Seismic Response Study of Asymmetric Systems with Linear and Nonlinear Fluid Viscous Dampers

By

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ABSTRACT
This investigation is concerned with application of nonlinear fluid viscous dampers for asymmetric-plan systems. For this purpose, the behavior of nonlinear fluid viscous dampers is summarized first followed by the effects of damper nonlinearity on seismic response of asymmetric systems. It is shown that the peak force in nonlinear damper tends to be smaller compared to linear damper. However, this is true only for values of $V/\dot{u}_d$ less than a certain threshold value; for larger values of $V/\dot{u}_d$, the force in nonlinear damper may become larger than that in the linear damper. While the damper nonlinearity tends to limit the damper force, it leads to smaller equivalent damping at velocities larger than the design velocity. The investigation on the seismic response of one-story, one-way asymmetric linear and nonlinear systems with linear and nonlinear fluid viscous dampers shows that the damper nonlinearity leads to only minor (less than 10%) reduction in edge deformations, base shear, and base torque except for linear systems and for nonlinear systems with long periods ($T_y > 0.5$ sec).

For short-period ($T_y < 0.5$ sec) linear and nonlinear systems, damper nonlinearity may be used to achieve significant reduction (of the order to 30%) in flexible-edge deformations. Furthermore, the effects of plan-asymmetry on the flexible-edge are significantly reduced, especially for short period systems. The effects on stiff-edge deformation, base shear, and total damping force are modified very little by the damper nonlinearity. The modification for the base torque and total damping torque is slightly larger.
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INTRODUCTION

Several recent studies have shown that supplemental fluid viscous damping effectively reduces seismic response of elastic and inelastic asymmetric-plan systems [1-5]. However, these investigations examined behavior of asymmetric-plan systems with linear fluid viscous dampers. Nonlinear fluid viscous dampers (velocity exponent less than one) have the apparent advantage of limiting the peak damper force at large velocities while still providing sufficient supplemental damping [5-8]; for linear dampers (velocity exponent equal to one), the damper force increases linearly with damper velocity [9-11].

Seismic response of asymmetric systems with nonlinear viscous and viscoelastic dampers has been the subject of a recent investigation [12]. It was found that structural response is weakly affected by damper nonlinearity, and nonlinear damper achieve essentially the same reduction in response but with much smaller damper force compared to linear dampers. Furthermore, earthquake response of the asymmetric systems with nonlinear dampers can be estimated with sufficient degree of accuracy by analyzing the same asymmetric systems with equivalent linear dampers. Simplified analysis procedure for asymmetric-plan systems with nonlinear dampers has also been developed [13].

While the aforementioned investigations have led to improved understanding of effects of damper nonlinearity on seismic response of asymmetric systems, they were limited to systems with lateral load resisting systems responding in the linear elastic range of behavior. It would also be useful to investigate these effects on systems in which lateral load resisting elements are deformed beyond the elastic limit.

Therefore, the main objectives of this research investigation are to (1) evaluate the effects of damper nonlinearity on seismic response of nonlinear asymmetric systems; and (2) evaluate how the effects of plan-asymmetry are influenced by the damper nonlinearity. For reference purposes, responses of linear systems are also included.

In order to understand the behavior of nonlinear fluid viscous dampers, summarized first is the theoretical background on nonlinear fluid viscous dampers. Based on the dissipated energy equivalence during one harmonic cycle, an equivalent damping ratio for systems with nonlinear viscous dampers is defined. The hysteretic behavior of nonlinear dampers is reviewed followed by the relationships between the damper force and damper velocities. In particular, the force-velocity relationships of nonlinear dampers that provide variable damping versus constant damping over a range of damper velocities are examined.

The effects of damper nonlinearity are investigated by comparing normalized values of seismic responses—edge deformations, base shear and torque, and total damping force and torque at the base—of one-story, one-way asymmetric linear and nonlinear systems having nonlinear fluid viscous dampers (velocity exponent $\alpha = 0.7$, 0.5, and 0.35) with those for systems having linear fluid viscous damper (velocity exponent $\alpha = 1$). Also examined are the ratios of these responses in systems with nonlinear damper and those in systems with linear dampers. The effects of plan-asymmetry are investigated by normalizing the response of asymmetric-plan systems with supplemental dampers with the response of the corresponding symmetric-plan system—a system with relative location and stiffness of all resisting elements as well as location, damping coefficient, and damping exponent of all supplemental dampers identical to those in the asymmetric-plan system but with a rotational degree-of-freedom restricted. These normalized responses in systems with
nonlinear dampers are compared with those in systems with linear dampers to examine how the damper nonlinearity modifies the effects of plan-asymmetry. The seismic responses are computed for a suite of 20 ground motions developed for the SAC studies [14].

