Nuclear Orientation of $^{253}$Es in Neodymium Ethylsulfate*

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Einsteinium-253 nuclei were oriented at low temperatures in a neodymium ethylsulfate lattice. From the temperature-dependent α-particle angular distribution a nuclear magnetic moment $\mu = (2.7 \pm 1.3) N$ was deduced. From the values for the angular distribution function at the lowest temperatures it was possible to test the predictions of the Mang shell-model theory for the relative phases and amplitudes of the α-partial waves. As predicted, the waves of angular momentum $L=0$ and 2 are in phase, and the $L=0$ and 4 waves are out of phase. The predicted wave amplitudes are in error, especially that of the $L=4$ wave. The predicted relative intensities (which are proportional to the amplitudes squared) for the $S$, $D$, and $G$ waves are 1.000:0.179:0.0052, whereas the relative intensities that best fit the experimental angular distribution are 1.000:0.216:0.0078.

INTRODUCTION

Spiers$^1$ predicted that anisotropic α-particle emission would take place from oriented nuclei. Subsequent nuclear-orientation experiments confirmed this prediction and also yielded information about the relative amplitudes and phases of the observed α partial waves.

Hill and Wheeler$^2$ made the first quantitative estimate of enhanced α-particle emission from the poles of prolately deformed spheroidal nuclei. Their reasoning can be understood with the aid of
Fig. 1. They assumed a uniform probability of $\alpha$-particle formation within the nuclear volume. If the barrier set up by the remaining nucleons is simply Coulombic beyond a given radius, then this barrier is both thinner and lower at the poles than it is at the equator, and tunneling is greatly enhanced at the poles. For moderate nuclear deformation they predicted a 16-fold increase of $\alpha$-particle intensity from the poles over that from the equator. However, the angular distribution of nuclear radiation is determined primarily by the requirement that angular momentum be conserved in the nuclear decay. The greatly enhanced polar emission predicted by Hill and Wheeler could occur only from a nucleus with a large component of angular momentum perpendicular to the nuclear symmetry axis.

Roberts, Dabbs, and co-workers\(^3\text{-}^5\) were the first to test the predictions of Hill and Wheeler experimentally. They oriented $^{233}\text{U}$ and $^{235}\text{U}$ in single crystals of $\text{UO}_2\text{Rb(NO}_3)_3$ and $^{237}\text{Np}$ in single crystals of $\text{NpO}_2\text{Rb(NO}_3)_3$. For all three nuclei they observed preferential emission perpendicular to the crystalline $c$ axis. In the $^{237}\text{Np}$ case, they were able to establish that the nuclear spins also tend to orient perpendicular to the $c$ axis. These two facts taken together imply preferential emission along the nuclear-spin vector (i.e., from the poles), thus confirming the predictions of Hill and Wheeler. The data for the uranium isotopes are consistent with this interpretation of the $^{237}\text{Np}$ results, although for uranium the direction of orientation was not established directly. Chasman and Rasmussen\(^6\) have discussed the difficulties in interpreting the $^{233}\text{U}$ data.

Navarro, Rasmussen, and Shirley (NRS)\(^7\) aligned trivalent $^{249}\text{Cf}$ in a single crystal of neodymium ethylsulfate $\text{Nd(C}_2\text{H}_5\text{SO}_4)_3\cdot9\text{H}_2\text{O}$, which we shall abbreviate below as NES; NRS and also Frankel\(^8\) aligned $^{255}\text{Es}$ in NES. For both isotopes the prediction of Hill and Wheeler was confirmed: Preferential $\alpha$-particle emission from the nuclear poles was observed. Preferential polar emission in these cases reflects the fact that the $S$ and $D \alpha$ waves (corresponding to orbital angular momenta $L = 0$ and $2$) are in phase, while preferential equatorial emission would imply that the $S$ and $D$ waves were out of phase. In this paper we report an extension of the $^{255}\text{Es}$ nuclear-orientation studies to determine the relative phase of the $S$ and $G$ ($L = 4$) waves. In a future paper nuclear orientation studies of $^{241}\text{Am}$ and $^{252}\text{Fm}$ in NES will be reported.

**THEORY**

Nuclear orientation has been treated in a number of review articles,\(^9\text{-}^{13}\) and we shall discuss only those aspects of the method that are especially pertinent to $\alpha$-particle angular distributions. The angular-distribution function is essentially a consequence of the law of conservation of angular momentum applied to nuclear radiations. For $\alpha$ particles the angular-distribution function can be written as

$$W(\theta) = 1 + \sum_{k>0} \sum_{L, L'} a_L a_{L'} \cos(\phi_L - \phi_{L'}) b_k(LL'I,I) \times F_s(LL'I,I) B_s(I_s, T) P_2(\cos \theta) / \sum_L |a_L|^2,$$

(1)

where $a_L$ is the relative amplitude and $\phi_L$ the relative phase of the $L$-alpha wave. The parameter $F_s$ is familiar from $\gamma$-ray angular-correlation theory, and $b_k(LL'I,I)$ is a particle parameter which reflects the fact that $\alpha$ particles are spin-zero bosons. Only even-$k$ terms are nonzero for parity-conserving processes.

The orientation parameters $B_s$ are statistical tensors which describe the populations of the nuclear magnetic substates. They contain all the solid-state physics information, and the entire temperature dependence in $W(\theta)$. The $P_s$'s are the Legendre polynomials with the angle $\theta$ measured between the crystalline $c$ axis and the direction of $\alpha$-particle emission.

