

How to Publish Counterexamples in 1 2 3 Easy Steps

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Inspired by Rick Trebino's [How to Publish a Scientific Comment in 1 2 3 Easy Steps](#), and encouraged by my friend and colleague Ron Fox's treatise on [Science and Integrity](#), I was motivated to record one of my own recent experiences in mathematics.

In some sense, there is a significant difference between mathematics and other sciences - we mathematicians are fond of modestly referring to our field as "the Queen of the Sciences" - and that difference is largely embodied in the difference between a *theory* and a *theorem*. Theories come and go – geocentric theories of the solar system were eventually replaced by heliocentric theories, and one Big Bang theory by another. Theorems, on the other hand, are more or less eternal. The Pythagorean Theorem, the Central Limit Theorem, and the Fundamental Theorem of Algebra, re-verified by successive generations of scholars, are accepted as true by essentially all scientists. There are no competing theories once the hypotheses are agreed upon. Thus there are essentially no debates about whether or not theorems are true, and published "Comments" about the truth of established theorems are almost nonexistent.

This perceived permanence of theorems has a major drawback – the blind acceptance of a published false theorem that has not been retracted is often the basis for future false theorems. But we mathematicians also make at least our share of mistakes, some of which find their way into print, and when that happens, correction of the record is imperative. In mathematics, however, the correction does not occur in the form of a Comment or Debate or Opinion, but rather in the form of a Counterexample. A Counterexample, in short, is a formal proof, via an example, that a certain mathematical claim or theorem is false.

But even in the Queen of the Sciences, publishing Counterexamples is not always easy, for exactly the same reasons that Trebino found was the case for publishing Comments in physics. I am recording these events to further support Trebino's findings, and in the hope that young scientists may learn from our mistakes, and not be discouraged when they dare speak out.

As Ron Fox pointed out to me, the present article is complementary to Trebino's, in that Trebino was trying to correct erroneous statements published about his own research, and I was trying to correct research errors published by other scientists. Also, in contrast to Trebino's article, this chronology contains exact dates, quotes, and journal names, and is a single sequence of concrete events, not a composite. Hence, the reader may find the humor here slightly more subtle (and dark), since much of it must be gleaned from the actual words of the participants. With a tip of the hat to Rick Trebino, here goes:

Suppose that you discover serious mathematical errors in a widely-circulated journal published by, say, the *American Mathematical Society*. The official *AMS Code of Ethics* <http://www.ams.org/secretary/ethics.html> proudly proclaims, in part,

"The public reputation for honesty and integrity of the mathematical community and of the Society is its collective treasure and its publication record is its legacy...

mathematicians have certain responsibilities, which include...To correct in a timely way or to withdraw work that is erroneous".

So, after writing out and checking your Counterexamples, you assume that correction of those published errors in the Queen of the Sciences will consist of Three Easy Steps:

1. You politely notify the authors of the errors, providing concrete counterexamples, and the authors then confirm (and perhaps contest some of) the errors.
2. You and the authors contact the Editor, who verifies the errors with an Associate Editor and an expert referee.
3. The Editor publishes your counterexample proofs in a timely manner in the same journal where the false proofs appeared.

Easy as π , no? Dream on!

The Players:

Au1. First author, a professor of political science at a leading U.S. university

Au2. Second author, an associate professor of mathematics at a smaller U.S. university

Au3. Third author, an assistant professor of economics at a European university

Ed1. Editor of the *Notices of the American Mathematical Society*, a professor of mathematics at a large U.S. university

Ed2. Associate Editor of the *Notices*, a professor of mathematics at a leading U.S. university, and member of the National Academy of Sciences

The Events:

December 2006

Under the editorial direction of Ed1, the *Notices of the American Mathematical Society*, perhaps the most widely-read *AMS* journal, publishes an article by Au1-2-3. The article is widely promoted by an official *AMS News Release*, podcast in *Scientific American (Online)*, and *The Discovery Channel*, among others.