THEORETICAL BACKGROUND

Force in Nonlinear Fluid Viscous Damper

The force in a nonlinear fluid viscous damper is given by

\[ f_D = C_\alpha \text{sgn}(\dot{u})|\dot{u}|^\alpha \]  

(1)

in which \( C_\alpha \) is the damper coefficient, \( \dot{u} \) is the damper velocity, \( \text{sgn}(\cdot) \) is the signum function, and \( \alpha \) is the damper exponent ranging in values from 0.2 to 1 for seismic applications [6, 8, 10, 15]. For \( \alpha = 1 \), equation (1) becomes \( f_D = C_\alpha \dot{u} \) which represents force in a linear damper. Therefore, exponent \( \alpha \) is representative of the nonlinearity of a fluid viscous damper.

For a single-degree-of-freedom (SDF) system with mass \( m \), stiffness \( k \), and a nonlinear fluid viscous damper defined by equation (1), the supplemental damping ratio \( \zeta_{sd} \) is defined based on the concept of equivalent linear viscous damping [6, 10, 11, 15] as follows:

\[ \zeta_{sd} = \frac{E_D}{4\pi E_{so}} = \frac{E_D}{2\pi ku_o^2} \]  

(2)

where \( E_{so} \) is the elastic energy stored at the maximum system displacement, \( u_o \). The energy dissipated by the damper \( E_D \) is usually computed as that during one cycle of harmonic motion \( u = u_o \sin \omega t \) at \( \omega = \omega_n \) (\( \omega_n \) = natural frequency of the SDF system) and is given by [6, 8, 10, 15]:

\[ E_D = \pi \beta_\alpha C_\alpha \omega_n^\alpha u_o^{1+\alpha} \]  

(3)

where the constant \( \beta_\alpha \) is

\[ \beta_\alpha = \frac{2^{\frac{1+\alpha}{\alpha}} \Gamma^2(1+\alpha/2)}{\pi \Gamma(2+\alpha)} \]  

(4)

and \( \Gamma(\cdot) \) is the gamma function. Equation (1) can also be written in an alternative but equivalent form [16]. Utilizing equation (3) in equation (2) gives \( \zeta_{sd} \) as a function of the peak displacement \( u_o \):

\[ \zeta_{sd} = \frac{\beta_\alpha C_\alpha}{2m\omega_n^2} (\omega_n u_o)^{\alpha-1} \]  

(5)

Therefore, for a given value of supplemental damping ratio, \( \zeta_{sd} \), the damper coefficient of a nonlinear damper with damper exponent of \( \alpha \) can be calculated as
For a linear damper with $\alpha = 1$, equation (6) gives $C_1 = 2m\omega_n\zeta_{sd}$ implying that the damping coefficients of a nonlinear and linear damper, both with same damping ratio, $\zeta_{sd}$, are related as:

$$C_a = \frac{2m\omega_n\zeta_{sd}(\omega_n u_a)^{1-\alpha}}{\beta_a}$$

(6)

Utilizing equation (7), equation (1) can be re-written as:

$$\frac{f_D(t)}{f_D(\alpha = 1)} = \frac{1}{\beta_a} \left( \frac{\omega_n u_a}{\dot{u}_a} \right)^{1-\alpha} \text{sgn}(\dot{u}) |\dot{u}|^\alpha$$

(8)

and the peak value of the damper force is given as:

$$\frac{f_{D_0}(\alpha)}{f_{D_0}(\alpha = 1)} = \frac{1}{\beta_a} \left( \frac{V}{\dot{u}_a} \right)^{1-\alpha}$$

(9)

in which $V = \omega_n u_a$ is the pseudo-velocity for the SDF system.

It is usual to define $C_a$ when the system is subjected to harmonic motion with peak displacement equal to the design displacement $u_{des}$. For this case, the damper force is given by:

$$\frac{f_{D_0}(\alpha)}{C_a \omega_n u_{des}} = \frac{1}{\beta_a} \left( \frac{\dot{u}_a}{\dot{u}_{des}} \right)^\alpha$$

(10)

It is useful to emphasize that equation (10) represents the relationship between force and velocity of a nonlinear damper for which the damping coefficient $C_a$ is defined at $u_{des}$. Therefore, the damping ratio is equal to $\zeta_{sd}$ only at displacement equal to $u_{des}$; for displacements (and hence velocities) either lower or higher than $u_{des}$, the equivalent damping ratio would not be equal to $\zeta_{sd}$. For systems in which the damping ratio is equal to $\zeta_{sd}$ at all displacements, the damper force is given by:

$$\frac{f_{D_0}(\alpha)}{C_a \omega_n u_{des}} = \frac{1}{\beta_a} \left( \frac{\dot{u}_a}{\dot{u}_{des}} \right)$$

(11)