Because the product of the particle parameter and an $F$ coefficient are proportional to the product of a $3j$ symbol and a $6j$ symbol

$$b_k(LL'I,I)F_s(LL'I,I) = (-1)^{I_s+1} \sqrt{(2L+1)(2L'+1)(2I+1)(2k+1)} I_s^k \times \begin{pmatrix} L & L' & k \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} I & I_s & I_s \end{pmatrix},$$

(2)
the angular distribution can be written as
\[ W(\theta) = 1 + A_2(\ell L'I' I)B_2(I, T)P_2(\cos \theta) + A_4(\ell L'I' I)B_4(I, T)P_4(\cos \theta), \]
which shows that each angular-distribution coefficient can be factored into two terms. The summation over \( k \) is restricted to \( k = 2, 4 \) for this case because \( A_2(\ell L'I' I) \) is less than 0.02 \( A_4(\ell L'I' I) \) and hence, can be neglected. Higher-order terms are zero because of the properties of the \( 6j \) symbol. The \( A_4 \) term depends specifically on the decaying nucleus, while the \( B_4 \) term depends on both the (hyperfine) interaction of the nucleus with its environment and the temperature. We will treat these two terms in order, first considering the \( \alpha \)-decaying nucleus and then consider the solid-state aspects.

The conservation of angular momentum restricts \( L \) to \( |I_1 - I_2| \leq L \leq I_1 + I_2 \), where \( I_1 \) and \( I_2 \) are the spins of the parent and daughter nucleus, respectively. The conservation of parity further restricts \( L \) to even (odd) values if the parent and daughter have the same (opposite) parity. The relative amplitudes of the \( \alpha \) waves can be obtained from the reduced transition probabilities calculated according to the prescription of Bohr, Fröman, and Mottelson (BFM).\(^{14}\) The projection \( K \) of total angular momentum along the nuclear symmetry axis is nearly a constant of motion for most spheroidal nuclei. For odd-mass nuclei, transitions for which \( \Delta K = 0 \) are favored: These transitions do not involve the last odd nucleon and do not require the breaking of pairs. Mang, Poggenburg, and Rasmussen\(^{15}\) have shown that pairing correlations have a decisive influence on \( \alpha \)-decay transition rates.

As a consequence of the Wigner-Eckart theorem, the transition matrix elements for a given \( \alpha \)-partial wave populating various rotational states of the daughter will vary, according to the BFM model, as the ratio of Clebsch-Gordan coefficients. The reduced transition probability or decay constant \( \lambda = (\langle n | 2n \rangle \langle n | 2n \rangle) \) will then be proportional to the product of (the square of a hindrance factor averaged from neighboring even-even nuclei) times (a barrier penetration factor). The predicted partial wave intensities for the favored transitions of \(^{253}\)Es are given in Table I. A partial decay scheme, as adapted previously,\(^{16}\) is given in Fig. 2.

The BFM prediction is exact only in the limit of infinite nuclear moment of inertia and vanishing nuclear quadrupole moment. Because the actinides typically have large quadrupole moments, deviations from the BFM predictions are expected. More accurate values for the decay constants and hence, for the partial wave amplitudes can be obtained by solving either a three-dimensional non-

![Image](image.png)

**FIG. 2.** Partial decay scheme for \(^{253}\)Es adapted from Ref. 16. The \( \alpha \)-particle populations are shown in parenthesis.

**TABLE I.** Intensities for partial waves in favored \( \alpha \) transitions to the ground-state band in \(^{249}\)Bk (Bohr, Fröman, and Mottelson theory).

<table>
<thead>
<tr>
<th>( E_f ) (keV)</th>
<th>( I_f^\pi )</th>
<th>( S )</th>
<th>( D )</th>
<th>( G )</th>
<th>( \sum(%) )</th>
<th>Measured intensity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \frac{3}{2}^+ )</td>
<td>79.6</td>
<td>10.0</td>
<td>0.127</td>
<td>89.7</td>
<td>90</td>
</tr>
<tr>
<td>41.7</td>
<td>( \frac{7}{2}^+ )</td>
<td>5.92</td>
<td>0.327</td>
<td>6.24</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td>93.8</td>
<td>( \frac{11}{2}^+ )</td>
<td>0.88</td>
<td>0.267</td>
<td>1.15</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>155.8</td>
<td>( \frac{13}{2}^+ )</td>
<td>0.083</td>
<td>0.083</td>
<td>0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>229.3</td>
<td>( \frac{15}{2}^+ )</td>
<td>0.0083</td>
<td>0.0083</td>
<td>0.014</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

separable Schrödinger equation or a set of coupled differential equations for the radial wave functions. The shell-model theory of \( \alpha \) decay as developed by Mang,\(^{17}\) Mang and Rasmussen,\(^{18}\) and Zeh and Mang\(^{19}\) has been used most extensively for the calculation of decay constants, and therefore we will not consider other (more exact but less extensive) direct numerical integrations of the coupled differential equations that have been performed.\(^{20,21}\)

Their\(^{17-19}\) approach is essentially a nucleon overlap model in that the decay constant is proportional to the probability that the wave functions of the
daughter nucleus and $\alpha$ particle are contained in the wave function of the parent nucleus. The shell-model theory of $\alpha$ decay gives the decay constant as the product of a reduced width $\gamma_{1fL}^{1/2}$, which contains all information about the $\alpha$-formation process times a penetrability factor $P_{\nu}(\epsilon)$, which accounts for the penetration of the predominantly Coulombic barrier by the $\alpha$ particle. Thus

$$\lambda = \frac{1}{\hbar} \sum_{fL} P_{\nu}(\epsilon) \gamma_{1fL}^{1/2},$$

(4)

where $\epsilon$ is the $\alpha$-particle kinetic energy.