6/1/07

The Editor of *SIAM Review*, a major publication of the *Society for Industrial and Applied Mathematics*, asks me to review a different paper on the same general topic by Au1-2-3, who cite their own December *Notices* article.

7/11/07

I review the *SIAM* submission, find serious mathematical errors, and send my report on the *SIAM Review* submission, including detailed proofs of the errors in the form of concrete counterexamples to the main results. The *SIAM* Editor agrees with my findings, sends the authors Au1-2-3 a copy of my report containing the counterexamples, and rejects the article.

[ASIDE] Not to worry – persistence is essential in publishing. Four months later (see [11/26/07](#)), Au1-2-3 publish the same article, essentially without change, even after the authors know there are errors in it (see [8/27/07](#)).

Curious as to how the errors arose in the first place, I study the original December 2006 *Notices* article cited by Au1-2-3, and find many of the same errors.

7/26/2007

After double-checking my findings with two experts in the field, I email Au2:

“Dear [Au2], It appears that there may be some serious mathematical errors in your December 2006 in the *Notices* (since you are the mathematician on the team, I am writing you directly, rather than [Au1]). I would like to talk to you on the phone, and ask a few questions... Thanks in advance. Regards Ted Hill”

Later same day

I do not receive a direct reply from Au2, but instead from Au1, who emails me:

“You're quite right that there was at least one error, which we attempted to correct in the attached Corrigendum [sic]. Of course, we would like to hear of any other errors that you might have found.”

Still later same day

I email Au1:

“Thanks for the quick reply, and the draft of your correction. Attached are some notes I drafted - maybe I am missing something (but then so are two colleagues who also checked it for me). I will try to study your proposed correction soon”

7/30/07

I email Au1-2-3 detailed proofs of errors in their Corrigendum, and tell them I sent the Editors of the *Notices* a draft of my counterexamples “to help them coordinate the correction process (assuming that they agree corrections need to be made).” I also send my latest counterexamples to a third expert in the field, who also confirms my findings.

7/31/07

Au1 emails me:

“My coauthors and I have discussed [your counterexamples], and most, we think, are based on misconceptions or misinterpretations of our results.”

8/2/07

I email the Authors that:

“I sent you a detailed mathematical account of the errors in your article. I have no reason to believe there is anything erroneous in what I sent you, and indeed had it checked by three other mathematical experts. ...you state that you think that misconceptions and misinterpretations are the basis of my criticisms. Please give me detailed arguments to support your claim, just as I have already done for you.”

Later same day

Authors reply:

“Your request for feedback would be a reasonable one if you had not submitted your comments to the *Notices*...We will respond to the editor of the *Notices* if asked to do so.”

8/3/07

I email Ed1 that:

“I have been in contact with Professors [Au1-2-3] about my corrections to their December 2006 article in the *Notices*. This communication has been unproductive ... Therefore, please consider my 7/27/07 [Counterexamples] submission for publication in the *Notices*.”

8/20/07

I email Authors that:

“I have looked over your Corrigendum (attached), and found what appear to be several significant logical errors”, which I then describe in detail.

8/27/07

Au2 emails me:

“I would like to write a new corrigendum... [since] I do think that some of your comments have merit and should be addressed in print”.

He tells me he also agrees that their *SIAM Review* submission contained similar errors, and mentions that Au1 alleged a previous “plagiarism” issue(!) involving me, but Au1 did not give him details.

[ASIDE] Science and mathematics is sometimes much like football and war – the best defense is often perceived to be a good offense. Attack, attack, attack.

8/30/07

Au2 emails me once more:

“I wanted to thank you again for reading our work closely. We appreciate the time that you have spent on this! (I'll speak for [Au1 and Au3] on this one.) And, we will certainly thank you in print if given the chance.”

[ASIDE] But given numerous opportunities to thank me in print later (see [11/26/07](#) and [October 2008](#)) they never did. Still waiting.