Behavior of Nonlinear Fluid Viscous Dampers

Figure (1a) presents the force-displacement response (or hysteresis loops) of linear ($\alpha = 1$) and nonlinear ($\alpha = 0.35$ and 0) fluid viscous dampers with equivalent damping ratio $\zeta_{sd}$ when subjected to harmonic motion. The hysteresis loop for the linear damper ($\alpha = 1$) is well known- elliptical shape whereas that of nonlinear damper with $\alpha = 0$ (friction damper) is rectangular; the shape for nonlinear damper with $0 < \alpha < 1$ fall between these two
Because all hysteresis loops enclose same area (\(\zeta_{nl}\) was defined based on equal energy dissipation or equal area), the peak damper force in nonlinear damper (\(\alpha < 1\)) is less than that for the linear damper (\(\alpha = 1\)). For systems subjected to nonlinear damper (\(\alpha < 1\)) is less than that for the linear damper (\(\alpha = 1\)). For systems subjected to harmonic motion, equation (9) simplifies to \(f_{D_0}(\alpha) / f_{D_0}(\alpha = 1) = 1 / \beta\), which gives 0.785 (= \(\pi / 4\)) for \(\alpha = 0\) and 0.866 for \(\alpha = 0.35\). This indicates that the peak damper force in friction damper is about 22% and 13% less in nonlinear dampers with \(\alpha = 0\) and 0.35, respectively, compared to the linear damper.

The reductions in peak damper force noted in Figure (1a) occur for systems subjected to harmonic motion; similar level of reduction may not occur when the same system is subjected to ground motions. To investigate this, plotted in Figure (1b) is the relationship between the damper force and ratio of the pseudo-velocity and peak velocity, \(V / \dot{u}_o\) (equation 9). Note that for systems subjected to harmonic motions, \(V / \dot{u}_o = 1\), and the peak force in nonlinear damper is less than that in the linear damper (Figure 1b). For values of \(V / \dot{u}_o\) larger

Figure 1. Behavior of linear and nonlinear fluid viscous dampers.
than a certain threshold value, the force in nonlinear damper may become larger than that in
the linear damper. The threshold value occurs for \( V / \dot{u}_o = \beta_u^{\alpha (1-\alpha)} \) (= 1.27, 1.25, and 1.23, for
\( \alpha = 0, 0.7, \) and 0.35, respectively). For earthquake ground motions, the ratio \( V / \dot{u}_o \) can be
larger than one for very-short period systems [11: Sec 6.12], and as a result the force in
nonlinear damper may be larger than in linear damper.

Figure (1c) presents the relationship of equation (10) for nonlinear damper with its
damping coefficient defined to give equivalent \( \zeta_{sd} \) at peak displacement of \( u_{des} \). At value of
\( \dot{u}_o / \dot{u}_{des} \) larger than certain threshold value, the peak force in nonlinear damper is less than
that in the linear damper; the threshold value of \( \dot{u}_o / \dot{u}_{des} \) is less than one and depends on \( \alpha \).
Furthermore, the peak force tends to reach a upper bound in nonlinear damper with
increasing values of \( \dot{u}_o / \dot{u}_{des} \), as apparent from flattening of the curves for \( \alpha < 1 \); the force in
linear damper keeps on increasing linearly with \( \dot{u}_o / \dot{u}_{des} \). This indicates that damper
nonlinearity tends to limit the damper force at velocities in excess of the design velocity. This
behavior is generally cited as a major advantage of nonlinear dampers over the linear
dampers [7, 8]. However, it must be noted that for values of \( \dot{u}_o / \dot{u}_{des} > 1 \), the equivalent
damping provided by the nonlinear damper is smaller compared to its design value (Fig. 1d).
For \( \dot{u}_o / \dot{u}_{des} < 1 \), on the other hand, nonlinear damper provide larger damping ratio compared
to the design value.

The behavior of nonlinear dampers that provide damping ratio equal to selected \( \zeta_{sd} \)
value at all displacements and velocities (equation 11) is plotted in Figure (1e). It is apparent
from these results that the peak force increases linearly for linear as well as nonlinear
dampers. However, the rate of increase is lower for nonlinear dampers compared to the linear
dampers. As expected, the damping ratio remains the same for all values of \( \dot{u}_o / \dot{u}_{des} \) (Figure 1f).

**SYSTEM, GROUND MOTIONS, AND RESPONSE STATISTICS**

**System**
The system considered was the idealized one-story building of Figure 2 consisting of a rigid
deck supported by structural elements (wall, columns, moment-frames, braced-frames, etc.),
and fluid viscous dampers incorporated into the bracing system. The mass properties of the
system were assumed to be symmetric about both the \( X \)- and \( Y \)-axes whereas the stiffness and
the damper properties were considered to be symmetric only about the \( X \)-axis.

The center of mass (CM) of the system was defined as the centroid of inertia forces
when the system is subjected to a uniform translational acceleration in the direction under
consideration. Since the mass was uniformly distributed about both the \( X \)- and \( Y \)-axes, the
CM coincided with the geometric center of the deck.

The center of rigidity (CR) was defined as the point on the deck through which
application of a static horizontal force causes no rotation of the deck. The lack of symmetry
in the stiffness properties about the \( Y \)-axis was characterized by the stiffness eccentricities, \( e \),
deefined as the distance between the CM and the CR. With both CM and CR defined, the edge
that is on the same side of the CM as the CR was denoted as the stiff edge and the other edge was designated as the flexible edge (Figure 2a).

