

## ALEXANDER'S SUBBASE LEMMA

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This note provides a very simple proof of Alexander's subbase lemma, from the point of view of Nonstandard Analysis. There is no direct appeal to Zorn's lemma or equivalent principle (as in [1]). This set theoretic principle is, of course, embodied in the construction of the nonstandard extension  ${}^*X$ .

Notation and terminology is that of A. Hurd and P. Loeb [2].

**Lemma** (Alexander). *Let  $(X, T)$  be a topological space and  $\mathcal{S}$  a subbase of closed sets. If every family of closed sets in  $\mathcal{S}$  with the finite intersection property has nonempty intersection, then  $(X, T)$  is compact.*

*Proof.* Recall that  $(X, T)$  is compact iff  ${}^*X = \bigcup_{x \in X} \mu(x)$  [2, Theorem (2.9), Chapter III] and that the monad of  $x$  is  $\mu(x) = \bigcap \{ {}^*G \mid x \in G, X - G \in \mathcal{S} \}$  [2, Proposition (1.4) of Chapter III]. Let  $\alpha \in {}^*X$ . Consider  $\mathcal{F} = \{ F \mid F \in \mathcal{S}, \alpha \in {}^*F \}$ . Then  $\mathcal{F}$  has the finite intersection property and, by assumption, there is a point  $x$  such that  $x \in \bigcap \{ F \mid F \in \mathcal{F} \}$ . We show that  $\alpha \in \mu(x)$ : if  $x \in G$  and  $X - G \in \mathcal{S}$ , then  $\alpha \notin {}^*(X - G)$ , by our choice of  $x$ , hence  $\alpha \in {}^*G$ , as required.

### REFERENCES

1. J. L. Kelley, *General topology*, Van Nostrand, New York, 1955.
2. A. E. Hurd and P. A. Loeb, *An introduction to nonstandard real analysis*, Academic Press, 1985.

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