

Analysis and Testing of Heat Transfer through Honeycomb Panels

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This project attempts to simulate accurately the thermal conductivity of honeycomb panels in the normal direction. Due to the large empty space of the honeycomb core, the thermal radiation mode of heat transfer was modeled along with conduction. Using Newton's Method to solve for a steady state model of heat moving through the honeycomb panel, the theoretical effective thermal conduction of the honeycomb panel was found, ranging from 1.03 to 1.07 Q/m/K for a heat input of 2.5 W to 11.8 W. An experimental model was designed to test the theoretical results, using a cold plate and a heat plate to find the effective conductance of six samples, each with different colored face sheets or core thicknesses. The experimental data revealed that the analytical results underestimated the conductance, showing a range of difference from 0.31% to 90%. Further analysis regarding the radiation effects is needed to reproduce accurately the effective thermal conductance of the honeycomb panel.

I. Introduction

For spacecraft and aircraft design, the mass is one of the biggest factors. Engineers find ways to reduce the mass in as many components as possible. One of the heaviest components is the structure. Engineers, in order to reduce mass, have used sandwiched composite structures, or more specifically honeycomb panels, to save weight while keeping the spacecraft structurally intact. Honeycomb panels consist of three parts: two face sheets and a honeycomb core. The honeycomb core is an arrangement of thin connected cells, usually hexagons, which are sandwiched between the two face sheets. An example is shown in Fig. 1. The core provides normal strength of the structure, and the face sheets provide tensile strength. This light-weight composite allows for large loading while keeping mass low. However, thermally the honeycomb panel is not as efficient.

In a spacecraft, electronic components are unable to get rid of heat by themselves because the vacuum environment disallows any convection or conduction into the environment. The only ways for the heat to move around away from the components are conducting to other parts of the spacecraft and radiating out of the spacecraft. The reasons honeycomb panels are structurally attractive also make it thermally inefficient. Because of the core, the honeycomb panel is mostly empty space. As such, when heat travels through the core, most of it is conducted through the thin walls of the cells, which have a very low area of conductance. This requires a large temperature difference between the two face sheets to move the heat through the core. Also, because of the empty space, radiation heat transfer is also a factor. Compared to conduction, radiation is a very poor way to move heat around. Also, view factors are needed to determine how much heat is radiated. If the heat transfer is too poor, components will overheat themselves and be unusable. Hence, knowing how much heat can be moved through the panel is a necessary piece of information when designing thermal subsystems.

The purpose of this experiment is to determine a way to find the effective conductivity of an aluminum honeycomb panel when heat is moving through it. It also determines how much heat is moved by radiation rather



Figure 1. A Piece of Honeycomb Panel

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than conduction. This project uses a theoretical model using thermodynamics principles and numerical methods to determine the theoretical effective conductivity. The theoretical model is validated by using an experiment to find the practical effective conductivity. The theoretical model uses a method similar to the Swan and Pittman method to determine the theoretical effective conductance.¹ For this experiment, the effective conductance across the honeycomb panel is not considered, as it is unpractical.

II. Analysis

Determining the effective thermal conductance is a complicated process. Because the core consists of hexagonal cells, radiation and conduction are the two modes which heat uses to move through the plate. Also, the view factor within the cells themselves determines how effective the radiative heat transfer is. To simplify the process, the following assumptions are used. First, the face sheets of the panel are extremely thin, so that the temperature difference through them is negligible. Second, there is no convection heat transfer inside the panel, as the experiment will take place inside a still environment. Third, the cell walls of the core are thin so that the temperature gradient across them is negligible.¹ Fourth, the thermal properties of the materials used do not change with the temperature.² Fifth, the thermal effects of the bonding agent between the core and the face sheets are considered negligible. Sixth, the heat transfer functions are nonlinear due to the thermal radiation mode. With these assumptions, a one-dimensional analysis can be used to determine the effective thermal conductivity.

To consider the effects of the radiation and view factors, a finite difference method is used. The panel is divided into seven layers. The first and last layers are top and bottom of the panel and encompasses the face sheets, while the layers in between are purely honeycomb core. Also, for view factor calculations, it is assumed that the hexagonal cells are cylinders for simplifications.

Between each layer, conduction is possible. If there are m layers, then the heat transfer through conduction from layer m to layer $m-1$ can be calculated by,

$$Q_{cond} = \frac{kA_{hc}}{l} (T_{m-1} - T_m) \quad (1)$$

where Q_{cond} is the heat transferred through conduction, k is the conductivity of the material, A_{hc} is the touching surface area between the two layers, l is the heat path distance between two adjacent layers, and T is the temperature of the respective layer. The heat path between each layer is from the center to center of each layer. For the face sheets, however, this path is half of the length between the intermediate layers. This comes from the assumption that the heat conduction through face sheet is negligible because of the large area to path length ratio from the center of the face sheet to the surface of the face sheet. Also, the area of the heat conduction is calculated by multiplying the total area of the honey comb and the ratio of the honeycomb core to the bulk material. In the presence of air, another term is needed to find the heat transfer into the layer of air. This is calculated by,

$$Q_{cond} = \frac{k_a A_a}{l_a} (T_1 - T_7) \quad (2)$$

where l_a is the length from the one face sheet to the other, A_a is the area of the hexagon cell, and k_a is the conductivity of air. The heat conduction through the air compared to the conduction through the cell walls is small, but not negligible. However, since air has an effective transmission of one, radiation heat transfer can be done inside the honeycomb core.