The penetrability factor is usually calculated using the WKB approximation. The reduced width is given by

$$\gamma_{1fL}^{1/2} = (\hbar^2/2M)R_0 |g(I_f I_L; R_0)|^2,$$

(5)

where $g(I_f I_L; R_0)$ is a time-dependent probability amplitude measuring the probability that the wave function of the parent contains an $\alpha$ particle and a daughter of the specified quantum numbers at a relative distance $R_0$; it is related to the magnitude of the single-particle wave functions at the nuclear surface.

Details about the calculation of transition probabilities are given by Poggenburg and by Poggenburg, Mang, and Rasmussen (PMR). An abbreviated table for the main $\alpha$ transitions of $^{252}$Es is given as Tables II and III. The intensities were derived from the normalized transition probabilities given in Ref. 22. The shell-model theory predicts relative phases of the partial waves in addition to wave amplitudes which are proportional to the square root of normalized transition probabilities.

The relative phases of the $L$ waves deserve some comment. As shown by Preston, the relative phases can be obtained by the inward numerical integration of the coupled differential equations because of the requirement that the imaginary part of the wave function vanish at the nuclear surface for a quasistationary state. This condition permits two values for $\phi_L$, one near zero and one near $\pi$. Brussard and Tolhoek concluded that the phase shift due to penetration through the Coulombic barrier would shift the $S$ and $D$ waves by less than $1\%$. Rasmussen and Hansen estimated that because of the quadrupole coupling between the outgoing $\alpha$ particle and the rotational states of the daughter nucleus the $D$ wave lags the $S$ wave by $7^\circ$ at infinity for the odd-mass neighbors of $^{242}$Cm. The quadrupole phase shift is largest for a weak wave coupled to a strong wave, but fortunately in that case the interference term is small. Also the phase shift can be in either direction so the shifts may tend to cancel. We will assume that the waves are either exactly in phase or exactly out of phase and hence,

$$\cos(\phi_L - \phi_L) = \pm 1.$$

We now consider some solid-state aspects of this research. The similarity between the chemical properties of the actinides and lanthanides implies a similarity between the electronic structure of the actinides and the extensively studied lanthanides. This similarity was used to facilitate data interpretation.

For a free actinide ion, $J$ remains a good quantum number although $L$ and $S$ do not because the spin-orbit interaction causes a breakdown of Russell-Saunders coupling. An intermediate coupling calculation using extrapolated values for the electrostatic and spin-orbit interaction constants gave the $E^3$ free ion wave function. It is $79\% \gamma_d$. In

### Table II. Intensities and phases for partial waves in favored $\alpha$ transitions to the ground-state band in $^{249}$Bk (Mang shell-model theory).

<table>
<thead>
<tr>
<th>$I_f^T$</th>
<th>$S$</th>
<th>$D$</th>
<th>$G$</th>
<th>$I$</th>
<th>$\sum(%)$</th>
<th>Measured intensity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}^+$</td>
<td>-83.47</td>
<td>-8.84</td>
<td>0.079</td>
<td>0.0002</td>
<td>92.49</td>
<td>90</td>
</tr>
<tr>
<td>$\frac{3}{2}^+$</td>
<td>-5.259</td>
<td>0.197</td>
<td>0.001</td>
<td>5.457</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td>$\frac{5}{2}^+$</td>
<td>-0.782</td>
<td>0.158</td>
<td>0.004</td>
<td>0.944</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>$\frac{7}{2}^+$</td>
<td>0.050</td>
<td>0.054</td>
<td>0.003</td>
<td>0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{9}{2}^+$</td>
<td>0.0050</td>
<td>0.0017</td>
<td>0.0067</td>
<td>0.014</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table III. Intensities and phases for partial waves in $\alpha$ transitions to the first excited rotational band in $^{249}$Bk (Mang shell-model theory).

<table>
<thead>
<tr>
<th>$E_f$ (keV)</th>
<th>$I_f^T$</th>
<th>$F$</th>
<th>$H$</th>
<th>$\sum(%)$</th>
<th>Measured intensity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.8</td>
<td>$\frac{1}{2}^-$</td>
<td>-0.39</td>
<td>-0.006</td>
<td>0.40</td>
<td>0.8</td>
</tr>
<tr>
<td>39.6</td>
<td>$\frac{3}{2}^-$</td>
<td>-0.057</td>
<td>-0.008</td>
<td>0.58</td>
<td>0.7</td>
</tr>
<tr>
<td>82.6</td>
<td>$\frac{5}{2}^-$</td>
<td>-0.33</td>
<td>-0.04</td>
<td>0.37</td>
<td>0.7</td>
</tr>
</tbody>
</table>
the lanthanides and actinides the spin-orbit interaction is stronger than the crystal field (CF) interaction which is in turn stronger than the hyperfine (hf) interaction. A CF of lower than cubic symmetry will partially or completely remove the \((2J+1)\)-fold electronic degeneracy of the free ion giving a series of CF levels which are in turn split by the hf interaction.

The CF Hamiltonian can be written as

\[
3\mathcal{H}_{\text{CF}} = \sum_i \sum_{k \neq k'} B_i^k (C_i^k)^2,
\]

where \(B_i^k\) is a CF parameter which is proportional to the more familiar \(A_i^k(y^k)\) parameter. \(C_i^k\) is a tensor operator; the summation involving \(i\) is taken over all electrons of interest. For the rare-earth ethylsulfates the only CF parameters that affect the electronic states are \(B_{a^3}, B_{a^4}, B_{b^4}\), and \(B_{b^6}\).