8/31/07

I email Ed1 and also Ed2 (who has worked in the field of the Au1-2-3 papers) with concrete counterexamples to the theorems and claims in the original *Notices* article, as well as counterexamples to statements and proofs in the Au1-2-3 Corrigendum.

9/2/07

Ed1 emails me suggesting I should publish the Counterexamples elsewhere, and on the math arXiv, and then the *Notices* can publish a short correction note.

[ASIDE] I send the Editor of the *Notices* my Counterexamples to theorems that appeared in his journal under his watch, and his solution for me is to publish the Counterexamples *elsewhere?!?!.*

9/4/07

I respond that:

“In light of the media attention the [Au1-2-3] article is receiving (*AMS News Release, Scientific American, The Discovery Channel* etc), it is therefore imperative that the *Notices* (not some second-rate journal or on-line arXiv that many *Notices* readers like me rarely reads) publish the Counterexamples, and publish them expeditiously, perhaps also asking [Au1-2-3] to retract the paper.”

10/6/07

Au2 emails me:

“I had lunch with [Au1] a couple of weeks ago and he said that he was not interested in writing a joint corrigendum (or whatever it would be called) which would include your comments, examples, criticisms, etc. He believes that we should respond once your final comments appear or are posted.”

[ASIDE] Apparently Au1 wants the truth decided by debate, and volunteers to make the closing argument.

10/12/07

I email Ed1 and Ed2, copied to Au1-2-3:

“[Au1-2-3]'s refusal to correct their article or retract it in a timely manner (it may take years for my Counterexamples to be published, if ever), is in direct violation of the *AMS Code of Ethics*”.

I include the exact statement of the *COE* quoted in the beginning of this article, and state that

“It has now been nearly three months since I brought these errors to the attention of the authors and the *Notices*. Since the authors have gone on the record saying they will not correct their paper until my Counterexamples are published, it is time for the *Notices* editors to make a clean decision - accept or reject my submission. If the *Notices* wants to reject my Counterexamples after three months delay, without refereeing it, and knowing full well that publication of the Counterexamples elsewhere could take years,

so be it. That will be part of the record. ...It is the professional duty of the authors (and editors), not the person who discovered the errors, to see that faulty statements and proofs are corrected as soon as possible.”

10/18/07

To clarify the issues, I email Ed1-2 and Au1-2-3 specifying:

“a dozen errors in the original article, and attached detailed arguments and counterexamples to support each of those claims.”

I also state:

“The option to have the *Notices* simply refer to un-refereed second-party URL’s for the Counterexamples and/or Corrections is not acceptable -- websites’ contents may be changed at any time, they do not carry the *Notices*’ stamp of approval that the original article received, and they may not be available to all readers of the *Notices*, hardcopies of which reach some very remote locations (and as [Ed2]’s note says, ALL those *Notices* readers who read part or all of that paper deserve notification of any significant errors in the paper).

“Nor is a *Letter to the Editor* the place to publish corrections, whether the publication be *Newsweek*, *New York Times* or the *Notices* -- the journal itself must acknowledge the errors, not some private individual or individuals.”

[ASIDE] These were wasted words on my part - Ed1’s Final Solution, a YEAR LATER (see **October 2008**), is that he himself publishes a *Note* in the *Letters to the Editor* (i.e., a *Letter to the Editor from the Editor*... also see **AFTERMATH** below).

Later same day

Ed2 emails Au1-2-3 and me:

“I propose that the four of your [sic] put your heads together, as scientists (not as humans who may have previous history), with the sole purpose of enlightening your audience. The issues should be mathematical, and usually mathematicians can reach agreement on what is correct and what isn’t.”

[ASIDE] It is the Queen of Science, after all – see Intro.

10/20/07

Au1 emails Ed2:

“We think Ted’s critique is ***without merit*** [emphasis added].”