In the honeycomb core, radiation is coupled with the conduction as a heat path. However, radiation is not a linear function of temperature difference like conduction, which makes the heat transfer between each layer to be a nonlinear function. To find how much heat is transferred through radiation, a single cell is analyzed. Assuming that the cell is a gray body, the heat transfer from surface m to surface n is,

$$Q_{rad,m \rightarrow n} = \varepsilon_n \varepsilon_m A_m \sigma F_{m \rightarrow n} (T_n^4 - T_m^4) \quad (3)$$

where Q_{rad} is the heat transferred through radiation, ε is the emissivity of the respective surface area, A is the surface area of the respective layer, σ is the Boltzmann-Stefan constant, F is the view factor from surface m to surface n , and T is temperature of the surface.³ The equation is applied from one layer to all the other layers, as each layer is visible to each other. Each radiation term is multiplied by the number cells in the honeycomb panel to calculate the

total heat transferred through radiation. Since the material used for the core is aluminum, the assumed emissivity is .09. However, since some of the samples have face sheets covered in black binder paint, the emissivity for those is .9 instead. The view factor is how much the each surface is visible to each other. Calculating the view factor requires the surface areas, the distance between them, and the angle between the two. These variables make each view factor calculated between each layer to be unique. The calculation of the view factor between two surface area can be calculated as,

$$F_{1-2} = \frac{\iint \cos \theta_1 \cos \theta_2 dA_1 dA_2}{\pi r^2} \quad (4)$$

where θ_1 and θ_2 are the angles between the their respective surface normal and the ray between the two surface area, dA_1 and dA_2 is the differential area of the their respective surface area, and r is the distance between the center of the two surface areas. However, assumption of a cylindrical cell allows a simpler calculation, which has been calculated by Buschman and Pittman.⁴ Since the cylinders are separated into layers, three types of view factors needs to be calculated: the view factor between the top and bottom layers, which are disks, the view factor between the top or bottom layer with the sides of the cylinders at different layers, and the view factor between the sides of cylinders at different layers. The view factor from top or bottom to the other layers, it can be calculated as,

$$F_{1-2} = \frac{1}{4} \left\{ \left(1 + \frac{H_2}{H_1} \right) \left[4 + (H_1 + H_2)^2 \right]^{1/2} - (H_1 + 2H_2) - \frac{H_2}{H_1} (4 + H_2^2)^{1/2} \right\} \quad (5)$$

where H_1 is the ratio of the distance between the top or bottom layer and the closest edge of the cylinder section to the radius of the cylinder, and H_2 is the ratio of the distance between the closest edge of the cylinder section and the farthest edge to the radius of the cylinder. The view factor between each cylindrical section is calculated as,

$$F_{1-2} = \frac{1}{4(L_3 - L_2)} \left[2L_1(L_3 - L_2) + (L_3 - L_1)X(L_3 - L_1) - (L_2 - L_1)X(L_2 - L_1) - L_3X(L_3) + L_2X(L_2) \right] \quad (6)$$

$$X(L) = (L^2 + 4)^2$$

where L_1 is ratio of the height of one section to the radius of the cylinder, L_2 is the ratio of the distance between the bottom of the one section and the bottom of the other section to the radius of the cylinder, and L_3 is the ratio of the distance between the bottom of one section and the top of the other section to the ratio of the cylinder. For the radiation transfer between face sheets, the view factor is calculated as two parallel disks of the same radius, which is,

$$F_{1-2} = \frac{1}{2} \left[\left(\frac{[2R^2 + 1]}{R^2} \right) - \sqrt{\left(\frac{[2R^2 + 1]}{R^2} \right)^2 - 4} \right] \quad (7)$$

where R is the ratio of the radius of the disks to the distance between the disks. By combining the total heat transfer from radiation and conduction, the total heat transfer can be obtained. The following equations represent the total heat transfer for each layer:

$$Q_m = Q_{in} + \frac{kA_{hc}}{l} (T_{m+1} - T_m) + \frac{k_a A_a}{l_a} (T_1 - T_7) + N \sum_{n=1}^7 \varepsilon_n \varepsilon_m A_m \sigma F_{m \rightarrow n} (T_n^4 - T_m^4) \quad m=1 \quad (8)$$

$$Q_m = \frac{kA_{hc}}{l} (T_m - T_{m-1}) - \frac{kA_{hc}}{l} (T_{m+1} - T_m) + N \sum_{n=1}^7 \varepsilon_n \varepsilon_m A_m \sigma F_{m \rightarrow n} (T_n^4 - T_m^4) \quad m=2, 3, \dots, 6 \quad (9)$$

$$Q_m = 1000(T_C - T_7) - \frac{kA_{hc}}{l}(T_m - T_{m-1}) - \frac{k_a A_a}{l_a}(T_7 - T_1) + N \sum_{n=1}^7 \varepsilon_n \varepsilon_m A_m \sigma F_{m \rightarrow n} (T_n^4 - T_m^4) \quad m=7 \quad (10)$$

where N is the total number of honeycomb cells inside the core, T_C is the temperature of the cold plate, and Q_{in} is the heat applied onto the face sheet. The last equation has a term which represents that cold plate absorbing all the heat from the lower layer without changing temperature or needing a large temperature difference. The resulting Jacobian matrix for the system of equations above can be found in the appendix. The equations are similar to one found in Swanson and Pittman paper.¹