For \(N\) electrons the nonrelativistic magnetic hyperfine Hamiltonian for a free ion is

\[
3\mathcal{H}_{\text{mhf}} = \frac{2 \mu_B \mu_N}{I} \sum_{j=1}^{N} \sum_{m=1}^{2} \frac{8 \pi}{3} \sum_i \delta(r_i) \mathcal{H}_{j i},
\]

with \(\mathcal{H}_{j i} = \mathcal{T}_{j} \mathcal{I}_{i} - \sqrt{10}(5 \mathcal{C}_{(m_i)^2})\). The last term in Eq. (7) arises only if unpaired \(s\) electrons are present or if core-polarization effects are important.

For an ion at a site of axial symmetry the hf interaction can be described by a spin Hamiltonian

\[
3\mathcal{H}_{\text{hf}} = A_{||} S_{x} + B(J_{S_x} + I_{S_y}) + P[J_{S_z} - \frac{1}{2}(I(I + 1))],
\]

where \(A\) and \(B\) are magnetic hf interaction parameters and \(P\) is the quadrupole coupling constant. The hyperfine interaction can be adequately treated as a perturbation on the CF energy levels which were obtained by diagonalizing the combined electrostatic, spin-orbit, and crystal-field interaction matrices.

Again for an ion at a lattice site of axial symmetry the magnetic hyperfine interaction tensor has two components, one along the axis and one perpendicular to it. Then

\[
A_{||} = A = 4 \mu_B \mu_N \mu \langle r^3 \rangle \delta \langle + | J_+ | + \rangle \langle N | J \rangle / I,
\]

\[
A_{\perp} = B = 4 \mu_B \mu_N \mu \langle r^3 \rangle \delta \langle + | J_- | - \rangle \langle N | J \rangle / I,
\]

\(|+\rangle\) and \(|-\rangle\) represent the two components of the doubly degenerate CF state under consideration and

\[
\langle f^N S L J | \sum N_i \beta f^N S' L' J \rangle \langle f^N S L J | f^N S' L' J \rangle.
\]

The discussion of the electric hyperfine interaction and the quadrupole coupling constant will be deferred until a future paper because the \(^{253}\text{Es}\) data can be interpreted in terms of the magnetic hf interaction constants alone.

**EXPERIMENTAL TECHNIQUES**

The experimental work was performed independently by Navarro, Frankel, and Soinski over a nine year period. Various modifications of technique and apparatus occurred; we report only those techniques used most recently.

The \(\text{NES}\) crystals were grown from saturated solutions at both room temperature and \(277\) \(\text{K}\); growth at the lower temperature produced more perfect crystals. The crystals were weighed approximately \(3\) \(\text{g}\). One face of the crystal was sanded flat at a \(45^\circ\) angle with respect to the crystalline \(c\) axis. The radioactive trivalent ion in the form of either the nitrate or chloride salt was dissolved in several micro liters of saturated \(\text{NES}\) solution, \(2\mu\)l of this solution were applied to the center of the sanded face and allowed to remain for \(20\) sec before removal. The process was repeated until approximately \(2 \times 10^6\) dis/min of activity remained on the crystal. Those crystals exhibiting the highest saturation anisotropy were prepared with the fewest applications (ideally only one) of radioactive solution.

A \(\text{NES}\) crystal was mounted in a glass cryostat chamber shown in Fig. 3. The compressed manganese ammonium sulfate pill serves as a "getter" for residual \(^4\text{He}\) exchange gas. The chromium-potassium-alum-glycerine slurry cools upon adiabatic demagnetization to approximately the same temperature as \(\text{NES}\) and decreases the heat leak down the \(2\)-mm-diam Pyrex glass support rod.

The nylon filament prevents the \(\text{NES}\) crystal from hitting the cryostat walls due to vibrations. The detectors are along and perpendicular to the \(\text{NES}\) \(c\) axis. The error in positioning is estimated to be \(\pm 2^\circ\). Because the second- and fourth-order Legendre polynomials are relatively flat at \(0^\circ\) and \(90^\circ\), the errors introduced by misalignment are small.

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**FIG. 3.** Experimental chamber for nuclear orientation studies of \(\alpha\)-emitting isotopes.
Silicon semiconductor detectors were used to detect the $\alpha$ particles. Gold was evaporated on the front and back faces of 1-cm $\times$ 1-cm $\times$ 0.4-mm silicon wafers. An electrically conducting epoxy was used to fasten each detector to a molybdenum mounting strip and to secure a fine gold wire to the front electrode. Approximately 125-V reverse bias was applied to collect the charge carriers at the gold electrodes.

The final temperature reached after adiabatic demagnetization was determined from the initial values of $H/T$ using the temperature scale of Blok, Shirley, and Stone. The ratio of the initial magnetic field (measured with a rotating coil gaussmeter) to the pumped helium-bath temperature (measured with a dibutylphthalate filled manometer) determines the magnitude of the entropy of the electronic spin system and also the final temperature reached upon adiabatic demagnetization to zero applied magnetic field. Final temperatures measured in this manner are accurate to ±6%.

Following adiabatic demagnetization, several full spectrum "cold" counts were taken. The NES crystal was then warmed to the 4He bath temperature (approximately 1 K) and a series of "warm" counts were taken for normalization. Because there is an anisotropic angular distribution at 1 K, these warm counts were renormalized to 4 K where the angular distribution is isotropic.