The center of supplemental damping (CSD) was defined as the centroid of damper forces when the system is subjected to a uniform translational velocity in the direction under consideration. The lack of symmetry in the damper properties about the Y-axis was characterized by the supplemental damping eccentricity, \( e_{sd} \), defined as the distance between the CM and the CSD.

The corresponding symmetric-plan system was defined as a system with coincidental CM, CR, and CSD, but with relative locations and stiffnesses of all resisting elements as well as locations and damping coefficients of all supplemental dampers identical to those in the asymmetric-plan system. In other words, the corresponding symmetric-plan system is identical to the asymmetric-plan system but with rotational degree of freedom restrained.

The lateral force resisting system consists of six-elements, three each in the x- and y-directions (Figure 2a). The middle element in each direction is located at the CM and the two outermost elements are equidistant from the CM; their location is based on the system parameters described in the next section. Since the stiffness eccentricity in the y-direction is zero, the three elements in the x-direction have equal stiffness, i.e., \( k_{x1} = k_{x2} = k_{x3} \). In the y-direction, elements 2 and 3 possess equal stiffness, i.e., \( k_{y2} = k_{y3} \), while the stiffness of element 1 is larger than those of elements 2 and 3; the relative values depend on the stiffness eccentricity in the x-direction.

The supplemental damping distribution consists of six-dampers, three each in the x- and y-directions (Figure 2b). The two outermost dampers in each direction are located at the two edges and the middle damper is located at the CM. The total damping in the x- and y-directions are assumed to be equal. Furthermore, the damper distribution in the x-direction is assumed to be symmetric, i.e., \( C_{x1} = C_{x2} = C_{x3} \). In the y-direction, \( C_{y2} = C_{y3} \), and \( C_{y1} > C_{y2} \) or \( C_{y3} \); the relative values are decided based on the damping eccentricity in the x-direction. The procedure to compute the damper coefficients based on the damper parameters is described in Appendix I.
Ground Motions

The sets of 20 ground motion records were assembled for Los Angeles, Seattle, and Boston representing probabilities of exceedance of 2%, 10%, and 50% in 50 years (return periods of 2475, 475, and 72 years, respectively) [14]. The 10% probability of exceedance in 50 years set of records developed for Los Angeles are used in this investigation (Table 1). The 25%-damped pseudo-acceleration and displacement response spectrum for each ground motion is shown in Figures 3a and 3b, respectively. Also shown is the median response spectrum, developed according to the procedure presented in the next section.

Table 1. Basic characteristics of ground motion considered.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Record Information</th>
<th>Duration (sec)</th>
<th>Magnitude $M_w$</th>
<th>R (km)</th>
<th>PGA (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LA01</td>
<td>Imperial Valley, 1940</td>
<td>39.38</td>
<td>6.9</td>
<td>10.0</td>
<td>0.46</td>
</tr>
<tr>
<td>LA02</td>
<td>Imperial Valley, 1940</td>
<td>39.38</td>
<td>6.9</td>
<td>10.0</td>
<td>0.68</td>
</tr>
<tr>
<td>LA03</td>
<td>Imperial valley, 1979</td>
<td>39.38</td>
<td>6.5</td>
<td>4.1</td>
<td>0.39</td>
</tr>
<tr>
<td>LA04</td>
<td>Imperial valley, 1979</td>
<td>39.38</td>
<td>6.5</td>
<td>4.1</td>
<td>0.49</td>
</tr>
<tr>
<td>LA05</td>
<td>Imperial valley, 1979</td>
<td>39.38</td>
<td>6.5</td>
<td>1.2</td>
<td>0.30</td>
</tr>
<tr>
<td>LA06</td>
<td>Imperial valley, 1979</td>
<td>39.38</td>
<td>6.5</td>
<td>1.2</td>
<td>0.23</td>
</tr>
<tr>
<td>LA07</td>
<td>Landers, 1992</td>
<td>79.98</td>
<td>7.3</td>
<td>36.0</td>
<td>0.42</td>
</tr>
<tr>
<td>LA08</td>
<td>Landers, 1992</td>
<td>79.98</td>
<td>7.3</td>
<td>36.0</td>
<td>0.43</td>
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<tr>
<td>LA09</td>
<td>Landers, 1992</td>
<td>79.98</td>
<td>7.3</td>
<td>25.0</td>
<td>0.52</td>
</tr>
<tr>
<td>LA10</td>
<td>Landers, 1992</td>
<td>79.98</td>
<td>7.3</td>
<td>25.0</td>
<td>0.36</td>
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<tr>
<td>LA11</td>
<td>Loma Prieta, 1989</td>
<td>39.98</td>
<td>7.0</td>
<td>12.4</td>
<td>0.67</td>
</tr>
<tr>
<td>LA12</td>
<td>Loma Prieta, 1989</td>
<td>39.98</td>
<td>7.0</td>
<td>12.4</td>
<td>0.97</td>
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<tr>
<td>LA13</td>
<td>Northridge, 1994, Newhall</td>
<td>59.98</td>
<td>6.7</td>
<td>6.7</td>
<td>0.68</td>
</tr>
<tr>
<td>LA14</td>
<td>Northridge, 1994, Newhall</td>
<td>59.98</td>
<td>6.7</td>
<td>6.7</td>
<td>0.66</td>
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<tr>
<td>LA15</td>
<td>Northridge, 1994, Rinaldi</td>
<td>14.95</td>
<td>6.7</td>
<td>7.5</td>
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<tr>
<td>LA16</td>
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<td>14.95</td>
<td>6.7</td>
<td>7.5</td>
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<td>LA17</td>
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<td>6.0</td>
<td>6.7</td>
<td>1.02</td>
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<td>LA20</td>
<td>North Palm Springs, 1986</td>
<td>59.98</td>
<td>6.0</td>
<td>6.7</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Response Statistics