Equation 7, 8, and 9 represents a nonlinear system of equation that can be used to solve numerically for a steady state. By setting the system to zero, the steady state temperature can be found. The system can then be solved using the Newton Method, where

$$T_{k+1} = T_k + J(T)^{-1} f(T) \quad (11)$$

where T is a matrix of the temperatures of the layers, k is the iterative step, J(T) is the Jacobian matrix of the system of equations, and f(T) is the system of equations. The Newton Method is a numerical method that finds approximations of the roots of the equations. The method finds the roots by approximating the function with a tangent line, then finding the x-intercept of that tangent. By doing the same thing over and over again until the solution converges, a close approximation of the roots can be found. So when the system of equation converges, the steady state temperatures are found, and the effective conductivity can be found. However, for a system of non-linear functions, a Jacobian matrix is needed, which is the first order partial derivative of the system of equations. The theoretical effective thermal conductance can then be found with,

$$k_{eff,th} = \frac{Q_{in} t}{(T_{1,ss} - T_{7,ss}) A} \quad (12)$$

where t is the thickness of the honeycomb panel, $T_{1,ss}$ is the steady state temperature of the top layer, $T_{7,ss}$ is the steady state temperature of the bottom layer, and A is the area of the honeycomb panel. A code written in Matlab was used to solve the system of equations, using Matlab's inbuilt Newton's Method solver, fsolve. With this theoretical model, an experimental model is needed to verify it.

III. Apparatus and Procedures

To confirm the theoretical model, the following experiment was run. Six honeycomb samples were used, with the dimensions and configuration shown in Table 1. The samples were constructed from cores and face sheets obtained from AASC's scrap materials. Only two properties were set as variables: height and whether the face sheets were bare or painted black. The areas of the cores slightly differ from each other, but are within acceptable bounds. The cores and face sheets were bonded together with Aeropoxy PR2032 laminating resin and PH3660 hardener, and were pressed for about 24 hours. For the purpose of simplicity, the emissivity of the black paint is assumed to be .9, while the emissivity of the bare aluminum face sheets and the honeycomb core is .09. The material of the core is assumed to be 5056 aluminum alloy.

Table 1. Configuration and Measurements of the Honeycomb Panel Samples.

Samples	Measurements			Uncertainty		
	Length (in)	Width (in)	Thickness (in)	Length (in)	Width (in)	Thickness (in)
Tall Bare	3.036	3.034	2.01	0.0000254	0.0000254	0.0000254
Tall Black	2.79	2.999	2.01	0.0000254	0.0000254	0.0000254
Med Bare	2.906	2.988	1.513	0.0000254	0.0000254	0.0000254
Med Black	3.0162	2.97	1.51	0.0000254	0.0000254	0.0000254
Short Bare	2.774	2.959	0.885	0.0000254	0.0000254	0.0000254
Short Black	2.906	2.693	0.883	0.0000254	0.0000254	0.0000254

The samples were placed between a hot plate and a cold plate. The configuration can be seen in Fig. 2, with a picture of with Fig. 3. A hot plate was constructed by adhesively bonding a 110 V_{ac} 90 W flexible heater from McMaster-Carr to a 58.06 cm² piece of 0.3175 cm thick 6061 aluminum sheet metal. The heater has a resistance of 138.1 Ω. The heater was powered by Powerstat Variable Autotransformer Type 116B, where the output voltage could be change. The output voltage was tracked by a Fluke 17B multimeter. To find the current power output of the heater, the voltage output squared was divided by the resistance of the heater. The cold plate consists of a 0.3175 cm thick 6061 aluminum sheet metal with 15.24 cm long 6061 aluminum rods with 1.27 diameters attached to it by screws. The plate was then inserted inside a Styrofoam box filled with ice water. Since ice does not change temperature as it melts, the ice keeps the plate's temperature relatively the same. The box also makes sure that the samples do not lose heat due to convection from any cross winds. To reduce the amount of heat radiated out from the samples into the environment, MLI was constructed by using household aluminum foil. Four sheets of foil were bounded together on two edges, Four such piece were made, and arranged into a box. To measure the temperature, K-type thermocouples were used, one attached to the top of the hot plate, and another attached underneath the cold plate where the sample was placed. They were attached using electrical tape. The temperature was found using an Omega Model HH23 Microprocessor Thermostat. With these materials, testing can begin.

The sample was placed inside the Styrofoam box in the middle of the cold plate. The MLI was place around the sample. The hot plate is then place on top of the sample. The box is then close with a lid. The power is then turned on to about 20 V_{ac}, and left on until the temperatures reading do not change over time. The temperatures of top and bottom were then recorded. Then the voltage is increased to 30 V_{ac}, then to 40 V_{ac} with the same process. The process was then repeated with each sample. To obtain the effective thermal conductivity, a similar equation to Eq. 12 can be used,

$$k_{eff,exp} = \frac{P_{in,exp} t_{samp}}{(T_{top} - T_{bot}) A_{hc,samp}} \quad (13)$$

where $P_{in,exp}$ is the heat input of the heater going through the core, t_{samp} is the thickness of the sample, T_{top} is the temperature at the top of the heater, T_{bot} is the temperature at the bottom of cold plate, $A_{hc,samp}$ is the area of the honeycomb core. For this experiment, the binding agent is ignored due to the relative thinness to the core. The heat input can be found by dividing the power of the heater by the area of the heater plate to obtain the power density, P_{den} , and multiplying that by the area of honeycomb core sample. The temperature difference between the thermocouples and the sample through the plates is small also, so it is neglected. It is also assumed that the honeycomb itself is insulating the middle of the panel so that only a negligible amount of heat is lost through the atmosphere.