A typical pulse-height spectrum is shown in Fig. 4. The $\alpha$ detectors used had an energy resolution full width at half maximum of 100 keV or better for 5.5-MeV $\alpha$ particles. The broadening of the peak shown in Fig. 4 is due to energy losses within the NES crystal. The effect of (Rutherford) scattering on the measured anisotropy was determined empirically by dividing typical spectra into several segments and by calculating the anisotropy of each segment. The lowest-energy segments should show the smallest anisotropy if large-angle scattering is important; but, within statistics the anisotropy was constant across each spectrum. However as a safeguard, the lowest 25% of the spectra were not used in calculating anisotropies. The $^{249}$Bk daughter $\beta^-$ activity contributed counts to the lowest part of the energy spectrum but no corrections were necessary because that part of each spectrum was excluded from analysis.

Solid-angle correction factors were calculated by explicit numerical integration of the Legendre polynomials over both the finite source and detectors using a computer program written by Dr. William D. Brewer.

**RESULTS**

The experimental work on $^{253}$Es was carried out in three sets of measurements, which we shall denote by I, II, and III. I measurements were made by Navarro in 1961 and 1962; II measurements were made by Frankel in 1963; and III measurements were made by Soinski in 1968 and 1969. Data from a series of adiabatic demagnetizations performed by Soinski using one source and one set of two detectors are plotted in Fig. 5 and tabulated in Table IV. The statistical accuracies are given in parentheses. We designate any one such series of demagnetizations as a run. In discussing these data, there are two rather distinct "figures of merit," which are best treated separately; the saturation values of $W(0)$ and $W(\pi/2)$ and their temperature dependences.

The observed saturation values of $W(0)$ and $W(\pi/2)$ are affected by the degree to which $^{253}$Es$^{3+}$ grows into Nd$^{3+}$ lattice sites substitutionally, by scattering in the source crystal, and by the performance of the detectors. Values of $W(0)$ and

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**FIG. 4.** Typical axial counter pulse-height spectrum for $^{253}$Es in NES. The cold spectrum was taken at 0.011 K and the warm spectrum at 1 K.

**FIG. 5.** Experimental angular distribution of $^{253}$Es in NES as a function of the inverse temperature. The curves shown are theoretical based on different estimates for the relative intensities of the $\alpha$-partial waves.
$W(\pi/2)$ from I, II, and III are set out in Table V. In I the “effect” as measured by the $\pi/2$ detector, i.e., $1 - W(\pi/2)$ is relatively small; The same is true for the effect at the 0° detector, $W(0) - 1$, in II. In III, on the other hand, large effects are observed with both detectors. We interpret this to mean that III gave the most accurate saturation values for $W(0)$ and $W(\pi/2)$, because almost any error in a nuclear orientation experiment will act to reduce the effect observed. Inspection of the discrepancies in Table V suggests that two kinds of errors were present in I and II. First, the smaller effect on even the “better” detector in each case suggests a source-preparation problem: Perhaps the activity was too deep (leading to excessive scattering) or incompletely substituted into lattice sites. Secondly, the relatively large diminution of the effect in the “poorer” detector in each case indicates something more grossly wrong either with that detector or with $\alpha$ emission in that direction (e.g., more scattering in the source in one direction). In III extreme care was taken to grow $^{253}$Es only near the surface of the crystal. Also the detectors and associated circuits were far more reliable than in I and II. The large effects observed with both detectors in series III were obtained with several sources. For these reasons we take the saturation values of $W(0)$ and $W(\pi/2)$ from series III as being characteristic of $^{253}$Es in neodymium ethylsulfate.

The temperature dependences of $W(0)$ and $W(\pi/2)$ are affected by different variables than those that affect their saturation values. Immediately after demagnetization the small active volume of the NES crystal is warmed at an appreciable rate by radioactive heating. This can lead to a spurious apparent temperature dependence, with high-temperature points showing a reduced effect. The hyperfine structure constants derived in such cases are anomalously small. If $\gamma$-ray distributions are studied the activity can be distributed throughout the NES crystal, and the temperature can be monitored through the magnetic susceptibility. For $\alpha$-particle studies, however, the activity must be concentrated in a small volume on the surface, and the resulting intense self-heating can raise the local temperature well above that of the bulk crystal. From the change in $\alpha$-particle anisotropy with time, we found that in series III the spot warmed from $1/T = 90$ K$^{-1}$ to $1/T = 20$ K$^{-1}$ in 30 min. In series I and II the warmup rate was lower. Ironically, the stronger the source and the shallower its distribution in from the surface [two conditions conducive to reliable measurement of saturation values of $W(0)$ and $W(\pi/2)$], the worse will be the self-heating effect. Thus in our experiments, series I and II gave a temperature dependence in $W(0)$ and $W(\pi/2)$ characteristic of larger values of $A = 0.28 \pm 0.3$ cm$^{-1}$ than those that would be derived from series III ($A = 0.18 \pm 0.02$ cm$^{-1}$). The values of $A$ obtained from several runs are given in Table VI. The final adopted value of $A = 0.26 \pm 0.03$ cm$^{-1}$ was decided from the entries in Table VI on the basis of the above discussion.

**DISCUSSION**

The curves shown in Fig. 5 are theoretical; the dashed and solid curves differ because different relative amplitudes were used for the partial waves. We shall discuss these curves and the saturation values of $W(0)$ and $W(\pi/2)$ later; first we shall discuss the value for the hf interaction constant $A$ and the derivation of the magnetic moment of $^{253}$Es from $A$.

