Direct Displacement-Based Design Using Inelastic Design Spectrum

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Objectives

• Demonstrate application of inelastic design spectra to direct displacement-based design (DDBD)

• Demonstrate potential limitations of current DDBD that use elastic design spectra and equivalent linear systems
Equivalent Linear System: Period

- For bilinear systems

\[ T_{eq} = T_n \sqrt{\frac{\mu}{1 + \alpha \mu - \alpha}} \]

- For elasto-plastic systems

\[ T_{eq} = T_n \sqrt{\mu} \]
Equivalent Linear System: Damping

- For bilinear systems
  \[ \zeta_{eq} = \frac{2 (\mu - 1)(1 - \alpha)}{\pi \mu (1 + \alpha \mu - \alpha)} \]

- For elasto-plastic systems
  \[ \zeta_{eq} = \frac{2 (\mu - 1)}{\pi \mu} \]
Equivalent Linear System

\[ T_{eq} = T_n \]

\[ \alpha = 0, 0.05, 0.1, 0.2 \]

\[ \zeta_{eq} \]

\[ \alpha = 0, 0.05, 0.1, 0.2 \]
Substitute Damping

Gulkan & Sozen, Shibata & Sozen
(Takeda model for R/C structures)
Elastic Design Spectrum

Pseudo-velocity (log scale)

Elastic design spectrum

Peak ground acceleration, velocity, and displacement

Natural vibration period (log scale)

1/33 sec 33 Hz
1/8 sec 8 Hz
10 sec 1/10 Hz
33 sec 1/33 Hz
Elastic Design Spectrum

Pseudo-Acceleration

Deformation
DDBD Using Elastic Spectra: Step-by-Step Procedure

1. Estimate the yield deformation for the system
2. Establish acceptable plastic rotation, $\theta_p$
3. Determine design displacement and ductility factor: $u_m = u_y + h \theta_p$ and $\mu = u_m / u_y$
4. Estimate the total equivalent viscous damping:

\[ \zeta_{eq} = \frac{2}{\pi} \frac{(\mu - 1)(1 - \alpha)}{\mu(1 + \alpha \mu - \alpha)} \]

\[ \hat{\zeta}_{eq} = \zeta + \zeta_{eq} \]
DDBD Using Elastic Spectra: Step-by-Step Procedure

5. Enter deformation design spectrum and read $T_{eq}$. 
   ➔ Determine the secant stiffness

$$k_{sec} = \frac{2\pi^2}{T_{eq}^2} m$$
6. Determined the required yield strength:

\[ f_y = \frac{k_{sec} u_m}{1 + \alpha \mu - \alpha} \]
DDBD Using Elastic Spectra: Step-by-Step Procedure

7. Estimate member size and detail (reinforcement in R/C structures, connections in steel structures) to provide $f_y$.
   ➔ Calculate initial elastic stiffness $k$.
   ➔ Calculate yield deformation: $u_y = f_y / k$

8. Repeat steps 3 to 7 until a satisfactory solution is obtained.
Example

- R/C viaduct
- Superstructure weight = 190 kN/m
- Bent spacing = 39.6 m

\[ W = 7517 \text{ kN} \]

\[ k = \frac{3EI}{H^3} \]
Design Summary: DDBD Using Elastic Spectra

- Starting yield displacement = 4.5 cm.
- Convergence achieved after three iterations.
- The final design has:
  - Longitudinal column reinforcement = 1.3%
  - Initial stiffness = 95.17 kN/cm
  - Lateral yield strength = 839.7 kN
  - Yield displacement = 8.82 cm, Design displacement = 26.8 cm
  - Elastic period = 1.78 sec, Secant period = 3.14 sec
Inelastic Design Spectrum

Pseudo-velocity $V$ or $V_y$ (log scale)

Elastic design spectrum

$A = \alpha V u_{go}$

$V = \alpha V u_{go}$

$D = \gamma_0 u_{go}$

$V / \mu$

Inelastic design spectrum

Natural vibration period $T_n$ (log scale)

$1/33$ sec
$33$ Hz

$1/8$ sec
$8$ Hz

$10$ sec
$1/10$ Hz

$33$ sec
$1/33$ Hz

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Conversion of Elastic to Inelastic Design Spectrum