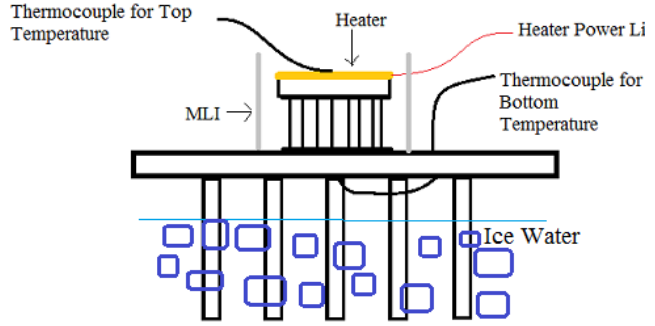


Figure 2. A Schematic of the Experimental Model

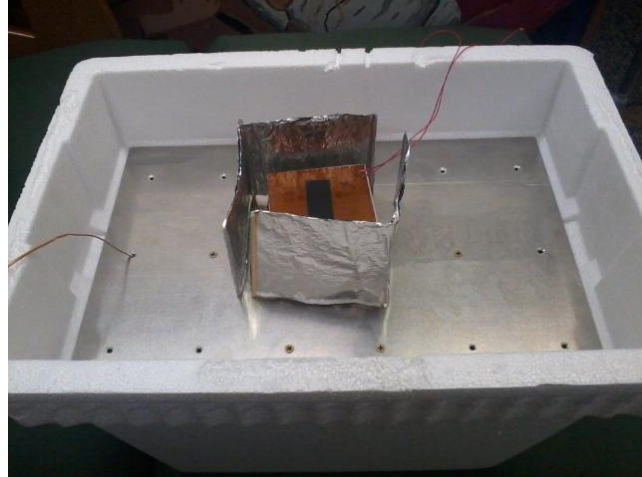


Figure 3. An picture of the experimental configuration

IV. Results

Once the experiment provided the data, they were compared to the effective conductance given by the theoretical model. The results of both are shown in Table 2, with Fig. 4 and Fig. 5 comparing the results of the theoretical model and the experimental results together. Table 3 shows the error calculated, using the analysis found in the appendix.

Table 2. The Effective Thermal Conduction of both the Theoretical Model and the Experimental Data

Samples	Heat Input (W)			$k_{\text{eff,exp}}$ (Q/m/K)			$k_{\text{eff,th}}$ (Q/m/K)			% Difference		
	1	2	3	1	2	3	1	2	3	1	2	3
Tall Bare	2.95	7.25	11.80	1.491	1.826	2.036	1.058	1.064	1.071	40.92	71.70	90.21
Tall Black	2.69	6.16	10.62	1.470	1.611	1.877	1.062	1.067	1.074	38.44	50.99	74.72
Med Bare	2.77	6.13	10.96	1.132	1.384	1.614	1.045	1.046	1.048	8.42	32.35	54.00
Med Black	2.91	6.48	11.37	1.171	1.430	1.542	1.049	1.051	1.054	11.72	36.10	46.40
Short Bare	2.68	5.80	10.31	0.861	1.030	1.040	1.036	1.036	1.036	-16.94	-0.63	0.31
Short Black	2.51	5.67	9.78	0.852	1.028	1.100	1.041	1.041	1.042	-18.17	-1.22	5.59

V. Discussion

For the theoretical model's results to be compared to the experimental data, observations of the trends and numbers are needed. As can be seen in Fig. 4, the theoretical effective thermal conductance of the panel increases as the heat input increases. With the shorter panels, the increase is not as noticeable. The change in conductance is due to the radiative effect, because as the greater heat input means that a higher temperature difference is needed between the face sheets. However, with the higher temperature, the radiation mode will have a higher effect on the overall heat path. The radiation allows the heat to transfer to the other side of the panel more easily than would

Table 3. Error of the Experimental Effective Conductance

Samples	k_{eff} (W/m/K)		
	1	2	3
Tall Bare	0.143354	0.132086	0.161651
Tall Black	0.140037	0.114791	0.148073
Med Bare	0.109373	0.103241	0.1291
Med Black	0.114278	0.106912	0.122728
Short Bare	0.096467	0.08341	0.084258
Short Black	0.096015	0.08253	0.089993