[Aside] If you can’t attack – deny, deny, deny. When Richard Pryor’s wife caught him *in flagrante delicto* with another woman, he just denied it. “Who you gonna believe,” he asked his wife, “me - or your lying eyes?”

Au1 continues:

“Because [sic] we have found a dialogue with Ted impossible, we think the only solution is for Ted to finalize his critique. We will then offer a point-by-point response,

and that will close matters. This critique and rebuttal could then be reviewed by you or other referees for possible publication. We do not want to get into an endless debate, which has already gone on too long. Finally, I would note that in a related area, game theory, there have been published exchanges, such as between Borel and von Neumann about which one discovered the minimax theorem. [Au2], [Au3], and I see no alternative but to have such an exchange and let this close matters. I hope you agree.”

[ASIDE] Note that Au2 had already confirmed some of the errors (see [8/27/07](#)); and that a year later, after many more denials, Au1-2-3 finally admit many of the errors in print (see [October 2008](#)).

10/21/07

Ed2 does not agree that a “Borel-von Neuman exchange” is the proper way to set the mathematical record straight. He emails Ed1, Au1-2-3 and me saying:

“I'd like to get the science settled first, and then we can better decide how to get this info to the audience”, and that he would like me to first identify one or two concrete issues to start with. He asks Au1-2-3 for their Corrigendum, and also states, “For example, as I understand cake cutting, it seems to me that the second objection of Ted's, a counterexample to Pareto optimality, is valid. Or am I missing something?”

I send my first counterexample issue.

10/22/07

Authors send Ed2 their Corrigendum to their original article, which Ed2 sends to me and asks whether it addresses my concerns.

Later same day

I email Ed2 that the Corrigendum also has significant errors, which I spell out with concrete counterexamples, and tell him that I had already sent those to Au1-2-3.

10/26/07

Authors send Ed2 a response to my first counterexample.

Ed2 emails me a copy of his reply to Authors – he doesn't buy the Richard Pryor Defense:

“But this is not a response to the counterexample. Ted refers to prose on page 1314, where you quite clearly say that cut-and-choose satisfies (2). As far as I can tell, it doesn't, and Ted is correct in what he wrote. ..to say that Ted's counterexample is "without merit" [see [10/20/07](#)] just seems flatly wrong. Perhaps it can be explained, but dismissal "without merit" doesn't work with me.”

Ed2 then asks me for a second concrete issue to focus on; I send it to him and Au1-2-3.

11/13/07

Authors email Ed2 their response to my second counterexample, stating:

“It is worth noting at the outset that we deliberately wrote an expository article, which the *Notices* encourages, without all the formality of a mathematics research paper.”, and conceding “Hill is correct that in the special case...” . They concluded “Our results are consistent with the definitions we used, *except for exceptional cases.*” [emphasis added].

[ASIDE] This unique and creative defense is admirable – except for one tiny detail. Using their logic, you could easily get your name in lights with exciting new results such as: *Theorem: All numbers are larger than three*, which is 100% correct except for exceptional cases.

11/16/07

I email Ed2 a detailed refutation of Au1-2-3's 11/13/07 "exceptional cases" response, including my reaction to their "expository article" defense, stating:

"Even for expository articles, the reader expects that standard technical terms such as "absolutely continuous" have the standard meaning, and the reader expects that if an expository article contains items clearly labeled "Theorems" and "Proofs", those should be more or less correct and complete."

11/26/07

Authors publish essentially the same paper they submitted to *SIAM Review* (see 7/11/07 above) in *Dagstuhl Seminar Proceedings 07261*, knowing that it contains errors I had found four months earlier in my referee's report (see 8/27/07).

They also publish a sequel to their original *Notices* article containing similar errors, in *Dagstuhl Seminar Proceedings 07261*.

[ASIDE] In contrast to Au2's promise to acknowledge my contribution (see 8/30/07 above), neither of the articles contain any thanks to me, let alone correction of the mathematical errors.

