Empennage Sizing and Aircraft Stability using Matlab

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This report entails the process and techniques on how to properly size the empennage of a low speed aircraft for a desired level of stability. First, the theory behind aircraft stability is presented. The process is then divided into four major parts which are discussed in detail. Each part explains how the aircraft is to be stabilized including the equations and analysis. The first part describes how to generate the basic empennage size based on static stability. The second part describes how to check the dynamic stability of the aircraft based on the equations of motion. The third part verifies the static stability but for all flight conditions. The final part describes how to augment the stability of the aircraft as well as create a simple and effective autopilot system. The techniques and equations are then implemented into a Matlab program and the outputs are discussed. The Matlab program is validated based on the empennage size and stability of the Ryan Navion aircraft. The report concludes by explaining the empennage design process of an example aircraft, OTG-3.

Nomenclature

A	=	State-Space Matrix
AC	=	Aerodynamic Center
AR	=	Aspect Ratio
a	=	Characteristic Equation Vectors
b	=	Wing Span (ft)
CG	=	Center of Gravity
C_L	=	Coefficient of Lift
$C_{L,o}$	=	Coefficient of Lift at Zero Angle of Attack
$C_{L,\alpha}$	=	Wing Lift Curve Slope
$C_{L,\beta}$	=	Rolling Moment due to Sideslip
C_M	=	Pitching Moment Coefficient
$C_{M,o}$	=	Pitching Moment at Zero Angle of Attack
$C_{M,\alpha}$	=	Pitching Moment due to Angle of Attack
$C_{M,\delta e}$	=	Pitching Moment due to Elevator Deflection
$C_{N,\beta}$	=	Yawing Moment due to Sideslip
$\bar{c}$	=	Mean Aerodynamic Chord (ft)
$c_{l,\alpha}$	=	2-D Wing Lift Curve Slope
g	=	Gravity
I	=	Mass Moments of Inertia
i	=	Incidence Angle
k	=	Feedback Control Gains
L	=	Rolling Moment
l_t	=	Tail Moment Arm (Distance from CG to tail AC)
M	=	Pitching Moment
m	=	Mass
N	=	Yawing Moment
NP	=	Neutral Point
p	=	Roll Angular Rate
Q	=	Dynamic Pressure
q	=	Pitch Angular Rate

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r = Yaw Angular Rate
S_H = Horizontal Tail Area (ft^2)
SM = Static Margin
S_V = Vertical Tail Area (ft^2)
S_w = Wing Area (ft^2)
T_2 = Minimum Time to Double or Half Amplitude
u = x-direction Velocity
u_o = Flight Velocity
V = Controllability Matrix
V_H = Horizontal Tail Volume Coefficient
V_V = Vertical Tail Volume Coefficient
v = y-direction Velocity
W = Transformation Matrix
w = z-direction Velocity
X = x-direction Aerodynamic Force
X_{ac} = Distance from Wing Leading Edge to AC (ft)
X_{cg} = Distance from Wing Leading Edge to CG (ft)
X_{NP} = Distance from Wing Leading Edge to NP (ft)
Y = y-direction Aerodynamic Force
Z = z-direction Aerodynamic Force

α = Angle of Attack (deg or rad)
β = Sideslip Angle (deg or rad)
ε = Downwash
ζ = Damping Ratio
η = Tail Efficiency Factor
θ = Flight Path Angle (deg or rad)
$\ddot{\theta}$ = Angular Acceleration
λ = Taper Ratio (tip chord/root chord)
σ = Sideslip Downwash
ω = Imaginary Part of a Pole
ω_n = Undamped Natural Frequency

Subscripts

a = Aileron
DR = Dutch Roll
e = Elevator
f = Fuselage
H = Horizontal Tail
o = Reference or Equilibrium
p = Phugoid
pwr = Power (Engine)
r = Rudder
sp = Short Period
t = Tail
T = Throttle
V = Vertical Tail
w = Wing

I. Introduction

IN aircraft design, the ability to effectively size the empennage for aircraft stability and control is of upmost importance. The empennage is the tail section of the aircraft including the horizontal and vertical stabilizers (also referred to as tails). Although the ailerons are not located on the empennage, they are a vital part of the control of the aircraft and are directly coupled with the empennage system. There are two axes of stability for an aircraft. The longitudinal stability (up and down) is controlled by the horizontal stabilizer while the lateral (or directional) stability (right and left) is controlled by the vertical stabilizer. Roll control is controlled by the ailerons; however this is coupled with directional stability. Before the empennage can be sized for acceptable control, it must be understood how the empennage stabilizes an aircraft.

For longitudinal stability, only the vertical forces are considered. The wings of an aircraft create a lifting force which is located at the wing's aerodynamic center, AC_w. This lifting vector creates a positive moment (pitch up) around the center of gravity, CG, which is usually located aft of the AC_w. This moment needs to be offset by an upward force behind the CG. The horizontal stabilizer is a small wing that creates an upward lifting force behind the CG. This stabilizer can be sized much smaller than the wing because there is a longer moment arm as it is located further away from the CG. The forces about the CG are shown found in Fig. 1.

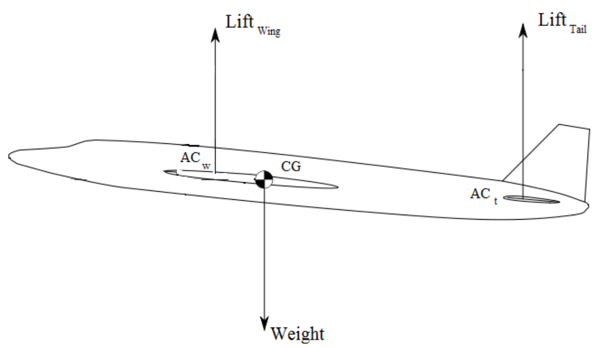


Figure 1. Longitudinal Forces around the Center of Gravity

The lifting forces can be summed around one point where the moment is equal to zero. This is known as the neutral point, NP. Equilibrium is when the neutral point is in the exact location as the center of gravity. However, this assumes that the aircraft is not at an angle of attack. If the aircraft is at an angle of attack, α, this would change the lifting characteristics of the wing and tail, shifting the neutral point. As the neutral point shifts, the moment about the CG changes. The change of moment about the CG is referred to as $C_{M,\alpha}$ (Coefficient of moment due to a change in angle of attack). Two $C_{M,\alpha}$ curves are plotted in Fig. 2, one for a stable aircraft and one for an unstable aircraft.