- Use available $R_y - \mu - T_n$ relationships
  - Newmark and hall
  - Krawinkler et al
  - Fajfar et al.
  - Miranda & Bertero

- Relationships based on nonlinear analysis of SDF systems
Constant Ductility Inelastic Design Spectrum

Newmark & Hall

Krawinkler et al.
Inelastic Design Spectrum

Pseudo-Acceleration

Deformation
DDBD Using Inelastic Spectra: Step-by-Step Procedure

1. Estimate the yield deformation for the system.
2. Establish acceptable plastic rotation, $\theta_p$.
3. Determine design displacement and ductility factor: $u_m = u_y + h \theta_p$ and $\mu = u_m / u_y$. 

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DDBD Using Elastic Spectra: Step-by-Step Procedure

4. Enter deformation design spectrum and read $T_n$.

- Determine the elastic stiffness

$$k = \frac{2\pi^2}{T_n^2} m$$
DDBD Using Inelastic Spectra: Step-by-Step Procedure

5. Determine the required yield strength
   \[ f_y = k u_y \]

6. Estimate member size and detail (reinforcement in R/C structures, connections in steel structures) to provide \( f_y \).
   \[ \Rightarrow \text{Calculate initial elastic stiffness } k. \]
   \[ \Rightarrow \text{Calculate yield deformation: } u_y = f_y / k \]

7. Repeat steps 3 to 6 until a satisfactory solution is obtained.
Design Summary: DDBD Using Inelastic Spectra

- Starting yield displacement = 4.5 cm.
- Convergence achieved after five iterations
- The final design has:
  - Longitudinal column reinforcement = 5.5%
  - Initial stiffness = 238.6 kN/cm
  - Lateral yield strength = 1907 kN
  - Yield displacement = 7.99 cm, Design displacement = 26.0 cm
  - Elastic period = 1.16 sec
Evaluation of Design: Inelastic Analysis

1. Calculate initial elastic period from $m$ and $k$
2. Determine $A$ from elastic design spectrum
   \[ \text{Elastic Design Force: } f_o = mA \]
3. From known $f_y$, calculate: $R_y = f_o / f_y$
4. Determine ductility demand $\mu$ from $R_y-\mu-T_n$ relationships
5. Calculate displacement and plastic rotation
   \[ u_m = (\mu / R_y)(T_n / 2\pi)^2A \]
   \[ \theta_p = (u_m-u_y)/h \]
Evaluation of Example Design: DDBD Using Elastic Spectra

- Demands from inelastic analysis of the design structure
  - $u_m = 39.7$ cm.
  - $\mu = 4.52$
  - $\theta_p = 0.0343$ rad.

- Design using elastic design spectrum
  - $u_m = 26.8$ cm (32.6% underestimation)
  - $\mu = 3.04$ (32.6% underestimation)
  - $\theta_p = 0.02$ rad
    (Demand exceeds acceptable value by > 72%)
Evaluation of Example Design: DDBD Using Inelastic Spectra

- Demands from inelastic analysis of the design structure
  - \( u_m = 25.9 \text{ cm} \)
  - \( \mu = 3.25 \)
  - \( \theta_p = 0.0199 \text{ rad} \)

- Design using elastic design spectrum
  - \( u_m = 26.0 \text{ cm} \)
  - \( \mu = 3.06 \)
  - \( \theta_p = 0.02 \text{ rad} \)
  - Predictions are nearly the same as the inelastic demands
Conclusions

• A direct displacement-based design procedure is presented
  ➔ Uses well-known inelastic design spectrum
  ➔ Provides displacement estimates consistent with those from inelastic analysis
  ➔ Produces design that satisfies the design criteria of acceptable plastic rotation
  ➔ The procedure is as simple as the current DDBD procedure using elastic design spectra
Conclusions

- DDBD procedure based on elastic design spectra
  - Uses equivalent linear systems
    - Secant stiffness and equivalent damping
  - Provides displacement estimate which can be significantly smaller than that from inelastic analysis
  - Plastic rotation demand may exceed the acceptable value
    - Leaves an erroneous impression that the allowable plastic rotation constraint has been satisfied