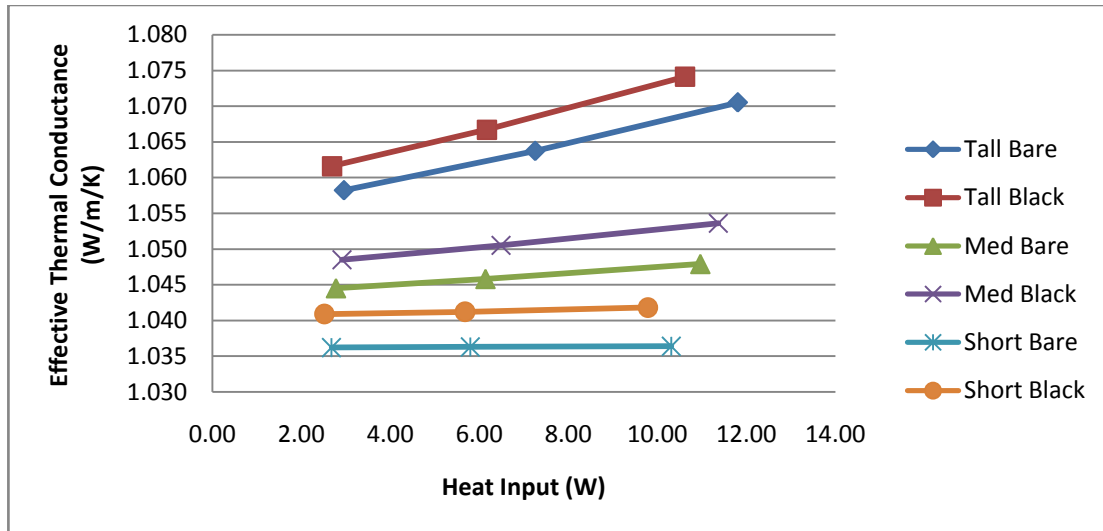


Figure 4. Theoretical Effective Conductance with Increasing Heat Input

conduction, due to the increase in thickness. It is also noticeable that as the panel gets taller, the higher the general effective conductance, as well as the increase in conduction due to greater heat input. The increases are due to the increase in the surface which radiation can flow to. Since the general shape of the cells do not change and the cell walls are thin, radiation becomes more of a factor as conduction becomes less effective in moving heat as if as there was no honeycomb core. Since the surface area which is conducted through the honeycomb core remains the same, the greater core thickness allows more heat to exchange due to radiation rather than conduction. With the black face sheets, however, the increase in emissivity of the face sheet has a small but noticeable effect on the conductivity, with about a .32-.54% increase. The highest increase comes from when the heat travelling through the panel is the greatest. This small difference indicates that changing the emissivity of the face sheets does not change the effects of radiation heat path except for the thicker cores. With these observations of the theoretical model, comparison with the experimental data is possible.

When compared to the experimental model, the theoretical results do not quite match. Although the effective conductivity does increase with higher heat input, the increase in thickness causes a greater increase than what the theoretical analysis expected. In fact, experimental data shows that, for the medium and tall samples, the effective thermal conductivity ranges 8-90% more than the theoretical results. The short samples, however, were within a 20% difference, sometimes being lower than the theoretical model. This larger difference indicates that thermal radiation is a bigger part of the effective thermal conductivity than initially thought. Also, the theoretical model needs to place a bigger emphasis on the role of the radiation heat path. However, simply just amplifying the radiation effect does not have the desired effect, as that would just increase the conductivity of the short samples also. Possibly the best change would be calculating the view factors without using the assumption that the hexagonal cells can be modeled as cylinders. The more accurate view factor might be able to adjust the radiation factor enough to simulate accurately the effective thermal conductance.

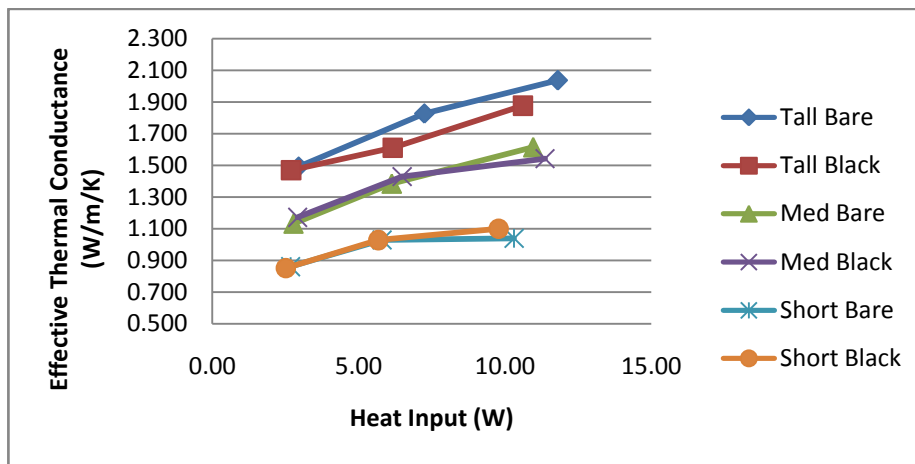


Figure 5. Experimental Effective Conductance with Increasing Heat Input

When comparing the bare and black face sheet samples, the trends found in the analytic model do not seem to match. For the most part, the black samples show a lower conductivity than the bare face. However, the difference is so small that they are within the errors found in Table 3. Higher precision temperature sensors might be needed to find the effective conductivity, but for practical purposes, there seems to be only a negligible difference between using bare face sheets and black face sheets. However, the thicker black face sheets diverge from this trend, showing a noticeably lower conductivity than the bare face sheets. The deviation might be due to human error, as these were the first samples to be tested, so the steady state temperatures might not have been properly recorded.

VI. Conclusion

The theoretical model did not accurately simulate the effective thermal conductivity of honeycomb panels. The model underestimated the effects radiation has on the conductivity, and requires a higher understanding of the process of calculating the view factors. However, the model did predict the general trends of the honeycomb panels. Higher heat input increases the effective thermal conductivity, though not enough to reduce the temperature difference. Thickening the core also increases the conductivity, much more than the using black paint on the face sheets. Also, painting black paint on the face sheets shows no practical effects on the conductivity. If the theoretical model were more accurate, then it could be used as a simple way to find the conductivity of the honeycomb panel by

implementing it into a simple GUI program, asking for certain inputs like cell size, core density, and other variables. This program could be useful to students working on projects that require the knowledge of the temperature difference between two sides of a honeycomb panel, like on a spacecraft that uses honeycomb panels for structure.

Acknowledgments

Thanks to David Esposto for advising me, to Dr. Jin Tso for helping me to rework my theoretical model when the previous one did not work, and to Cal Poly for allowing me to study there.