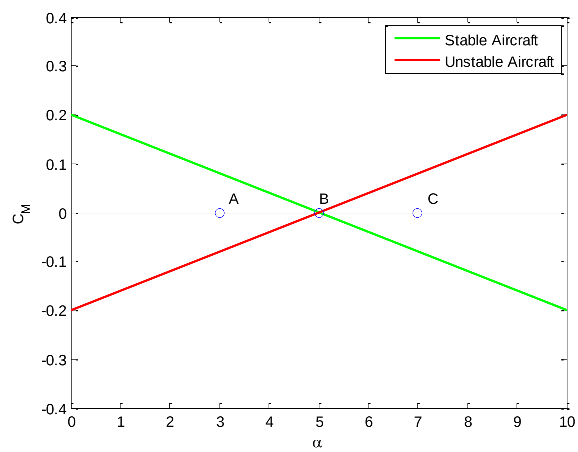


Figure 2. $C_{M,\alpha}$ Curves for Stable and Unstable Aircraft

In Fig. 2, point B is the equilibrium position where there moment acting about the CG is zero. If an aircraft were to suddenly increase its angle of attack (move from B to C), a stable aircraft would return the pitched up aircraft back to equilibrium by creating a negative pitching moment, returning the nose back down. This can only be done if there is a negative pitching moment as the angle of attack is increased. This is the case for the green line (stable aircraft). An unstable aircraft would continue to increase the angle of attack because of the positive pitch up moment (red line). Similarly, if the aircraft were to decrease its angle of attack (move from B to A), the stable aircraft will have a pitch up moment which will restore the aircraft back to equilibrium while the unstable aircraft will have a negative pitching moment further increasing the negative angle of attack. For positive stability, the aircraft must have a negative sloping $C_{M,\alpha}$ curve.

Similarly to longitudinal stability, lateral stability also desires that the aircraft return to its equilibrium position after it has been subjected to a disturbance. However, for the lateral case, only the right and left forces are considered and the equilibrium refers to yawing equilibrium. Yaw is when the aircraft's nose is pointed in a different direction than the direction the aircraft is actually traveling. This angle is called the sideslip angle, β. This can be caused from a crosswind and can be extremely dangerous during landing. To offset the yaw, a yawing moment, N, is created from the vertical tail. Figure 3 is an example of an aircraft in a sideslip with an offsetting yawing moment.

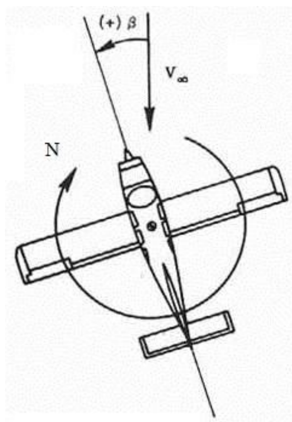


Figure 3. Aircraft in Sideslip, β, with an Offsetting Yawing Moment, N

Unlike longitudinal stability where an aircraft can be in equilibrium at any angle of attack, lateral equilibrium is usually at no sideslip (β = 0). The yawing moment changes as the sideslip angle changes similar to how pitching moment changes with angle of attack. This is known as $C_{N,\beta}$ (Yawing Moment due to sideslip, β). The vertical tail must be sized for a positive sloping $C_{N,\beta}$ so that an aircraft that is disturbed from its equilibrium position will create a restoring yawing moment to return the aircraft to equilibrium. This is shown in Fig. 4.

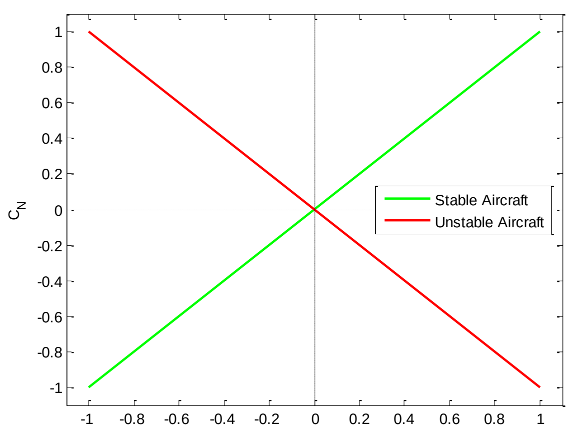


Figure 4. $C_{N,\beta}$ Curves for Stable and Unstable Aircraft

In addition to counteracting sideslip during crosswind landings, the vertical tail also serves other purposes. It protects against adverse yaw which is when the oppositely deflected ailerons create a yawing moment that opposes the turn direction. For multi-engine aircraft, if one engine were to fail, the working engine would create an undesirable yawing moment. The vertical tail is designed to stabilize the yawing moment created by the OEI (one engine inoperative) condition. The vertical tail must also be sized for adequate spin recovery.

Roll control is similar to both longitudinal and lateral stability. If the wings are perturbed from level flight, they must be able to return to level flight with a restoring rolling moment, L. However, the rolling moment changes as sideslip angle, β, changes which is known as $C_{L,\beta}$. For roll stability, $C_{L,\beta}$ must be negative.

It has now been determined how the empennage effectively stabilizes the aircraft; however, there has been no mention on how to size the empennage for the desired stability. The sizing process is broken into many smaller steps and cannot happen all in one instant. The following sections outline the steps on how to effectively size the empennage for adequate stability and how the steps are implemented into a Matlab computer program.

II. Overview of Matlab Code

As previously stated, empennage sizing is not an instantaneous process. It is highly dependent on other aircraft variables which may not be known until later in the aircraft design process. Also, as with all aircraft designs, the process is iterative and the design is always changing. This Matlab code needs to be easily incorporated into the design process to facilitate rapid design changes. Since the empennage sizing is dependent on other aircraft variables which may not be known, it is beneficial for the designer to break up the empennage design into sequential steps. This will allow the designer to size the empennage based on the known aircraft variables. The Matlab code and empennage sizing process is broken up into four steps: Basic Empennage Sizing, Detailed Empennage Sizing, Longitudinal Static Stability, and Stability Augmentation and Autopilot.

III. Basic Empennage Sizing

At the beginning of conceptual aircraft design, after the wing characteristics have been determined, the fuselage shape has been determined, and the weights have been estimated, the designer may begin the empennage design. The designer has multiple empennage options to consider. The three most popular are: T-tail, V-tail, and conventional tail.

