introducing the experimental portion of the lab, 40% of student teams left out the top horizontal member in Alt A in their final analysis. With the new lab in place, less than 20% of the teams had that error.

By conducting the closed-form analysis on similar structures with different properties, the students can see the benefits of symbolic manipulation. Since they often experience numerical or algebraic errors initially, and corrections are much easier to make with symbolic equations, they can better appreciate why this approach is recommended.

Unfortunately, the rigid joints in the PASCO Structures system are hard to accommodate in the simplified analysis of this lab. As a result, the students' analysis may predict deflection less than half that of the physical parts. While this is somewhat frustrating (and is a point requiring improvement), it can be used to point out the limitations of simplified physical tests (e.g., joints, boundary conditions, loading, measurement error). Basically, the students need to be reminded that the purpose of the simple test is to check the analysis, not the other way around. The test shows if their analysis is in the 'ballpark'.

After completing their closed-form analysis, the ABAQUS FEA usually proceeds smoothly. Although the students may have difficulty with some of the program inputs, they have much more confidence in whether or not the results they obtain make sense. For this lab, the closed-form analysis and ABAQUS predictions are generally within 5%. This encouraging result helps improve the students' confidence in their analysis as well as their appreciation of numerical analysis.

Another key benefit of the combined approach is that the students get to experience three alternative methods of analyzing the same system. They can compare (and are asked to do so in their final report) the strengths and weaknesses of each approach. Given the potential for error in each, they learn that it is good to check your results with another method. By checking their own closed-form results with two other methods, they develop the skills to do their own error-checking in the future.

Students also benefit from directly seeing how the structure deforms. This happens in the FEA, but since they are used to computer simulations showing anything the programmer desires (this is the video game generation!), the physical tests are far more effective at convincing the students that the analysis makes sense. Other benefits are that the mini-project is low-cost to implement and requires no additional time in lab. It builds on their work in earlier design classes, and shows how future classes (finite element analysis) can help them.

There are, of course, some weaknesses to the approach. One of the major ones is the difference between the closed-form analysis and the test results. Although this can be explained in terms of the test limitations and the simplifications of the analysis, it would be more satisfying for the students to see a better match. Another weakness is that this lab takes some more time than that

required for just the closed-form analysis. However, by replacing the FEA tutorial with the FEA on this mini-project, no additional time is required for the two topics.

### Future Enhancements

This mini-project is a work in progress. Based on recent experiences, there are a few changes planned for the next time the course is taught:

- Develop a better set of alternative structures. Currently, students find it is easy to choose between the two alternatives simply by using their engineering judgment. Although this is encouraging, it also means that the rest of the exercise is simply a matter of proving what they already know. A better set of alternatives would involve a less obvious choice. In addition, predicting the actual deflection of the PASCO Structures elements has been problematic. In the future, this must be improved so students can have greater confidence in their results.
- Incorporate experimental stress analysis. Although the students use strain gages in their junior-level design class, in the interests of time, this has been left out of the current lab. Based on recent experience, it seems like it would be a good addition.
- Develop and implement a good assessment tool to quantify learning enhancements provided by the new lab format. Currently, the benefits have only been captured qualitatively. For this to be more useful to other faculty, the results should be quantified.

### Conclusion

This paper presented a project incorporating simple experimentation, closed-form analysis, and finite element modeling to enhance student understanding of energy methods for predicting structural deflection. Simple models based on low-cost PASCO Structural Systems elements were tested with point loading conditions. Hand calculations based on Castigliano's theorem were performed to predict total deflection. Finite element analysis was completed using an ABAQUS CAE academic license. The addition of the experiments and the FEA to the existing Castigliano analysis lab had minimal effects on the lab cost, and by offsetting a later FEA lab, contributed no additional time.

By building and testing simple models of a structural problem and then analyzing that problem in closed-form, students were able to see how local deformations contribute to global deflections. They also remembered this when it came time to analyze the problem, by considering strain energy in all members. Performing numerical analysis of the same problem served to confirm the students' hand calculation results, and further reinforce the importance of local structural deformation.

When using this integrated approach, the author observed greater student engagement in the problem, improved discussions, and fewer errors in analysis results. Students commented informally and in end-of-term evaluations on the strengths of the combined approach. Quantification of the benefits is planned in the future.

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