Appendix

Raw Data

	1				2				3			
Samples	V _{ac}	T _{top} [C]	T _{bot} [C]	dT [C]	V _{ac}	T _{top} [C]	T _{bot} [C]	dT [C]	V _{ac}	T _{top} [C]	T _{bot} [C]	dT [C]
Tall Bare	20.25	22.7	5.7	17	31.74	42.1	8	34.1	40.5	59.5	9.7	49.8
Tall Black	20.28	23.9	6.6	17.3	30.71	41.3	5.1	36.2	40.3	60.5	7	53.5
Med Bare	20.22	20.3	3.5	16.8	30.07	35.3	4.9	30.4	40.2	52.8	6.2	46.6
Med Black	20.4	20.1	3.6	16.5	30.44	34.8	4.7	30.1	40.3	55.6	6.7	48.9
Short Bare	20.43	16.7	3.5	13.2	30.07	28.4	4.5	23.9	40.1	49.7	7.6	42.1
Short Black	20.27	17	3.9	13.1	30.46	30	5.5	24.5	40	47.5	8	39.5

R _{heater}	138.1	olms	(+/-)	0.9905
th _{plate}	0.1205	in	(+/-)	0.001
th _{heater}	0.12625	in	(+/-)	0.001
L _{heater}	3.041	in	(+/-)	0.001
W _{heater}	3.048	in	(+/-)	0.001
A _{heater}	9.268968	in ²	(+/-)	0.0004645

Absolute Errors

Samples	Area (m ²)	Power (W)			Power Density (W/m ²)			Heat Input (W)		
		1	2	3	1	2	3	1	2	3
Tall Bare	2.76912E-06	0.188166	0.454506	0.89267	31.48131	76.04294	149.328	0.187089	0.451914	0.88743
Tall Black	2.64265E-06	0.188711	0.425917	0.88474	31.57249	71.25954	148.0007	0.170439	0.384684	0.79895
Med Bare	2.68909E-06	0.187622	0.40862	0.88078	31.39026	68.36558	147.3393	0.175853	0.382996	0.82541
Med Black	2.73097E-06	0.190899	0.418576	0.88474	31.93852	70.03134	148.0007	0.184591	0.404752	0.85537
Short Bare	2.61674E-06	0.191448	0.40862	0.87684	32.03036	68.36558	146.6793	0.169627	0.362051	0.77678
Short Black	2.55609E-06	0.188529	0.419117	0.8729	31.54208	70.12195	146.0207	0.159259	0.354052	0.73727

Samples	A*dT/t			k _{eff} (W/m/K)		
	1	2	3	1	2	3
Tall Bare	0.142990884	0.14553	0.147764	0.143354	0.132086	0.161651
Tall Black	0.130111519	0.131807	0.134049	0.140037	0.114791	0.148073
Med Bare	0.178400355	0.180804	0.183581	0.109373	0.103241	0.1291
Med Black	0.184400234	0.186794	0.190274	0.114278	0.106912	0.122728
Short Bare	0.287463292	0.290468	0.296409	0.096467	0.08341	0.084258
Short Black	0.274849175	0.278151	0.282812	0.096015	0.08253	0.089993

Sample Calculations

Given:

V_{ac}=20.25 V_{ac}, T_{top}=22.7 C, T_{bot}=5.7, L=3.036 in=0.07711 m, W=3.034 in=0.0771 m, t=2.01 in =0.0511 m, R_{heater}=138.1 Ω, A_{heater}=9.269 in=0.00598 m

$$P = \frac{V^2}{R} = 2.969 W$$

$$P_{den} = \frac{P}{A_{heater}} = 496.54 \text{ W/m}^2$$

$$P_{input} = P_{den} * W * L = 2.951 \text{ W}$$

$$k_{eff} = \frac{P_{input}}{WL(T_{top} - T_{bot})} * t = 1.491 \frac{Q}{mC}$$

Error Analysis

$$\Delta A = \sqrt{(\Delta WL)^2 + (\Delta LW)^2}$$

$$\Delta P = \sqrt{\left(\frac{2\Delta V_{ac} * V_{ac}}{R_{heater}}\right)^2 + \left(\frac{\Delta R_{heater} * V_{ac}^2}{R_{heater}^2}\right)^2}$$

$$\Delta P_{den} = P_{den} \sqrt{\left(\frac{\Delta A_{heater}}{A_{heater}}\right)^2 + \left(\frac{\Delta P}{P}\right)^2}$$

$$\Delta P_{in,exp} = P_{in,exp} \sqrt{\left(\frac{\Delta P_{den}}{P_{den}}\right)^2 + \left(\frac{\Delta A_{hc}}{A_{hc}}\right)^2}$$

$$\Delta k_{eff,exp} = k_{eff,exp} \sqrt{\left(\frac{\Delta t_{samp}}{t_{samp}}\right)^2 + \left(\frac{\Delta A_{hc}}{A_{hc}}\right)^2 + \left(\frac{\Delta T_{top}}{T_{top}}\right)^2 + \left(\frac{\Delta T_{bot}}{T_{bot}}\right)^2 + \left(\frac{\Delta P_{in,exp}}{P_{in,exp}}\right)^2}$$