12/15/07

To clarify where things stand, Ed2 asks me to provide him and Au1-2-3 with a summary of what mathematical points I think we have agreed on, and what points we have not. I email them a succinct list of ten concrete points, and say:

"If you believe any are incorrect, please let me know which ones, and what the correction should be."

Au1-2-3 do not respond to me, but email Ed2 requesting a decision on his part now that "the math, as you wished, has been sorted out".

12/29/07

I email Ed2:

"After nearly six months, [Au1-2-3] are still dismissing 9 out of 10 of my concrete points, including CX 1 that you carefully analyzed, CX 4 where their Nov 13 response admitted "Hill is correct" (but claimed the case is "exceptional"), and CX 5 where the Nov 13 response said "...as illustrated by this counterexample [but] this case is exceptional". According to their latest response, however, they somehow feel that now "the math, as you wished, has been sorted out"! ..."

I ask Ed2 to have my Counterexample paper formally refereed.

2/14/08

Ed2 emails me that:

"I'm afraid I've run out of steam for this endeavor. I think most of the evidence is in your favor, and [Au1] has basically stonewalled. ... It isn't the sort of thing the *Notices* (it is not a journal of record, nor a research journal) will want to do much about in the way of correction. Perhaps they should, but they won't want to spend more pages than [Au1-2-3] did in order to explain what was wrong. ... You know, correctly, in your heart that you are right on most or all of the issues, and that I'm pretty much on your side as far as I've gotten into things. That's not much, but..."

[ASIDE] Ahh, I feel better already, knowing correctly in my heart...

2/20/08

I email Ed1, informing him that Ed2 suggested that my earlier submission of Counterexamples might be viewed as unacceptable because it is too long, and remind him of the *AMS Code of Ethics* statement on integrity and correction of errors (see [10/12/07](#)).

I attach a much shorter summary of the Counterexamples, and ask that it be considered for publication in the *Notices*.

6/13/08

Under pressure, Ed1 finally has the revised shorter Counterexamples refereed, and sends Au1-2-3 and me the referee's report, telling us:

"The (anonymous, of course) referee was suggested by [Ed2] as someone all involved would mutually respect, and I believe that upon reading the report appended below all will agree an excellent job has been done."

After spelling out detailed analyses of the points, the referee concludes:

"I agree that Hill's concerns are justified, and that the authors of [Au1-2-3] might want to address Hill's concerns. I will leave it up to the Editor to decide exactly how that should be done."

7/11/08

Ed1 emails me that he has rejected for publication both my original and my streamlined submissions of the Counterexamples, and says:

"I've decided that the way to inform the *Notices* readers about cake-cutting issues is via an "*Editor's Note*" on the subject that will appear in the October issue".

[ASIDE] Recall my explicit and uncontested objection to this solution in ([10/18/07](#)).

Ed1 appended a "courtesy copy" of his proposed *Note*, which acknowledged "serious mathematical errors" in the original Au1-2-3 article, "which have been checked by an expert referee, [and] verify the following mistakes..."

Ed1's proposed *Note* also included final responses by Au1-2-3 (i.e., allowing them the last word in the "exchange", just as they requested (see [10/6/07](#) and [10/20/07](#)). It also included the Editor's amazing policy statement

“a final editorial comment: precision and exposition are not necessarily incompatible, although when they do conflict the *Notices* tends to favor the second, or rather encourage authors to do so”.

[ASIDE] This authoritarian decree surprised my mathematician colleagues and me, especially since the *Notices* is an *AMS* journal, and its editors supposedly adhere to the *AMS Code of Ethics* (see [10/12/07](#), and [AFTERMATH](#) below). But he is the Editor, so what do we have to say about it? In fact, what do the Associate Editors have to say about it? I’ll wager ten-to-one odds that Ed1 did not check with his Associate Editors when he made this statement, and certainly not with the Associate Editor who is well known (e.g. Wikipedia) for saying

"A physics paper, like a newspaper article, is not meant for posterity; dotting of i's and crossing of t's is meant to happen after the fact, and is not in any case the 'real' work of a physicist. A mathematics paper, on the other hand, is supposed to be a work of art: perfect, complete, and beautiful ... [Otherwise] mathematics can only go in one direction: toward the profane."