Figure 5. T-tail, V-tail, and Conventional tail

A T-tail is commonly used on aircraft where the engines are located on the rear of the fuselage or on high-wing aircraft when the horizontal stabilizer may be located in the wing wake. However, T-tails require larger structural components in the vertical tail to support the horizontal tail. Larger structural components lead to increased weight and increased complication which is undesirable in aircraft design. V-tails are not commonly used, but are still a viable possibility. Since there are only two surfaces instead of three, there is less surface area which leads to less drag. A smaller surface area will also require less structural support and weight. However, according to Raymer[1] and Purser[2], the surfaces must be sized larger than separate horizontal and vertical surfaces to obtain satisfactory stability and control because of adverse roll-yaw coupling. This is when the V-tail causes an undesirable rolling moment in the opposite direction of the desired turn. For simplicity, this empennage analysis will only consider conventional tails.

A. Design Process

To begin the empennage sizing, the designer must know what variables are needed from aircraft. The designer must also know how these variables affect the tail size and stability. To determine the needed variables, the designer must determine what acceptable level of stability is desired and how to achieve this. For longitudinal stability, it was stated in §I that a negative $C_{M,\alpha}$ was needed for positive stability. To derive the $C_{M,\alpha}$ equation, the aircraft must be broken into components. These components are the wing, fuselage, tail, and power effects. Each component makes a contribution to the pitching moment about the CG. The component contributions are summed together to determine one equation for the pitching moment. Many elementary stability textbooks do this lengthy derivation, including Pamadi[3], which is where this $C_{M,\alpha}$ equation is derived from,

$$C_{M_\alpha} = C_{L_{\alpha,w}}\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right) + C_{M_{\alpha,f}} + C_{M_{\alpha,pwr}} - \eta V_H C_{L_{\alpha,t}}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \qquad \text{Eq. (1)}$$

where $C_{L,\alpha,w}$ is the wing lift curve solve, X_{cg} is the CG location in feet from the leading edge of the wing, X_{ac} is the wing aerodynamic center location from the leading edge of the wing in feet, $\bar{c}$ is the mean aerodynamic chord (ft), $C_{M,\alpha,f}$ is the pitching moment caused by the fuselage, $C_{M,\alpha,pwr}$ is the pitching moment caused by the engine, η is the tail efficiency factor, V_H is the horizontal tail volume coefficient, $C_{L,\alpha,t}$ is the tail lift curve slope, and $d\varepsilon/d\alpha$, the downwash term, is given as

$$\frac{d\varepsilon}{d\alpha} = \frac{2C_{L_{\alpha,w}}}{\pi AR} \qquad \text{Eq. (2)}$$

where AR is the aspect ratio of the wing. Usually, the wing and fuselage contributions to $C_{M,\alpha}$ are destabilizing (positive) and are offset by the tail (negative). To breakdown the tail contribution term; the tail efficiency factor, η, is the ratio of dynamic pressures of the tail and the wing. This can range between 0.8 and 1.2 and in most cases is assumed to be unity ($\eta = 1$). The volume coefficient, V_H, is defined as

$$V_H = \frac{S_H l_t}{S_w \bar{c}} \qquad \text{Eq. (3)}$$

where S_H is the horizontal tail area, l_t is the moment arm (distance from aerodynamic center of the tail to the CG), and S_w is the wing area. V_H ranges between 0.3 and 1.1 depending on the aircraft. Most aircraft of the same size and category fit into similar volume coefficient ranges. For positive stability, a large V_H, $C_{L,\alpha,t}$ and a small $C_{L,\alpha,w}$ is desired (since the downwash term will usually be less than 1). $C_{L,\alpha,t}$ is determined by taking the 2-D lift curve slope of the airfoil used on the tail ($c_{l,\alpha}$) and accounting for the 3-D effects,

$$C_{L_{\alpha,t}} = \frac{c_{l_\alpha}}{1+\left(\frac{c_{l_\alpha}}{\pi AR_H}\right)} \qquad \text{Eq. (4)}$$

where AR_H is the horizontal tail aspect ratio. Common horizontal tail aspect ratios range from 2 to 5 but are usually between 3 and 4. For the optimal aspect ratio, the designer would want to choose the aspect ratio that is the lightest weight or that gives the best lifting performance.

To ensure that the aircraft has positive stability, the CG point at which $C_{M,\alpha}$ equals zero is desired. By setting $C_{M,\alpha} = 0$ in Eq. (1) and solving for X_{cg}, the neutral point, NP, can be solved for,

$$\frac{X_{NP}}{\bar{c}} = \frac{X_{ac}}{\bar{c}} - \frac{C_{M_{\alpha,f}}}{C_{L_{\alpha,w}}} + \eta V_H \frac{C_{L_{\alpha,t}}}{C_{L_{\alpha,w}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \qquad \text{Eq. (5)}$$

where X_{NP} is the location of the neutral point from the leading edge of the wing in feet. If the center of gravity reaches this point, the aircraft will be neutrally stable. If the CG moves beyond this point, the aircraft will be unstable since it will have a positive $C_{M,\alpha}$. The CG location should always be located forward of the neutral point. The difference between the CG location and the NP is known as the static margin (SM),

$$SM = \frac{X_{NP}}{\bar{c}} - \frac{X_{cg}}{\bar{c}} \qquad \text{Eq. (6)}$$

The static margin is always in terms of percent mean aerodynamic chord. A stable aircraft will always have a positive static margin. Most aircraft have a static margin of approximately 5-10%. Aircraft that have extremely large static margins (upwards of 30%) are too stable and are extremely sluggish when trying to perform maneuvers such as climbing and turning.

Empennage design begins by determining an appropriate static margin and calculating the required tail area to meet this value. From the above equations, only eight variables are needed to size the horizontal tail. Table 1 is a list of the eight needed variables.

Table 1. Eight Needed Variables for Horizontal Tail Sizing

S_w	$\bar{c}$	AR	l_t
X_{cg}	X_{ac}	$C_{L,\alpha,w}$	$C_{M,\alpha,f,pwr}$

At the beginning of aircraft design, $C_{M,\alpha,f}$ and $C_{M,\alpha,pwr}$ are usually unknown and can be estimated from similar aircraft. Methods in "Flight Stability and Automatic Control" by Nelson[4] describe how to estimate $C_{M,\alpha,f}$ once the fuselage shape is fully defined. With the known variables and the desired static margin, the designer must then determine the appropriate aspect ratio for the horizontal tail to determine the lift curve slope. For this analysis and code, the airfoil is assumed to be a NACA 0012 which is a common airfoil for horizontal stabilizers. It has a 2-D lift curve slope ($c_{l,\alpha}$) of 0.1 per deg. Equations (1) through (6) can then be solved for the required horizontal tail area (S_H) needed to stabilize the aircraft at the desired static margin. Usually, it is easier to solve for the volume coefficient (V_H) and then solve for the tail area.