Baker and Bleaney$^{32}$ observed the paramagnetic...
resonance spectrum of Ho\(^{3+}\), the 4\(^{10}\) analog of Es\(^{3+}\), as a 1\(^{2}\) impurity in a crystal of yttrium ethylsulfate. The ground CF state was a non-Kramers doublet characterized by \(J_z = \pm 7\) with small admixtures of \(J_z = \pm 6\) and \(\pm 5\). A singlet characterized by \(J_z = +6, 0,\) and \(-6\) lay nearby in energy. The hf interaction constants for \(^{166}\)Ho were \(A = 0.334(1)\) cm\(^{-1}\), \(B = 0.02\) cm\(^{-1}\), \(P = 10^{-3}\) cm\(^{-1}\). The dominant term in the CF Hamiltonian is \(A I Z S z\), which results in alignment along the crystalline \(c\) axis with the nuclear magnetic substate \(I_x = \pm 1\) lying lowest in energy.

The temperature dependence of \(W(0)\) and \(W(\pi/2)\) for \(^{253}\)Es indicates that the \(A I Z S z\) term is also dominant in the hfs of the lowest electronic CF state of Es\(^{3+}\) in the ethylsulfate lattice. It should therefore in principle be possible to derive a value for the nuclear magnetic moment \(\mu\) of \(^{253}\)Es. To do so, a description of the CF electronic ground state and a value for \(\langle r^{-3}\rangle_{\text{CF}}\) are required in addition to the value for \(A\).

NRS\(^7\) used extrapolated values for the Es\(^{3+}\) CF parameters in the NES lattice. However, their extrapolation was based on Gruber’s CF parameters for Am\(^{3+}\) in LaCl\(_3\).\(^{35}\) As pointed out by Conway, Gruber misinterpreted his data, and therefore his CF parameters may be in error. The only reliable CF parameters for an actinide ion at a site of trigonal symmetry are those of Krupke and Gruber for Np\(^{3+}\) in LaBr\(_3\).\(^{35}\) The CF parameters for Es\(^{3+}\) in NES were estimated in two different ways. In the first method, we assumed that the \(A I Z S z\) parameters are the same for analogous lanthanides and actinides and that the only difference is contained in the radial integrals. Using Hufner’s CF parameters for Ho\(^{3+}\) in holmium ethylsulfate\(^{36}\) and ratios of radial integrals obtained from Hartree-Fock nonrelativistic wave functions without exchange,\(^{37}\) the Es\(^{3+}\) CF parameters in cm\(^{-1}\) are \(B_6^z = 430, B_6^y = -1380, B_6^z = -1120,\) and \(B_6^z = 960\). For Ho\(^{3+}\) in both holmium ethylsulfate\(^{36}\) and LaCl\(_3\) (Ref. 38), the first excited state \(J = 7\) is admixed into the ground state \(J = 8\) by the CF interaction. Evaluation of \(\mathcal{H}_{\text{CF}}\) for Es\(^{3+}\) with the inclusion of both the \(J = 8\) and \(J = 7\) wave functions\(^{38}\) gave a singlet as the ground state. Because alignment would not occur if a singlet were lowest, \(B_6^y\) was increased in magnitude in order to bring a doublet lowest in energy. Using \(B_6^z = 550\) cm\(^{-1}\), the ground CF state is \(0.939(\pm 7) + 0.305(\pm 1) + 0.153(\pm 5)\).

A second estimate for the Es\(^{3+}\) CF parameters was obtained from the values for Ho\(^{3+}\) in HoES,\(^{36}\) Np\(^{3+}\) in LaBr\(_3\) (Ref. 35) (the CF parameters for Np\(^{3+}\) in the isoelectronic lattices LaBr\(_3\) and LaCl\(_3\) should be approximately the same),\(^{40,41}\) and interpolated values for Pm\(^{3+}\) in LaCl\(_3\).\(^{42}\) The CF parameters obtained are \(B_6^z = -1440, B_6^y = -760,\) and \(B_6^z = 650\). \(B_6^y\) is the most difficult parameter to obtain by this procedure because \(B_6^y\) for Np\(^{3+}\) in LaBr\(_3\) is negative as a result of a large ligand (\(v = 5\) shell) overlap exchange charge.\(^{43}\) Again it was necessary to vary \(B_6^z\) in order to get a doublet lowest in energy. Using \(B_6^z = 600\) cm\(^{-1}\), the ground CF state is \(0.967(\pm 7) + 0.225(\pm 1) + 0.120(\pm 5)\).

The difficulties in extracting \(\langle r^{-3}\rangle_{\text{CF}}\) from experimental data have been discussed by several authors.\(^{44-49}\) Because of the lack of experimental data for Es\(^{3+}\), we take a value of \(\langle r^{-3}\rangle_{\text{CF}} = 10.92\) a.u., which was obtained using relativistic self-consistent Dirac–Fock wave functions.\(^{50}\) The nuclear magnetic moment was calculated using Eq. (9a) which can be written as

\[
\mu = \frac{\langle f^6 S L J || J_\text{g} || f^6 S ' L ' J ' \rangle}{\langle f^6 S L J \rangle \sum_{J'} \langle f^6 S ' L ' J ' \rangle},
\]

(10)

The two values for \(|\mu|\) resulting from the two sets of CF parameters are 2.79(32) and 2.63(30) \(\mu_N\). The errors quoted in parenthesis reflect the statistical uncertainty in the value of \(A\). These values are substantially lower than the value of 4.9 \(\mu_N\) previously reported\(^9\) because of a change in the electronic ground CF state obtained with a different choice of CF parameters.