Matlab Code

main.m

```
clc; clear; close all
```

```
%Daniel Nguyen
%Senior Project Code
%Analysis and Testing of Heat Transfer through Honeycomb Panels
```

```
intm=.0254; %Conversion from inches to meters
```

```
CS=1/4*intm; %Cell Size (m)
H=[2 1.5 3/4]*intm; %Core thickness (m)
```

```
RHO_HC=1.6*16.01846; %Core density (kg/m^3)
```

```
for z=1:length(CS)
    for v=1:length(H);
```

```
        khc=138; %Core Bulk Material Thermal Conductivity (Q/m/K)
        cs=CS(z); %Input Cell size
        h=H(v); %Input Thickness
        th_f=.0070*intm; %Thickness of Cell Walls (m)
        th_fs=.015*intm; %Thickness of Face Sheets (m)
        rho_hc_m=2700; %Density of Honeycomb Core Bulk Material (m)
        rho_hc=RHO_HC(z); %Input Core Density
        totA=(3*intm)^2; %Area simulated (m^3)
```

```
%Length of hexagon side
        l=cs/sqrt(3);
        %maximal diameter of a hexagon
        t=2*l;
```

```

r=t/2;

%   rad_control=0;
ncells=floor(totA/t/cs); %Number of cells

hex_area=1^2*3*sqrt(3)/2; %Area of Cells Top/Bottom
cond_area=totA*(rho_hc/rho_hc_m); %Conduction area

ele=5; %Number of Intermediate Area

sec=2+ele; %Total Number of layers

hsec=h/sec; %Thickness divided by layers

hc_rad_a=2*pi*r*hsec; %Radiation area

FVdtd=disktdisk(r,h); %View factor from top to botom

view=ele-1;
for i=1:ele
    %View factor from layer to another with sides
    FVcyl(i,1)=sep_cyl(hsec,hsec*i,h/sec*(i+1),r)

    %View factor from one layer to top or bottom
    FVcyltdi(i,1)=base_cyl(hsec,hsec*(i-1),r);
end

FVhc=zeros(ele,ele); %View factor place holder

%Insert View factor into matrix
for i=1:ele
    for j=1:ele
        if j==i
            FVhc(i,j)=0;
        else
            FVhc(i,j)=FVcyl(abs(j-i));
        end
    end
end

global FV A1 Aa L K Qin radareas EM nm cells cold kair
kair=.0275; %Thermal Conduction of air (Q/m/K)

cells=ncells;
K=khc;
L=h;
nm=ele;

%Input Emissivity of layers for bare faces
em1=ones(sec,1)*.09;
em1(1)=.09;
em1(sec)=.09;

%Input Emissivity of layers for black faces
em2=ones(sec,1)*.09;
em2(1)=.9;
em2(sec)=.9;

EM=em1; %Emmissivity of Bare Face Sheet Samples

```

```

FV1=[FVcyltdi,FVhc,flipud(FVcyltdi)];

FV=[0,FVcyltdi',FVdtd;FV1;FVdtd,flipud(FVcyltdi'),0];

fs_cond=cond_area/hsec*khc;
dcfs_cond=cond_area/hsec*khc*2;

cond=ones(sec,1)*fs_cond;
A1=cond_area;
Aa=totA-A1;

%Heat Input
if v==1
    qin=[2.951 7.249 11.803
         2.688 6.165 10.616];
elseif v==2
    qin=[2.773 6.134 10.962
         2.912 6.485 11.366];
elseif v==3
    qin=[2.676 5.798 10.311
         2.512 5.672 9.782];
end

areas=ones(sec,1)*hc_rad_a;
areas(1)=hex_area;
areas(sec)=hex_area;

radareas=areas;
cold=273; %Cold Plate temperature

%Calculate Effective conductance with bare face sheet samples
for p=1:length(qin)
    Qin=qin(1,p); %Heat Input
    T0=[287 284 283 282 280 277 274];
    options=optimset('Display','iter');
    %Solve using Newton-Ralphson Method using HC_ss
    [T,Tval,exitflag]=fsolve(@HC_ss,T0,options);
    %Effective Conductance of Bare Face Sheet Samples (Q/m/K)
    keff(v,p)=Qin*h/totA/(T(1)-T(7));

    Thot(z,v,p)=T(1);
    Tcold(z,v,p)=T(7);
end

EM=em2; %Emmissivty of Black Face Sheet Samples
%Calculate Effective conductance with black face sheet samples
for p=1:length(qin)
    Qin=qin(2,p); %Heat Input
    T0=[287 284 283 282 280 277 274];
    options=optimset('Display','iter');
    %Solve using Newton-Ralphson Method using HC_ss
    [T2,Tval,exitflag]=fsolve(@HC_ss,T0,options);
    %Effective Conductance of Black Face sheet Samples (Q/m/K)
    keff2(v,p)=Qin*h/totA/(T2(1)-T2(7));

    Thot2(z,v,p)=T(1);
    Tcold2(z,v,p)=T(7);
end
end
end

```

```

%Each row represents different thickness, while each column represents
%different heat input.
display(keff)
display(keff2)

```

sep_cyl.m

```

function FV = sep_cyl(l1,l2,l3,r)
%View factor from one section of the cylinder to another

```

```

L1=l1/r;
L2=l2/r;
L3=l3/r;

```

```

tm1=2*L2*(L3-L2);
tm2=(L3-L1)*x1(L3-L1);
tm3=(L2-L1)*x1(L2-L1);
tm4=L3*x1(L3);
tm5=L2*x1(L2);

```

```

FV=1/(4*(L3-L2))*(tm1+tm2-tm3-tm4+tm5);

```

```

end

```

disktdisk.m

```

function FV = disktdisk(r,a)
%View Factor from bottom to top of cylinder

```

```

R=r/a;
X=(2*R^2+1)/R^2;

```

```

FV=.5*(X-(X^2-5)^.5);

```

```

end

```

base_cyl.m

```

function FV = base_cyl(h1,h2,r)
%View factor from section of the cylinder to the base

```

```

H1=h1/r;
H2=h2/r;

```

```

tm1=(1+H2/H1)*(4+(H1+H2)^2)^.5;
tm2=H1+2*H2;
tm3=H2/H1*(4+H2^2)^.5;

```

```

FV=.25*(tm1-tm2-tm3);

```

```

end

```

HC_ss.m

```

function F = HC_ss(T)

```

```

%Heat Transfer Equation for Layer 1 to Layer 7

```

```

global FV Qin A1 L K radareas EM nm cells cold kair Aa
bol=5.67e-8; %Boltzman-Stefan Constant

```

```

%Layer 1

```

```

eq1=Qin-nm*K*A1^2/L*(T(1)-T(2))-kair*Aa/L*(T(1)-
T(7))+cells*bol*EM(1)*radareas(1)*((T(2)^4-T(1)^4)...