Next, the vertical tail sizing is performed by determining the appropriate tail size for an acceptable $C_{N,\beta}$. According to Roskam[5], an acceptable $C_{N,\beta}$ is 0.001 per deg or 0.057 per radian. $C_{N,\beta}$ is given as

$$C_{N_\beta}=C_{N_{\beta,wf}}+C_{L_{\alpha,v}}V_V\left(1+\frac{d\sigma}{d\beta}\right) \quad \text{Eq. (7)}$$

$$\left(1+\frac{d\sigma}{d\beta}\right)=0.724+\frac{\left(3.06*\frac{S_v}{S}\right)}{1+0.009AR} \quad \text{Eq.(8)}$$

where $C_{N,\beta,wf}$ is the $C_{N,\beta}$ due to the wing and fuselage. Similar to $C_{M,\alpha,f}$ and $C_{M,\alpha\ pwr}$, $C_{N,\beta,wf}$ can be estimated at the beginning of the aircraft design. $C_{L,\alpha,v}$ is the vertical tail lift curve slope. Again, a NACA 0012 airfoil is used. Equation (4) can be used to determine $C_{L,\alpha,v}$ depending on the vertical tail aspect ratio, AR_V. V_V is the vertical tail volume coefficient and is given as

$$V_V=\frac{S_v l_t}{S_w\, b} \quad \text{Eq. (9)}$$

where S_V is the vertical tail area and b is the wing span. $d\sigma/d\beta$ is the sideslip downwash term and is accounted for in Eq. (8). Similar to horizontal tail sizing, the designer must pick a vertical tail aspect ratio to calculate the lift curve slope. Aspect ratios for the vertical tail vary, but commonly an aspect ratio of 1.5 is chosen. Equations (7) through (9) can be solved for the vertical tail area based on the desired $C_{N,\beta}$. These equations are often solved numerically.

B. How the Code Works

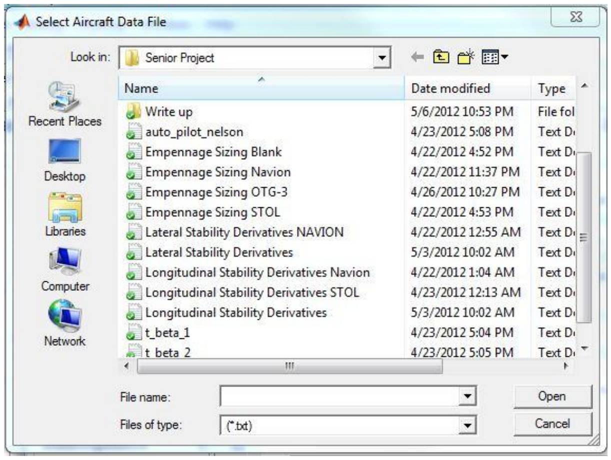
The Basic Empennage Sizing code begins by asking the user to select the aircraft data file (Step 1) which has the variables listed in Table 1 as well as $C_{N,\beta,wf}$. The user must fill out the first 9 variables in this file for the program to run. An example for the aircraft data file can be found in the Appendix (Empennage Sizing Blank.txt). The data file will then be read into Matlab. The program then prompts the user for the desired static margin and horizontal tail taper ratio, λ_H (Step 2). The taper ratio will have an effect on the wing lift curve slope in Eq. (4); however, for this analysis, it is assumed to be negligible. The user will then be asked to “Select Aircraft Type” and will be given 8 options (Step 3). Each option corresponds to a horizontal tail aspect ratio and historical volume coefficients which are used as a starting point in the numerical calculations of V_H and V_V. The final option is for the user to enter the desired horizontal tail aspect ratio, V_H, and V_V. The options and their values are given in Table 2.

Table 2. “Select Aircraft Type” Menu Options

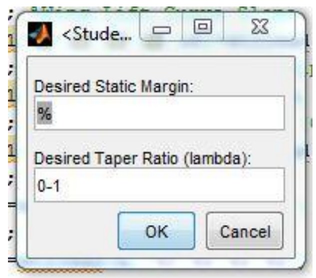
Airplane	V_H	V_V	AR_H
General Aviation – 1 engine	0.7	0.04	4
General Aviation – 2 engine	0.8	0.07	3
Twin Turbo-prop	0.9	0.08	3
Military Cargo	1.0	0.08	3.5
Sailplane	0.5	0.02	4
Agriculture	0.5	0.04	3.5
Homebuilt	0.5	0.04	3
Enter Desired AR_H, V_H, V_V	-	-	-

The user will then be asked to enter the desired vertical tail aspect ratio as well as the vertical tail taper ratio, λ_V (Step 4). The taper ratio for the vertical tail is usually 0.5 while horizontal tails range between 0.5 and 1. The code will then calculate the tail areas and dimensions for the desired stability. The Matlab command window will then display the dimensions for the horizontal and vertical tail including estimated elevator and rudder sizes. Two plots will be displayed which are the dimensions of the horizontal and vertical stabilizers. The user will then be asked to display the 3-D plot of the entire empennage. An example of these outputs will be shown in the next section.

Step 1)

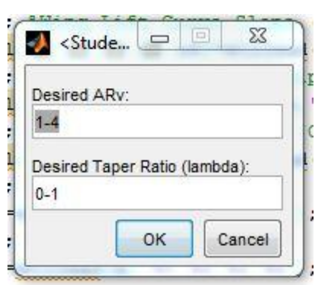


Step 2)



Step 3)

Step 4)



C. Code Validation

To validate the code, a test case was run using the Ryan Navion Aircraft. In Nelson[4], the Navion aircraft is used as an example to determine tail sizes in the same manner as explained in the previous section. The code was validated against the results in Nelson. The inputs are shown in Table 3 and the outputs as they appeared in the Matlab command window are shown below.