Two estimates of the \(^{253}\)Es nuclear magnetic moment have been made by Lamm.\(^{51}\) Assuming \(g_s = g_s^\text{free},\) \(\mu = +4.239\) \(\mu_N\). However, better agreement with the moments of odd-mass nuclei is obtained by using the semiempirical quenching factor \(g_s = 0.6g_s^\text{free}.\(^{52}\) Then \(\mu = +3.650\) \(\mu_N\), in better agreement with the values given above. The major source of error in our value for the nuclear magnetic moment is the uncertainty of the ground crystal-field state. Both of the predicted ground states have a large \(J_z = \pm 7\) component which gives a small derived value for \(\mu\). No reasonable choice of CF parameters gave a coefficient for \(|\pm 7\rangle\) of approximately 0.85 which would yield a moment of 3.6 \(\mu_N\). Therefore, we report a nuclear magnetic moment \(|\mu| = 2.7 \pm 1.3\) \(\mu_N\). The error reflects the statistical accuracy of the measured value of \(A\) and the uncertainties in both the CF parameters and the radial integral \(\langle r^{-3}\rangle_{\text{CF}}\).

Additional indirect measurements of the \(^{253}\)Es nuclear magnetic moment have been made recently. Worden et al.\(^{53}\) report a value of 5.1(13) \(\mu_N\) which was derived from the hfs observed in the \(^{253}\)Es emission spectrum. The moment was calculated using the Goudsmit–Fermi–Segre formula to which large and uncertain relativistic corrections had to be made. Edelstein\(^{54}\) reports a value of \(+3.62(50)\) \(\mu_N\) based on the analysis of the EPR spectrum of divalent \(^{253}\)Es in CaF\(_2\).\(^{55}\) Edelstein’s value should be more accurate than either our value or that of
Worden et al.

Now we turn to an investigation of the saturation behavior of $W(0)$ and $W(\pi/2)$. The limiting values $B_4(1/T=\infty)=+1.528$ and $B_4(1/T=\infty)+0.798$ are essentially realized in this experiment. Using the relative intensities and phases given by the Mang theory (Tables II and III), the angular-distribution function would be

$$W(\theta) = 1 + 0.575Q_2B_2P_2(\cos \theta) - 0.0287Q_4B_4P_4(\cos \theta),$$

where $Q_2$ and $Q_4$ are solid-angle correction factors. For the run summarized in Tables II and III, $Q_2(0)=0.946, Q_2(\pi/2)=0.962,$ and $Q_4(\pi/2)=0.877.$ In calculating the $Q_k$'s we assumed that the radioactivity was uniformly distributed throughout the spot. Since the activity would tend to concentrate at the center of the spot, the corrections may be too large; that is, the $Q_k$'s should perhaps be larger in magnitude. The justification for dropping the $P_4(\cos \theta)$ term is that $B_4(1/T=\infty)=0.174$ and $A_4=-0.0025.$ The 0.4% of the $\alpha$ decays not considered in the calculation of $W(\theta)$ were assumed to give the same angular distribution as the decays that were included.

Equation (11) is plotted as the dashed curve in Fig. 5; the fit to the saturation (lowest temperature) values of the data is poor. The experimental result is

$$W(\theta) = 1 + 0.630(5)Q_2B_2P_2(\cos \theta) - 0.059(7)Q_4B_4P_4(\cos \theta).$$

An examination of Tables II and III reveals that the theoretical intensity to the $\frac{3}{2}^+$ level of $^{248}$Bk is 2.5% larger than the observed intensity. Therefore the partial wave intensities for the $\alpha$ waves populating this level were changed in order to both improve the fit to the data and bring the theoretical and observed intensities for this level into agreement. Since the $S$ wave contributes most of this intensity, the $S$-wave intensity was decreased by 6%. The $D$-wave intensity was increased by 22% [mainly to fit the $P_2(\cos \theta)$ term] and the $G$-wave intensity was increased 220% [mainly to fit the $P_4(\cos \theta)$ term]. The resulting angular-distribution function would be

$$W(\theta) = 1 + 0.632Q_2B_2P_2(\cos \theta) - 0.054Q_4B_4P_4(\cos \theta),$$

which is plotted as the solid curve in Fig. 5. The fit to the lowest-temperature data is greatly improved. The reason for the poor fit at higher temperatures was explained in the Results. Changing

FIG. 6. Temperature dependence of the (negative) coefficient of the $P_4(\cos \theta)$ term in the $^{253}$Es angular-distribution function. The curves shown are theoretical based on different estimates for the relative intensities of the $\alpha$-partial waves.

transition probabilities for decays to other levels in $^{248}$Bk would not be instructive. The ability to fit the experimental data by varying the $\alpha$-partial wave intensities indicates that the angular-distribution function is sensitive to the relative amplitudes of the partial waves and that quantitative information about the amplitudes can be obtained. Our results indicate that the Mang theory, as applied to $^{253}$Es, underestimates the intensities of both the $D$ and $G$ partial $\alpha$ waves. These higher angular momentum waves are probably enhanced in intensity by the interaction between the outgoing $\alpha$ particle and the daughter nucleus. The $S^-, D^-, G$-wave intensities that fitted the data best are given in Table VII.

TABLE VII. Relative $\alpha$ partial wave intensities for the decay of $^{253}$Es.