```

```

*EM(1)*FV(1,2)+(T(3)^4-T(1)^4)*EM(3)*FV(1,3)+(T(4)^4-T(1)^4)*EM(4)*FV(1,4)...
+(T(5)^4-T(1)^4)*EM(5)*FV(1,5)+(T(6)^4-T(1)^4)*EM(6)*FV(1,6)+(T(7)^4-...
T(1)^4)*EM(7)*FV(1,7));

%Layer 2
eq2=nm*K*A1*2/L*(T(1)-T(2))-nm*K*A1/L*(T(2)-
T(3))+cells*bol*EM(2)*radareas(2)*((T(1)^4-T(2)^4)...
*EM(1)*FV(2,1)+(T(3)^4-T(2)^4)*EM(3)*FV(2,3)+(T(4)^4-T(2)^4)*EM(4)*FV(2,4)...
+(T(5)^4-T(2)^4)*EM(5)*FV(2,5)+(T(6)^4-T(2)^4)*EM(6)*FV(2,6)+(T(7)^4-...
T(2)^4)*EM(7)*FV(2,7));

%Layer 3
eq3=nm*K*A1/L*(T(2)-T(3))-nm*K*A1/L*(T(3)-
T(4))+bol*cells*EM(3)*radareas(3)*((T(1)^4-T(3)^4)...
*EM(1)*FV(3,1)+(T(2)^4-T(3)^4)*EM(2)*FV(3,2)+(T(4)^4-T(3)^4)*EM(4)*FV(3,4)...
+(T(5)^4-T(3)^4)*EM(5)*FV(3,5)+(T(6)^4-T(3)^4)*EM(6)*FV(3,6)+(T(7)^4-...
T(3)^4)*EM(7)*FV(3,7));

%Layer 4
eq4=nm*K*A1/L*(T(3)-T(4))-nm*K*A1/L*(T(4)-
T(5))+bol*cells*EM(4)*radareas(4)*((T(1)^4-T(4)^4)...
*EM(1)*FV(4,1)+(T(2)^4-T(4)^4)*EM(2)*FV(4,2)+(T(3)^4-T(4)^4)*EM(3)*FV(4,3)...
+(T(5)^4-T(4)^4)*EM(5)*FV(4,5)+(T(6)^4-T(4)^4)*EM(6)*FV(4,6)+(T(7)^4-...
T(4)^4)*EM(7)*FV(4,7));

%Layer 5
eq5=nm*K*A1/L*(T(4)-T(5))-nm*K*A1/L*(T(5)-
T(6))+bol*cells*EM(5)*radareas(5)*((T(1)^4-T(5)^4)...
*EM(1)*FV(5,1)+(T(2)^4-T(5)^4)*EM(2)*FV(5,2)+(T(4)^4-T(5)^4)*EM(4)*FV(5,4)...
+(T(4)^4-T(5)^4)*EM(4)*FV(5,4)+(T(6)^4-T(5)^4)*EM(6)*FV(5,6)+(T(7)^4-...
T(5)^4)*EM(7)*FV(5,7));

%Layer 6
eq6=nm*K*A1*2/L*(T(5)-T(6))-nm*K*A1/L*(T(6)-
T(7))+bol*cells*EM(6)*radareas(6)*((T(1)^4-T(6)^4)...
*EM(1)*FV(6,1)+(T(2)^4-T(6)^4)*EM(2)*FV(6,2)+(T(4)^4-T(6)^4)*EM(4)*FV(6,4)...
+(T(5)^4-T(6)^4)*EM(5)*FV(6,5)+(T(3)^4-T(6)^4)*EM(4)*FV(6,4)+(T(7)^4-...
T(6)^4)*EM(7)*FV(6,7));

%Layer 7
eq7=- (T(7)-cold)*1000+nm*K*A1*2/L*(T(6)-T(7))+kair*Aa/L*(T(1)-
T(7))+bol*cells*EM(7)*radareas(7)*((T(1)^4-T(7)^4)...
*EM(1)*FV(7,1)+(T(2)^4-T(7)^4)*EM(2)*FV(7,2)+(T(4)^4-T(7)^4)*EM(4)*FV(7,4)...
+(T(5)^4-T(7)^4)*EM(5)*FV(7,5)+(T(6)^4-T(7)^4)*EM(6)*FV(7,6)+(T(3)^4-...
T(7)^4)*EM(3)*FV(7,3));

F=[eq1;eq2;eq3;eq4;eq5;eq6;eq7];
end

```

References

- ¹Swann, R.T. and Pittman, C.M., *Analysis of Effective Thermal Conductivities of Honeycomb-Core and Corrugated-Core Sandwich Panels*, 1961, NASA TN D.714
- ²Swann, R.T., *Heat Transfer and Thermal Stresses in Sandwich Panel*, 1958, NACA TN 4349
- ³Pisacane, V.L., *The Space Environment and its Effects on Space Systems*, AIAA Education Series, AIAA, Reston, VA, 2008, Chaps. 23
- ⁴Buschman, A.J., and Pittman, C.M., *Configuration Factors for Exchange of Radiant Energy Between Axisymmetrical Sections of Cylinder, Cones, and Hemispheres and Their Bases*, NASA TN D-944