Table 3. Inputs to Basic Empennage Sizing Code for Ryan Navion Aircraft

Static Margin	25%
Horizontal Taper (λ_H)	0.5
Aircraft Type	General Aviation - 1 Engine
Vertical Aspect Ratio	1.3
Vertical Taper (λ_V)	0.5

Matlab Command Window Output:

Horizontal Tail Sizing

Static Margin is 25%
Vh is 0.68113
ARh is 4
Horizontal Tail Area is 44.6479 ft^2
Horizontal Tail Span is 13.3638 ft
Horizontal Tail Root Chord is 4.4546 ft
Horizontal Tail Tip Chord is 2.2273 ft
Horizontal Tail Taper Ratio is 0.5
Horizontal Tail 1/4 Chord Sweep is 4.7636 deg
Horizontal Tail Max Thickness is 0.53455 ft
Elevator Area is 13.3944 ft^2 (Add Margin of Safety)
Elevator Span is 6.0137 ft (per side)
Elevator Root Chord is 1.6983 ft
Elevator Tip Chord is 0.8074 ft

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Vertical Tail Sizing

Vv is 0.041192
ARv is 1.3
Vertical Tail Area is 15.2915 ft^2
Vertical Tail Span is 4.4586 ft
Vertical Tail Root Chord is 4.5729 ft
Vertical Tail Tip Chord is 2.2865 ft
Vertical Tail Taper Ratio is 0.5
Vertical Tail 1/4 Chord Sweep is 21.0375 deg
Vertical Tail Max Thickness is 0.54875 ft
Rudder Area is 4.5875 ft^2 (Add Margin of Safety)
Rudder Span is 4.4586 ft
Rudder Chord is 1.0289 ft
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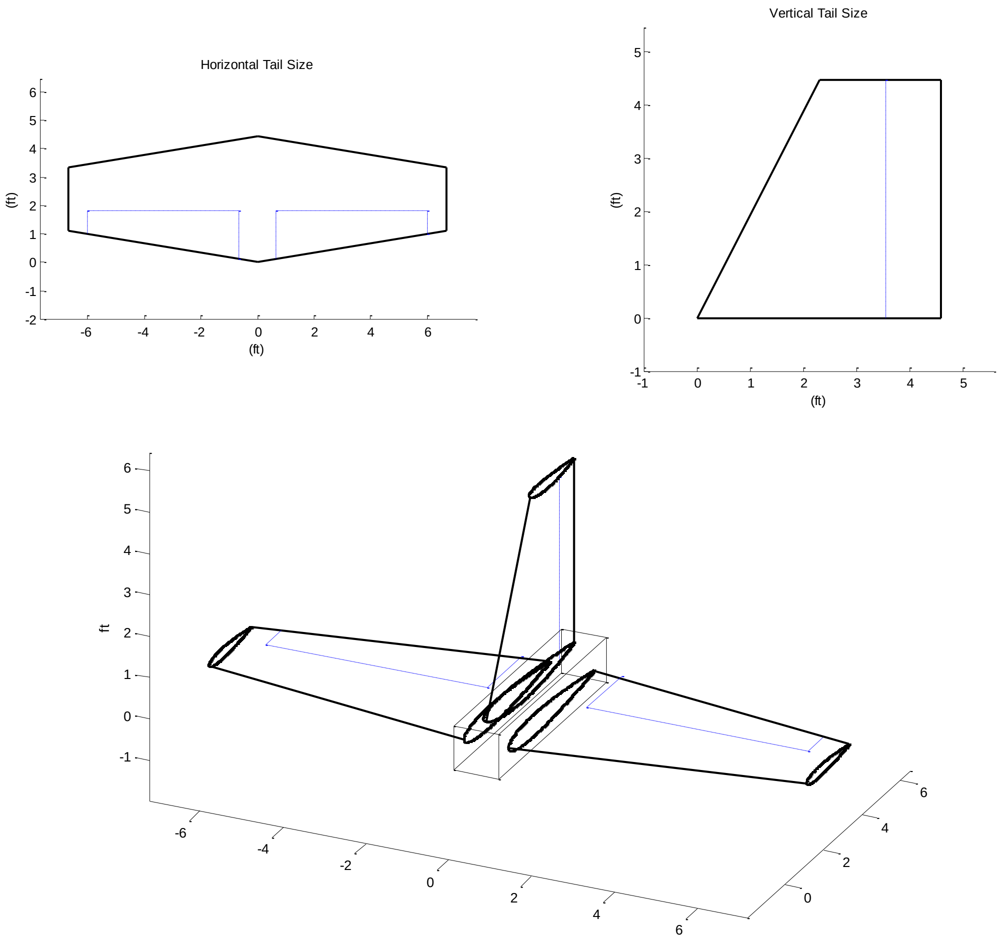


Figure 6. Basic Empennage Sizing Plots for Ryan Navion Aircraft Example

The comparison between the code's predicted values and the actual values is shown in Table 4.

Table 4. Basic Empennage Sizing Validation

	Navion - Actual	Code Prediction	% Diff
Horizontal Tail Area (S_H)	43 ft^2	44.65 ft^2	+ 3.8 %
V_H	0.66	0.68	+ 3.03 %
X_{NP}	0.55	0.545	- 9.09 %
Vertical Tail Area (S_V)	14.6 ft^2	15.3 ft^2	+ 4.8 %
V_V	0.04	0.041	+ 2.5 %

As it can be seen from the percent differences, the code accurately predicts both the horizontal and vertical tail areas for the Navion aircraft. Using the basic sizing techniques with only 9 known aircraft variables (two of which are estimated); the empennage can be accurately sized for stability.

IV. Detailed Empennage Sizing

Up until now, all the stability analysis has been static stability (i.e. the forces are not moving). In addition to static stability, the dynamic stability of the aircraft must be determined. Just because the aircraft is statically stable, this does not ensure dynamic stability. However, accurate dynamic stability requires detailed knowledge of the aircraft. Most importantly, the mass moments of inertia (I) are needed. These values are usually not fully defined until very late in the aircraft design process.