The dynamic response of each system to each of 20 ground motions is determined by response history analysis [11]. Presented in this paper are median values $\hat{x}$, defined as the geometric mean, of $n(=20)$ observed values of $x_i$ of the peak value of the structural response [17]:

\[ \hat{x} = \left( \prod_{i=1}^{n} x_i \right)^{1/n} \]
The linear elastic response of one-story, asymmetric-plan systems without supplemental
damping depends on (1) transverse vibration period, $T_y = 2\pi/\omega_y$ ($\omega_y$ = vibration frequency),
of the corresponding symmetric-plan system in the Y-direction; (2) normalized stiffness
eccentricity, $\bar{e} = e/a$ ($a$ = plan dimension perpendicular to the direction of ground motion);
(3) ratio of the torsional and transverse frequencies, $\Omega_0$; (4) aspect ratio of the deck, $a/d$;
and (5) mass and stiffness proportional damping constants, $\alpha_0$ and $\alpha_1$, which in turn depend
on the natural damping ratios in the two vibration modes of the system. Detailed description
of various parameters of a linear system is available elsewhere [1].

The additional parameters needed to include supplemental damping are: (1) supplemental damping ratio, $\zeta_{sd}$; (2) normalized supplemental damping eccentricity, $\bar{e}_{sd} = e_{sd}/a$; and (3) damper velocity exponent, $\alpha$. The supplemental damping ratio for systems with nonlinear dampers is defined by equation (5), in which $u_o$ is taken as the
deformation from the median elastic response spectrum for damping ratio equal to natural
damping plus the supplemental damping (Figure 3b). This approach differs from an earlier
investigation [18] that used an iterative procedure to define the damping ratio for systems
with nonlinear dampers. There are two reasons for adopting this approach. First, the
convergence in the iterative procedure may be difficult to obtain for systems with lateral load
resisting elements responding in the nonlinear range. This is especially true for short period
systems. Second, the iterative approach [18] leads to different damper coefficients to achieve
a selected damping ratio for different ground motions of an ensemble. For investigations
using an ensemble of ground motions, such as the present study, it may be useful to keep the
damper coefficients same for all ground motions.
For nonlinear systems, the strengths of lateral load resisting elements is defined as 
\[ f_x = k_x U_x \] and 
\[ f_y = k_y U_y \] in which 
\[ U_x = u_{ex} / R_x \] and 
\[ U_y = u_{ey} / R_y \] are the system yield displacements in the x- and y-directions, respectively, and 
\[ R_x \] and \[ R_y \] are the x- and y-direction strength reduction factors. It is useful to note here that strength distribution selected in this manner is consistent with the constant-D type distribution [19] and that advocated recently for asymmetric-plan systems [20]. While other strength distributions are possible, this distribution was selected for simplicity. The quantities \( u_{ex} \) and \( u_{ey} \) are taken as the x- and y-direction peak deformations of corresponding symmetric-plan system with linear viscous damping equal to natural damping plus the supplemental damping.

Responses are presented for the following values of system parameters: \( T_y \) in the range of 0.05 to 3 sec; \( \Omega_0 = 1; \) \( \bar{e} = 0.2; \) aspect ratio = 2; and \( \zeta = 5\% \) in all modes of the corresponding linear elastic symmetric-plan system. The parameters for the supplemental damping system were selected as: \( \zeta_{sd} = 20\%; \) \( \bar{e}_{sd} = -0.2; \) and \( \alpha = 1, 0.7, 0.5, \) and 0.35. For nonlinear systems, \( R_x \) and \( R_y = 4 \) were selected; the system is expected to be excited well into the inelastic range for this value of \( R_x \) and \( R_y \).

**Response Considered**

The following six response quantities are considered in this investigation: stiff- and flexible-edge deformations; base shear and base torque; and total damping force and damping torque at the base of the system. While edge deformation have been examined traditionally for asymmetric-plan systems [1], the various force quantities are included for the following reason. Behavior of linear and nonlinear fluid viscous dampers in the single-degree-of-freedom system subjected to harmonic motions indicates that the damper nonlinearity tends to limit the damper force for velocities in excess of the design velocity. While this assertion has been examined previously for symmetric systems subjected to ground motions, e.g., [16], it would be useful to examine it for asymmetric-plan systems where not only lateral forces but also torsional moments occur.