7/14/08

I email Ed1 and Au2 that the Au1-2-3 response quoted in his proposed *Editor's Note* contains additional new mathematical errors, and I again provide explicit counterexamples.

Later same day

Ed1 emails me:

“I'm not going to make further changes before I put the piece into production.”

[ASIDE] In other words, the Editor is going to publish mathematical claims he has reason to believe are false, simply because of the editorial inconvenience of checking and making the corrections. Truth be damned, deadlines are deadlines.

October 2008

I had only made two main requests to Ed1: that the *Notices* publish corrections “expeditiously” (see [9/4/07](#)); and that the *Notices* publish the corrections in a proper forum, specifically, not in *Letters to the Editor* (see [10/18/07](#)).

But when the *Notices* finally publishes a “correction”, more than a year has passed since I notified Ed1 of the errors, and the “correction” is published *by the Editor* as an *Editor's Note* buried in the bottom of its section *Letters to the Editor*.

[ASIDE] Editors can be creative too... But we can't complain - the *American Mathematical Society* has democratically elected officers, and as H. L. Mencken said, “Democracy is the theory that the common people know what they want and deserve to get it good and hard.”

The *Editor's October 2008 Letter to Himself* states, in part,

"Theodore P. Hill has found "serious mathematical errors" in the article ... by [Au1-2-3] which appeared in the December 2006 Notices and supplied analysis and detailed counterexamples ... which have been checked by an expert referee, verify the following mistakes..."

[ASIDE] The *Editor's Note* also contains a summary of many of the errors, and Au1-2-3's acknowledgement of those errors – the same errors sent them on **7/26/07**, denied by Au1-2-3 as "misconceptions" on **7/31/07**, partially confirmed as having "merit" by Au2 on **8/27/07**, denied by Au1-2-3 as "without merit" on **10/20/07**, partially confirmed by Ed2 on **10/26/07**, denied by Au1-2-3 as "exceptional cases" on **11/13/07**, and confirmed by a referee on **6/13/08**. ***It also contains the new mathematical errors*** I had pointed out both to the Ed1 and to Au1-2-3 (see **7/14/08**), but not one of the essential counterexamples. And no retractions – in fact, propagation of those errors is already in the mill...

"If two wrongs don't make a right, try three." (Author unknown)

May 2009

The *Notices* announces, *without disclaimer*, publication of a new book that contains essentially all of the same errors as the original Au1-2-3 2006 article in the *Notices*.

Fall 2009

More than two years after first submitting them, I am still trying to publish the Counterexamples.

AFTERMATH

At the end of his fascinating article, Trebino asks

"Okay, so the system is badly broken. How would I fix it?"

and he lists eleven suggestions. The scientific community does not exercise control over individual scientists, nor should it, but the community does have a say in who edits its journals. Regarding that fact, two of Trebino's suggestions seem particularly relevant here:

Trebino #4:

No journal editor should be allowed to edit a Comment on a paper that he allowed to be published. This is an obvious, unacceptable conflict of interest.

Trebino #11:

Finally, let's face it: most journal editors are simply too arrogant and have lost sight of the goal, which, it appears that I need to remind them of here, is to publish only truth. Perhaps they could be required to take a course or two in humility.

That would be a fine start. In the Real World, of course, that will not happen, so perhaps the best the rest of us working scientists can do is to follow Trebino's fine example, and share our experiences and continue to question authority. In the words of my historian friend Professor Hugh Hudson, "If we did not challenge accepted wisdom, if we didn't challenge authority," he said, "we would still be arguing the world is flat."