A. Design Process

Unlike static stability where the forces and moments are set equal to zero, dynamic stability sets the forces and moments equal to mass*acceleration (for forces) and I*$\ddot{\theta}$ (for moments) where $\ddot{\theta}$ is the angular acceleration. Now the equations of motion in all three directions must be determined. For aircraft, the equations are extremely ugly and non-linear. They are often simplified and linearized using small-disturbance perturbations. These linearized equations are given from Nelson,

Longitudinal Equations:

$$\left(\frac{d}{dt} - X_u\right)\Delta u - X_w \Delta w + (g\cos\theta_o)\Delta\theta = X_{\delta e}\Delta\delta_e + X_{\delta T}\Delta\delta_T \qquad \text{Eq. (10)}$$

$$-Z_u\Delta u + \left[(1 - Z_{\dot{w}})\frac{d}{dt} - Z_w\right]\Delta w - \left[(u_o - Z_q)\frac{d}{dt} - g\sin\theta_o\right]\Delta\theta = Z_{\delta e}\Delta\delta_e + Z_{\delta T}\Delta\delta_T \qquad \text{Eq. (11)}$$

$$-M_u\Delta u - \left(M_{\dot{w}}\frac{d}{dt} + M_w\right)\Delta w + \left(\frac{d^2}{dt^2} - M_q\frac{d}{dt}\right)\Delta\theta = M_{\delta e}\Delta\delta_e + M_{\delta T}\Delta\delta_T \qquad \text{Eq. (12)}$$

Lateral Equations:

$$\left(\frac{d}{dt} - Y_v\right)\Delta v - Y_p\Delta p + (u_o - Y_r)\Delta r - (g\cos\theta_o)\Delta\phi = Y_{\delta r}\Delta\delta_r \qquad \text{Eq. (13)}$$

$$-L_v\Delta v + \left(\frac{d}{dt} - L_p\right)\Delta p - \left(\frac{I_{xz}}{I_x}\frac{d}{dt} + L_r\right)\Delta r = L_{\delta a}\Delta\delta_a + L_{\delta r}\Delta\delta_r \qquad \text{Eq. (14)}$$

$$-N_v\Delta v + \left(\frac{d}{dt} - N_r\right)\Delta r - \left(\frac{I_{xz}}{I_z}\frac{d}{dt} + N_p\right)\Delta p = N_{\delta a}\Delta\delta_a + N_{\delta r}\Delta\delta_r \qquad \text{Eq. (15)}$$

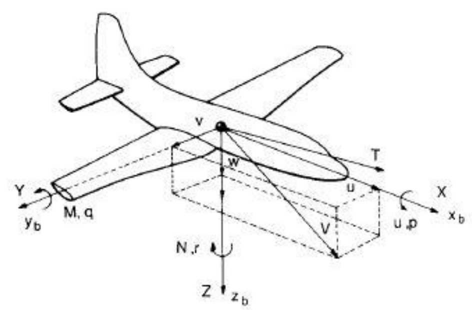


Figure 7. Axes, Forces, and Moments for an Aircraft

Table 5. Definition of Forces, Moments, and Velocity Components

	Roll Axis	**Pitch Axis**	**Yaw Axis**
Angular Rates	p	q	r
Velocity Components	u	v	w
Aerodynamic Force Components	X	Y	Z
Aerodynamic Moment Components	L	M	N
Moment of Inertia	I_x	I_y	I_z
Products of Inertia	I_{yz}	I_{xz}	I_{xy}

where δ_T is a change in throttle, δ_e is a change in elevator deflection, δ_a is a change in aileron deflection, and δr is a change in rudder deflection. These equations have also been simplified by only retaining the significant aerodynamic forces and moments and ignoring other motion variables.

Notice in the equations there are force and moment components that have a subscript from some other portion of the aircraft such as X_u or $N_{\delta a}$. These are called stability derivatives. For example, X_u is the change in the X force due to an increase in forward velocity, u. $N_{\delta a}$ is the change in yawing moment due to an aileron deflection. These stability derivatives allow for a simpler representation of the equations of motion. Many of the neglected forces and moments are represented as stability derivatives. When the stability derivatives are non-dimensionalized, they are given a capital C and are called stability coefficients. For example,

$$C_{X_u} = \frac{X_u u_o m}{Q S_w} \qquad \text{Eq. (16)}$$

where u_o is the reference flight velocity, m is the aircraft mass, and Q is the dynamic pressure. A complete summary of the non-dimensionlized stability derivatives are shown in the appendix. Using the linear equations of motion simplified with stability derivatives, the equations can be solved for the time response.

Solving the equations of motion is best done in state-space form. This is preferred because the state-space matrix is filled with stability derivatives only. From the state-space matrix, the stability of the aircraft can be determined. If the aircraft is stable, the time response will decay over time. An unstable aircraft will have an unbounded time response. The state-space matrices for both the longitudinal and lateral directions are given as,

Longitudinal: $(\dot{x} = Ax + Bu)$

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & u_o & 0 \\ M_u + M_{\dot{w}} Z_u & M_w + M_{\dot{w}} Z_u & M_q + M_{\dot{w}} u_o & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta \mathrm{q} \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} X_{\delta e} & X_{\delta T} \\ Z_{\delta e} & Z_{\delta T} \\ M_{\delta e} + M_{\dot{w}} Z_{\delta e} & M_{\delta T} + M_{\dot{w}} Z_{\delta T} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta e \\ \Delta\delta_T \end{bmatrix} \qquad \text{Eq. (17)}$$

Lateral: $(\dot{x} = Ax + Bu)$

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_o} & \frac{Y_p}{u_o} & -(1 - \frac{Y_r}{u_o}) & \frac{g\cos\theta_o}{u_o} \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta \mathrm{r} \\ \Delta\phi \end{bmatrix} + \begin{bmatrix} 0 & \frac{Y_{\delta e}}{u_o} \\ L_{\delta a} & L_{\delta r} \\ N_{\delta a} & N_{\delta r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta a \\ \Delta\delta_r \end{bmatrix} \qquad \text{Eq. (18)}$$

Many modern control theory books present techniques on how to solve these matrices to get the time response of the system. Instead of solving the system for the time response, the dynamic stability and lateral stability of the aircraft can be determine from the A state-space matrices. The characteristic equations for each direction can be determined by:

$$|A - \lambda I| = 0 \qquad \text{Eq. (19)}$$

where A is the state-space matrix for each direction, I is the identity matrix, and λ are the roots. For positive stability, all of the real parts of the roots must be negative. The roots are also known as the poles. If the poles are plotted on a real vs. imaginary axis, this is called a pole-zero map or root-locus. For stability, the poles must be in the left half plane. Figure 8 is an example of stable and unstable poles and the resulting time response showing the stability and instability.