In order to evaluate the effects of damper nonlinearity, response of the asymmetric-plan system with linear and nonlinear dampers is normalized by that of the corresponding symmetric system with linear damper. These values are defined as: 
\[ U'_x = u_{x,ASYM} / u_{x,SYM} \] for the stiff-edge deformation, 
\[ U'_y = u_{y,ASYM} / u_{y,SYM} \] for the flexible-edge deformation, 
\[ V'_b = V_{b,ASYM} / V_{b,SYM} \] for the base shear, 
\[ T'_b = T_{b,ASYM} / e V_{b,SYM} \] for the base torque, 
\[ V'_d = V_{d,ASYM} / V_{d,SYM} \] for the damping force at the base, and 
\[ T'_d = T_{d,ASYM} / e V_{d,SYM} \] for the damping torque at the base. Also examined is the ratio of the response of asymmetric system with nonlinear and linear damper.

In order to investigate the effects of plan asymmetry in system with nonlinear damper, response of the asymmetric-plan system with nonlinear dampers is normalized by that of the corresponding symmetric-plan system with same nonlinear dampers. These values are defined as: 
\[ U^*_x = u_{x,ASYM} / u_{x,SYM} \] for the stiff-edge deformation, 
\[ U^*_y = u_{y,ASYM} / u_{y,SYM} \] for the flexible-edge deformation, 
\[ V^*_b = V_{b,ASYM} / V_{b,SYM} \] for the base shear,
$T_b^* = \frac{T_{b,a,ASYM}}{V_{b,a,SYM}}$ for the base torque, $V_d^* = \frac{V_{d,a,ASYM}}{V_{d,a,SYM}}$ for the damping force at the base, and $T_d^* = \frac{T_{d,a,ASYM}}{V_{d,a,SYM}}$ for the damping torque at the base.

**EFFECTS OF DAMPER NONLINEARITY**

Figures 4 presents the median values of normalized responses – $U_b^*$, $U_f^*$, $V_b^*$, $T_b$, $V_d^*$, and $T_d^*$ – of the linear asymmetric systems with linear and nonlinear dampers. Figure 5 shows the ratio of the median values of the response of linear asymmetric-plan systems with nonlinear dampers and linear dampers. These results lead to the following observations.

![Graphs showing seismic response of elastic asymmetric systems with linear and nonlinear fluid viscous dampers.](image)

Figure 4. Seismic response of elastic asymmetric systems with linear and nonlinear fluid viscous dampers.

The damper nonlinearity leads to reduction in the stiffness- and flexible-edge deformations (Figures 4a and 4b). The reduction tends to increase with decreasing values of $\alpha$. The reduction for the stiff-edge is minimal: the reduction is, in general, less than 10% (Figure 5a) for the lowest value of $\alpha = 0.35$ considered in this investigation. The damper nonlinearity leads to larger reduction in the flexible-edge deformation (Figure 5b) compared to that in the stiff-edge deformation (Figure 5a). The reduction tends to be larger for short period systems and decreases as the system period increases (Figure 5b). The reduction may
be as large as 30% for short period systems ($T_y < 0.5$ sec). For longer period systems, the reduction may only be by less than 10%.

![Graphs](image)

**Figure 5. Ratio of seismic response of linear asymmetric systems with nonlinear ($\alpha = 0.7, 0.5, \text{ and } 0.35$) and linear ($\alpha = 1$) fluid viscous dampers.**

The damper nonlinearity has little influence on the base shear as apparent from curves for all values of $\alpha$ being nearly identical (Figure 4c). The ratio of base shear in systems with nonlinear and linear damper presented in Figure (5c) confirms this observation as the ratio is nearly equal to one over the entire period range. The base torque (Figure 4d) is reduced to a much larger degree compared to the base shear (Figure 4c). The largest reduction in the base torque is about 20% for $\alpha = 0.35$ and occurs for $T_y < 0.5$ sec (Figure 5d). For longer period systems ($T_y > 2$ sec) the reduction in base torque is minimal.

As observed previously for linear and nonlinear dampers subjected to harmonic motions, the total damper force in general reduces with reducing value of $\alpha$ (Figure 4e) in linear asymmetric systems subjected to ground motions. The percent reduction is about 15% for $\alpha = 0.35$ over the wide range of period values considered (Figure 5e). The total damping
torque at the base, however, may be increased slightly (Figure 4f); the increase may be as larger as 10% for \( \alpha = 0.35 \) (Figure 5f).

For systems with very short period (e.g., \( T_y \leq 0.1 \) sec for selected parameters), the total damping force may increase, instead of reducing, due to damper nonlinearity (Figure 4e). This increase may be as large as 25% (Figure 5e). Therefore, damping nonlinearity does not always reduce the damper force in asymmetric systems, as has been generally believed to occur for symmetric systems [6-10], and found in a recent investigation on asymmetric systems [12]. Furthermore, the increase in total damping torque for such systems may exceed 30% (Figure 5f).