<table>
<thead>
<tr>
<th></th>
<th>This work</th>
<th>Mang theory</th>
<th>BFM theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^2_0$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$a^2_2$</td>
<td>0.216</td>
<td>0.179</td>
<td>0.211</td>
</tr>
<tr>
<td>$a^2_4$</td>
<td>0.0078</td>
<td>0.0052</td>
<td>0.010</td>
</tr>
</tbody>
</table>
TABLE VIII. Coefficients $A_2$ and $A_4$ for the $^{253}$Es in NES angular-distribution function

\[ W(\theta) = 1 + A_2 Q_2 B_2 P_2(\cos \theta) + A_4 Q_4 B_4 P_4(\cos \theta). \]

<table>
<thead>
<tr>
<th></th>
<th>$A_2$</th>
<th>$A_4$</th>
<th>$W(0, 1/T \to \infty)$</th>
<th>$W(\pi/2, 1/T \to \infty)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>0.630(5)</td>
<td>-0.059(7)</td>
<td>1.872(7)</td>
<td>0.521(4)</td>
</tr>
<tr>
<td>BFM (S and G waves out of phase)</td>
<td>0.622</td>
<td>-0.026</td>
<td>1.882</td>
<td>0.536</td>
</tr>
<tr>
<td>BFM (S and G waves in phase)</td>
<td>0.700</td>
<td>-0.139</td>
<td>2.104</td>
<td>0.522</td>
</tr>
<tr>
<td>Mang theory (S and G waves predicted to be out of phase)</td>
<td>0.575</td>
<td>-0.029</td>
<td>1.812</td>
<td>1.570</td>
</tr>
<tr>
<td>Mang theory but with modified partial wave intensities</td>
<td>0.632</td>
<td>-0.054</td>
<td>1.878</td>
<td>0.521</td>
</tr>
</tbody>
</table>

modified intensities give the better fit to the data.

Using the intensities predicted by BFM as given in Table I and taking the $S$ and $D$ waves as in phase and the $S$ and $G$ waves as out of phase, as required by the experimental results, the angular-distribution function would be

\[ W(\theta) = 1 + 0.622 Q_2 B_2 P_2(\cos \theta) - 0.025 Q_4 B_4 P_4(\cos \theta). \]

(14)

The saturation anisotropies in this case are $W(0, 1/T \to \infty) = 1.882$ and $W(\pi/2, 1/T \to \infty) = 0.536$, giving fair agreement with experiment. For convenience we have tabulated the experimental and theoretical values of $A_2$ and $A_4$ in Table VIII.

Chasman and Rasmussen\(^t\) have considered the effect of quadrupole coupling between the outgoing $\alpha$ wave and the daughter nucleus on the relative intensities predicted by BFM. They concluded that the $D$-wave intensity is enhanced by 20\% for $^{233}$U. A similar correction may apply to $^{253}$Es, but, as shown in Table VII, the BFM value for $a_2^2$ is in good agreement with the experimental $a_2^2$ without this correction. No estimate has been made of $G$-wave intensity enhancement.

CONCLUSION

$^{233}$Es and $^{233}$Ho have similar electronic structure as is evidenced by their having the same nuclear orientation mechanism. Magnetic hyperfine splitting in both cases results from the interaction between the orbital and spin moments of the unpaired $f$ electrons and the nuclear magnetic dipole moment. The ions are of predominantly $^5I_3$ character, and, as expected, the $^{233}$Es electronic wave function contains less $^7I$ character than the $^{233}$Ho wave function.

Mang's shell-model $\alpha$-decay theory successfully predicts the relative phases of the $\alpha$-partial waves. The theoretical relative transition probabilities are not consistent with the experimentally determined wave amplitudes, but it is not clear whether the defect is in the theory itself or in the calculations of Poggenburg. The Fröman matrix method was used in calculating the barrier penetration factors. However, this method is not too good for solving coupled equations for a weak partial wave coupled to a strong wave. Clearly what is needed is a coupled-channel barrier penetration calculation such as the one performed by Rasmussen and Hansen for $^{242}$Cm. Such a calculation also gives the quadrupole coupling phase shifts and hence would remove the need for assuming that the $\alpha$-partial waves differ in phase by exactly 0 or $\pi$.

The disagreement between our value for the $^{253}$Es magnetic moment and other experimental values points to the need for additional ESR or optical spectrometric studies of salts of the actinides. A reexamination of the Am$^3^+$ in LaCl$_3$ spectra would be especially interesting to us. It should be pointed out that the trivalent lanthanide ion crystal spectra have been analyzed at various levels of sophistication and therefore the extrapolation from the Np$^{3+}$ CF parameters to those for Es$^{3+}$ on the basis of reported behavior in the lanthanides is open to question.

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Professor J. O. Rasmussen originally suggested this research and encouraged its completion. Dr. B. B. Cunningham and his co-workers supplied the separated radioactive isotopes used in this research. We are grateful to Dr. N. Edelstein for many helpful discussions and for the use of several computer programs.
Center, Dilliman, Quezon City, The Philippines.
27 The input parameters in wave numbers were 
\[ E^1 = 5300, \ E^2 = 23, \ E^3 = 480, \ \pm = -3700, \ \alpha = 78, \ \beta = 250, \ \gamma = 0.1. \]
28 The configuration interaction parameters, \( \alpha \), \( \beta \), and \( \gamma \) are discussed by W. T. Carnall, P. R. Field, and K. Rajnak, J. Chem. Phys. 49, 4424 (1968). The \( ^{23} \) Ne free ion wave function obtained is
\[
0.8880 |^{2S}_2 \rangle + 0.3846 |^{2D}_2 \rangle - 0.3846 |^{2D}_0 \rangle + 0.0968 |^{2L}_1 \rangle - 0.0287 |^{2S}_0 \rangle + 0.1016 |^{2L}_2 \rangle + 0.0341 |^{2L}_3 \rangle.
\]
29 The lower subscripts are the \( V \) quantum numbers which label the irreducible representations of the group \( G \).
54 N. Edelstein, to be published.