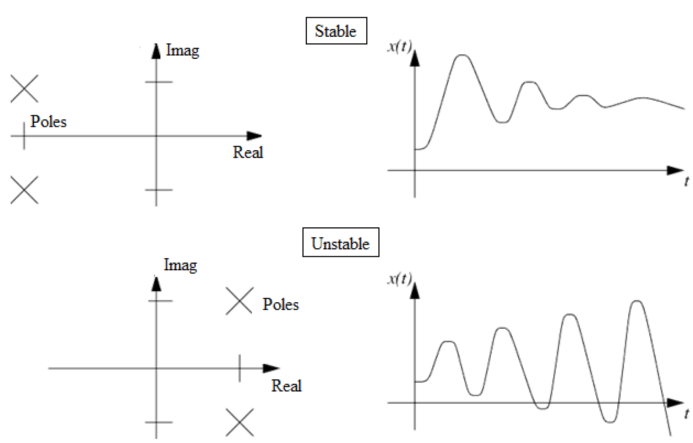


Figure 8. Stable and Unstable Root Locus and Time Response

One of the most beneficial reason for using state-space matrices is that the poles of system are simply the eigen values of the A state-space matrix. Dynamic stability of an aircraft is simply determining the stability derivatives, plugging them into the state space matrix, taking the eigen values, and then making sure they are all within the left half plane. The location of the poles relative to the real and imaginary axis determines how quickly the time response decays and the frequency of the oscillations.

Taking the eigen values of the longitudinal state-space matrix results in four poles (two sets of two poles). The poles closest to the origin are known as the phugoid or long period poles and the pair furthest from the origin are known as short period poles. From the poles, the damping ratio, ζ, and the natural frequency, ω_n, of the response can be determined by,

$$\zeta = \cos\,\theta \tag{Eq. (20)}$$

$$\omega_n = \frac{\omega}{\sqrt{1-\zeta^2}} \tag{Eq. (21)}$$

where θ is the angle between imaginary part of the pole (ω) and the real part of the pole ($\zeta*\omega_n$). By calculating the damping ratios, the flying level of the aircraft can be determined. The levels are as follows:

Level 1 – Flying qualities clearly adequate for the mission flight phase.
Level 2 – Flying qualities adequate to accomplish the mission flight phase but with some increase in pilot workload and/or degradation in mission effectiveness or both.
Level 3 – Flying qualities such that the airplane can be controlled safely but pilot workload is excessive and/or mission effectiveness is inadequate or both.

The FARs and MILSPECs generally base their regulations on the flying levels and damping ratios for stability requirements. Table 6 and 7 shows the damping ratios for the different flying levels.

Table 6. Phugoid Mode Flying Level Qualities

Level 1	$\zeta > 0.04$
Level 2	$\zeta > 0$
Level 3	$T_2 = 55$ s

Table 7. Short-Period Mode Flying Level Qualities

	ζ_{sp}	ζ_{sp}
	Min	Max
Level 1	0.35	1.30
Level 2	0.25	2.00
Level 3	0.15	-

where T_2 is the time to double or half the amplitude of the oscillation.

In the lateral direction, there is one pair of poles, known as the dutch roll mode, there is a pole (on the real axis) close to the origin known as the spiral mode, and the final pole is also located on the real axis, but much further away from the origin. This is known as the roll mode. Since the damping ratio cannot be calculated for the spiral mode, the flying levels are related to the minimum time to double or half in amplitude. The roll mode is calculated from the maximum roll time constants. The dutch roll flying qualities still depend on the damping ratio. The flying qualities for the lateral modes are shown in Table 8

Table 8. Lateral Flying Qualities

	Spiral Mode	Roll Mode	Dutch Roll
Level 1	T > 12s	$\tau < 1.0$	$\zeta > 0.19$
Level 2	T > 12s	$\tau < 1.4$	$\zeta > 0.08$
Level 3	T > 4s	$\tau < 10$	$\zeta > 0.02$

The designer can calculate the flying qualities and levels of the aircraft and then go back and make changes as necessary to correct for instability. The designer may also place the poles in the desired location for certain desired flying levels which will be discussed in the Stability Augmentation and Autopilot code.

B. How the Code Works

The Detail Empennage Sizing code begins by asking the user to select the aircraft data file. The same data file that was previously selected for the Basic Empennage Sizing code must be selected. For this code, everything in the file must be filled out. If any of the first 9 variables has changed, the first code needs to be rerun. The first code outputs variables that are used into the second code. It is important to run the first code with the correct aircraft and the correct file before running the second code.

After reading in the variables, the code will calculate all the stability derivatives for both the longitudinal and lateral directions. It will then put the stability derivatives into state-space form and find the eigen values of the matrices. From the eigen values, the flying quality levels will be determined. The flying qualities will be displayed in the command window and two text files will be created containing all of the stability derivatives and their units (one for longitudinal and one for lateral). These text files must be saved under a different name for each use so that the code will not overwrite the files. Two plots will be created which are the pole-zero maps (or root-locus) for each direction. From these, the dynamic stability can be verified if all the poles are in the left half plane. The code ends by outputting variables to be used in the next code.

C. Code Validation

The code was validated against the Navion aircraft as used in the previous section. The longitudinal and lateral stability derivatives were compared between the equations used in the code and the results in Nelson. The Matlab command window and plots are shown below:

```
Detailed Empennage Sizing

Longitudinal Flying Qualities

Phugoid Damping is Level 1
Phugoid Damping = 0.13353

Short Damping is Level 1
Short Period Damping = 0.68046

~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

Lateral Flying Qualities

Spiral Mode is Level 1
Spiral Mode time to double (or half) = 19.0838 seconds

Roll Mode is Level 1
Roll Mode time to double (or half) = 0.059173 seconds

Dutch Roll Mode is Level 1
Dutch Roll damping =  0.1984
```

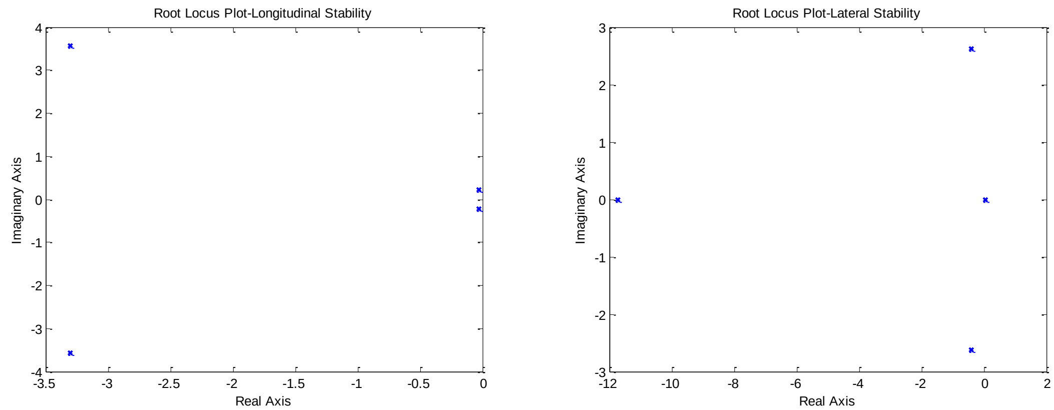


Figure 9. Longitudinal and Lateral Root-Locus Plots for Navion

From the command window, the aircraft is at a Level 1 flying quality for all modes in both the longitudinal and lateral direction. It can also be seen from the root locus plots that all the poles are in the left half plane showing the dynamic stability of the Navion aircraft.