In order to investigate how the effects of damper nonlinearity are modified by the system nonlinearity, the median values of normalized responses and the ratios are presented in Figures 6 and 7, respectively for nonlinear systems. These results lead to the following observations.

![Figure 6](image)

**Figure 6.** Seismic response of nonlinear asymmetric systems with linear and nonlinear fluid viscous dampers.

The trends for edge deformations in nonlinear systems (Figures 6a, 6b, 7a, and 7b) are in general similar to those for the linear systems observed earlier. The base shear may
reduce slightly for long period systems (Figures 6c) with the reduction being less than 10% (Figure 7c). The base torque, however, may increase significantly (Figure 6d) with the increase being larger than 20% over a wide range of periods for $\alpha = 0.35$ (Figure 7d). The trends for total damper force and damper torque (Figures 6e, 6f, 7e, and 7f) are, however, generally similar to those observed earlier for elastic systems (Figures 4e, 4f, 5e, and 5f).

![Graphs showing seismic response ratios](image)

Figure 7. Ratio of seismic response of nonlinear asymmetric systems with nonlinear ($\alpha = 0.7, 0.5, \text{and} 0.35$) and linear ($\alpha = 1$) fluid viscous dampers.

In summary, the damper nonlinearity leads to only minor (less than 10%) reduction in stiff edge deformation. The reduction in the flexible edge deformation of the order of 30% may be achieved for short period systems ($T_y < 0.5$ sec). For longer period systems, however, the reduction may only be by less than 10%. The base shear is essentially unaffected by the damper nonlinearity. The base torque may be reduced by up to 20% for short period linear and nonlinear systems but may be increased by up to 20% for longer period nonlinear systems. The damper nonlinearity leads to about 15% reduction in the total damping force for longer period systems. For very short period systems ($T_y \leq 0.1$ sec), however, the total damping force may increase by up to 25% due to damper nonlinearity. The total damping torque at the base increases slightly over the period range considered in this investigation.
EFFECTS OF PLAN-ASYMMETRY

Effects of plan-asymmetry are evaluated by examining the ratio of the response of the asymmetric-plan system and its corresponding symmetric-plan system; note that the corresponding symmetric-plan system will have the same damping ratio, $\zeta_{sd}$, and velocity exponent, $\alpha$, as the asymmetric-plan system. The median values of the ratio for six response quantities – $U_x^*$, $U_f^*$, $V_b^*$, $T_b^*$, $V_d^*$, and $T_d^*$ – are presented in Figures 8 and 9 for linear and nonlinear systems, respectively. These results lead to the following conclusions.

Figure 8. Effects of plan-asymmetry in seismic response of linear systems with linear and nonlinear fluid viscous dampers.

The plan-asymmetry generally has the effect of increasing deformation of the flexible edge, as apparent from the ratio being generally larger than one for this edge (Figures 8b and 9b). This observation is consistent with those of several previous investigations [21, 22]. The damper nonlinearity reduces this effect; increase in the flexible-edge deformation (Figures 8b and 9b) is smaller for lower values of $\alpha$. For example, median value of $U_f^*$ for a linear...
system with $T_y = 0.5$ sec may be reduced by about 18% due to damper nonlinearity (Figure 8b); $U_f^* = 1.388$ for linear damper ($\alpha = 1$) which reduces to $U_f^* = 1.133$ for nonlinear damper with $\alpha = 0.35$. The effects on the stiff-side elements are modified to a much smaller degree (Figures 8a and 9a) compared to the flexible-edge deformation.

The plan-asymmetry reduces the base shear of linear systems. The damper nonlinearity does not modify this trend as apparent from essentially identical curves for various values of $\alpha$ (Figures 8c). Since, the base shear in a nonlinear system is limited by its lateral strength, the plan-asymmetry appears to have no effect on the base shear in nonlinear systems; note that the ratio is essentially one over the entire period range (Figure 9c). As noted for linear systems, the damper nonlinearity does not further modify this trend for nonlinear systems (Figure 9c). The plan-asymmetry leads to base torque in asymmetric-plan systems, which is reduced slightly for linear systems and increased for nonlinear systems by the damper nonlinearity (Figures 8d and 9d).

![Figure 9](image)

Figure 9. Effects of plan-asymmetry in seismic response of nonlinear systems with linear and nonlinear fluid viscous dampers.

The plan-asymmetry generally tends to reduce the total damping force at the base in linear systems apparent from the ratio being smaller than one for most period values (Figure...
8e). The damper nonlinearity has the effect of reducing this effect, i.e., the reduction in total damping force at the base is smaller as $\alpha$ reduces. For nonlinear systems, the ratio is essentially one for all values of $\alpha$ over the entire period range (Figure 9e) indicating that effects of plan-asymmetry for such systems are minimal and they are not influenced by the damper nonlinearity. The plan-asymmetry as well as asymmetry in the damper distribution gives rise to the total damping torque at the base, which tends to increase with decreasing values of $\alpha$.

