ALEXANDER'S SUBBASE LEMMA

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This note provides a very simple proof of Alexander's subbase lemma, from the point of view of Nonstandard Analysis. There is no direct appeal to Zorn's lemma or equivalent principle (as in [1]). This set theoretic principle is, of course, embodied in the construction of the nonstandard extension *X.

Notation and terminology is that of A. Hurd and P. Loeb [2].

Lemma (Alexander). Let (X , T) be a topological space and \( S \) a subbase of closed sets. If every family of closed sets in \( S \) with the finite intersection property has nonempty intersection, then \( (X , T) \) is compact.

Proof. Recall that \( (X , T) \) is compact iff \( *X = \bigcup_{x \in X} \mu(x) \) [2, Theorem (2.9), Chapter III] and that the monad of \( x \) is \( \mu(x) = \bigcap \{ *G \mid x \in G , \ X - G \in \mathcal{P} \} \) [2, Proposition (1.4) of Chapter III]. Let \( \alpha \in *X \). Consider \( \mathcal{F} = \{ F \mid F \in \mathcal{P} , \ \alpha \in *F \} \). Then \( \mathcal{F} \) has the finite intersection property and, by assumption, there is a point \( x \) such that \( x \in \bigcap \{ F \mid F \in \mathcal{F} \} \). We show that \( \alpha \in \mu(x) \): if \( x \in G \) and \( X - G \in \mathcal{P} \), then \( \alpha \notin *(X - G) \), by our choice of \( x \), hence \( \alpha \in *G \), as required.

REFERENCES


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