In summary, the effects of plan-asymmetry on the flexible edge deformation are reduced by the damper nonlinearity. The effects on the stiff-edge deformation, base shear, and total damping force are modified very little by the damper nonlinearity. The modification for the base torque and total damping torque is slightly larger.

**CONCLUSIONS**

The investigation on the behavior of linear and nonlinear fluid viscous dampers subjected to harmonic motion led to the following conclusions, many of which are consistent with earlier observations [7-10, 15, 16, 18, 23]:

1. For harmonic motions, the peak force in nonlinear damper is smaller compared to linear damper. However, this is true only for $V/\dot{u}_o$ less that a certain threshold value, which is slightly larger than 1. For larger values of $V/\dot{u}_o$, the force in nonlinear damper may become larger than that in the linear damper.

2. The damper nonlinearity tends to limit the force in nonlinear dampers with $C_a$ defined at the design displacement $u_{des}$ and velocity $\dot{u}_{des}$ for velocities in excess of $\dot{u}_{des}$. However, such dampers provide smaller equivalent damping at higher velocities.

The investigation on seismic response of one-story, one-way asymmetric linear and nonlinear systems with linear and nonlinear fluid viscous dampers has led to the following conclusions:

1. The damper nonlinearity leads to only minor (less than 10%) reduction in stiff-edge deformation over the entire period range. The reduction of the order of 30% may be achieved in flexible-edge deformation for short period ($T_y < 0.5$ sec) systems; for longer period systems, the reduction in the flexible-edge deformation is comparable to that for the stiff-edge deformation.

2. The damper nonlinearity leads to minor reduction (less than 10%) in the base shear. The base torque may reduce by 20% for short period ($T_y < 0.5$ sec) linear and nonlinear systems, but may increase by 20% for longer period ($T_y > 0.5$ sec) nonlinear systems.

3. The damper nonlinearity leads to about 15% reduction in the total damping force for most periods. For very-short period ($T_y \leq 0.1$ sec) systems, however, the total damping force may increase, instead of reducing, with increase being as large as 25%. The damping nonlinearity may also lead to slight increase in the total damping torque at the base over the entire period range.
4. The effects of plan-asymmetry on the flexible edge deformation are significantly reduced by the damper nonlinearity, especially for short period systems. The effects on the stiff-edge deformation, base shear, and total damping force are modified very little by the damper nonlinearity. The modification for the base torque and total damping torque is slightly larger.

APPENDIX I. PLAN-WISE DISTRIBUTION OF NONLINEAR DAMPERS

The supplemental damping in systems with linear fluid viscous dampers can be characterized by three parameters [1]: (1) \( \zeta_{sd} = \frac{C_y}{2m\omega_y} \) representing the total damping provided by supplemental dampers; (2) \( \varepsilon_{sd} = \frac{e_{sd}}{a} \) representing x-direction eccentricity in the plan-wise distribution of supplemental damping; and (3) \( \bar{\rho}_{sd} = \sqrt{\frac{C_{sd}}{C_y}}/a \) representing plan-wise spread of the supplemental dampers. For systems with nonlinear dampers, only the first two parameters can be uniquely defined. Therefore, a pre-defined plan-wise spread of dampers is used in this investigation. Such distribution consists of six-dampers, three each in the x- and y-directions. The two outermost dampers in each direction are located at the two edges and the middle damper is located at the CM. The total damping in the x- and y-directions are assumed to be equal. Furthermore, the damper distribution in the x-direction is assumed to be symmetric, i.e., y-direction eccentricity in the supplemental damping is equal to zero. For systems with linear fluid viscous dampers, this distribution corresponds to \( \bar{\rho}_{sd} \) approximately equal to 0.37 for the selected parameters. With such a distribution, the damping coefficient of each of the elements is related to the first two parameters as:

\[
C_{x1} = C_{x2} = C_{x3} = \frac{(\omega_x u_{xo})^{1-a}}{\beta_a} \frac{2m\omega_x \zeta_{sd}}{3} \quad (I.1)
\]

\[
C_{y1} = \frac{(\omega_y u_{yo})^{1-a}}{\beta_a} \frac{2m\omega_y \zeta_{sd}}{3} \left(1 + \frac{4e_{sd}}{a}\right) \quad \text{and} \quad C_{y2} = C_{y3} = \frac{(\omega_y u_{yo})^{1-a}}{\beta_a} \frac{2m\omega_y \zeta_{sd}}{3} \left(1 - \frac{2e_{sd}}{a}\right) \quad (I.2)
\]

in which \( \omega_x \) and \( \omega_y \) are natural frequencies and \( u_{xo} \) and \( u_{yo} \) are the peak deformations of the corresponding symmetric-plan system in the x- and y-directions, respectively. Since, the systems in this investigation are subjected to ground motion in the y-direction only, \( u_{xo} \) is equal to zero. However, \( u_{xo} \) is taken to be equal to \( u_{yo} \) for the purpose of defining damping coefficients of x-dampers (equation I.1), i.e., \( u_{xo} = u_{yo} = u_o \).

REFERENCES