Next, the individual stability derivatives were compared. Table 9 and 10 are the comparisons of some of the important stability derivatives in each direction. The text file outputs for the Navion aircraft can be found in the appendix (Longitudinal Stability Derivatives Navion.txt and Lateral Stability Derivatives Navion.txt).

Table 9. Longitudinal Stability Derivatives Comparison

Stability Derivative	Code Results	Nelson	% Diff
X_u	-0.0674	-0.045	-49.8%
X_w	+0.0353	+0.036	-1.9%
Z_u	-0.3685	-0.369	+0.1%
Z_w	-2.0180	-2.01	-0.4%
M_q	-3.1278	-2.05	-53.6%

Table 10. Lateral Stability Derivatives Comparison

Stability Derivative	Code Results	Nelson	% Diff
N_p	-0.2904	-0.35	+17.0%
N_r	-0.5516	-0.76	+27.4%
N_β	+6.9274	+4.49	+54.3%
L_p	-11.7679	-8. 4	-40.1%
L_r	+2.3439	+2.19	+7.0%
$C_{N,\delta r}$	-0.0516	-0.072	+28.3%
$C_{L,\delta a}$	-0.134	-0.134	0%

The inaccuracy of some of the values could be explained by the estimation of some of the unknown variables needed to perform these calculations. Not all of the Navion's dimensions and variables were found within Nelson and other sources. Since the variables are on the right order of magnitude and not widely off, the method can be satisfactorily validated. If the user would like another source to compare the stability derivatives, AVL[6] is a great program at estimating these stability derivatives, but is much more complex.

V. Longitudinal Static Stability

The basic and detailed empennage sizing code only dealt with one CG location. However, the CG will always shift during flight as fuel in burned. It is necessary to make sure that the tail can properly stabilize an aircraft at the CG extremes during flight. Although this analysis is still considered static and not dynamic, it depends on the stability derivative ($C_{M,\delta e}$) which is a dynamic variable and was calculated in the previous code.

A. Design Process

A common visual approach to showing stability of an aircraft is through C_M vs α or C_M vs C_L chart. This chart plots the pitching moment about the CG at various angles of attack and C_L values. From this chart, the stability can be verified by checking that the line for the aircraft is negative sloping (refer to Fig. 2). This chart also shows the angle of attack or C_L value that the aircraft must fly at to meet its equilibrium trim condition ($C_{M,cg}$ = 0). To find the pitching moment about the CG ($C_{M,cg}$),

$$C_{M_{CG}} = C_{M_o} + C_{M_\alpha}\alpha \qquad \text{Eq. (22)}$$

where $C_{M,o}$ is the pitching moment at zero angle of attack, and $C_{M,\alpha}$ is from Eq. (1). To trim the aircraft at zero angle of attack, $C_{M,o}$ must be zero. $C_{M,o}$ can be found by summing up the contribution from the components of the aircraft similar to how $C_{M,\alpha}$ was derived,

$$C_{M_o} = C_{M_{o,w}} + C_{M_{o,t}} + C_{M_{o,f}} \qquad \text{Eq. (23)}$$

$$C_{M_{o,w}} = C_{M_{AC}} + C_{L_o}\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right) \qquad \text{Eq. (24)}$$

$$C_{M_{o,t}} = \eta V_H C_{L_{o,t}}(\varepsilon_o + i_w - i_t) \qquad \text{Eq. (25)}$$

$$\varepsilon_o = \frac{2C_{L_{o,w}}}{\pi AR} \qquad \text{Eq. (26)}$$

where $C_{M,o,f}$ is the zero angle of attack pitching moment from the fuselage, $C_{M,ac}$ is the coefficient of moment about the aerodynamic center, $C_{L,o}$ is the zero lift coefficient of lift, ε_o is the downwash angle (rad), i_w is the incidence angle of the wing (rad), and i_t is the tail incidence angle (rad). Changing the incidence angle of the tail allows the $C_{M,\alpha}$ curve to shift up or down but will not change the slope. However, this is only for one CG location. If the tail is set at an incidence angle for one CG case, it may create a negative $C_{M,o}$ at another CG location which will require the aircraft to fly at a negative angle of attack for trimmed flight. Most pilots prefer to fly at zero angle of attack, so the pitching moment curve must be shifted. The pitching moment curve can be shifted up or down by deflecting the elevators. Deflecting the elevators does induce extra drag known as trim drag. Airliners reduce trim drag by using an all moving horizontal tail to change the incidence angle during flight as the CG changes. For this analysis, the incidence angle will be set at a constant incidence angle to minimize the trim drag at the desired CG location. A trim triangle will be constructed to ensure the elevator deflections can trim the aircraft for the range of CG locations.

Instead of plotting numerous C_M vs α or C_M vs C_L curves for a range of CG locations, a trim triangle is constructed. A trim triangle takes a reference CG (usually the desired CG or middle CG point) and finds the pitching moment difference between the forward CG limit and the desired CG for all angles of attack. It then does the same for the aft CG limit and plots both of these lines. These lines form the two sides of the triangle where the x-axis is the angle of attack (α) and the y-axis is the difference in pitching moment (ΔC_M). The final side of the triangle is determined from the elevator stall or wing stall. The elevators have a deflection limit to which they will cause a stall on the horizontal tail. At this deflection, there is a maximum C_L value they can reach. The maximum C_L values will create the right side of the triangle. Inside the triangle is all of the pitching moment values that the aircraft is expected to see during flight as the CG shifts. The change in pitching moment due to the elevator deflections are then plotted on top of the triangle. The change in pitching moment due to the elevators can be calculated by,

$$\Delta C_M = C_{M,\delta e} * \delta_e \qquad \text{Eq. (27)}$$

where $C_{M,\delta e}$ is the coefficient of moment due to an elevator deflection and δ_e is the elevator deflection. If the triangle is within the maximum elevator deflections, the tail can properly stabilize the aircraft at all CG locations and at all angles of attack. Usually, the maximum elevator deflections are between 25 and 30 degrees in either direction. If parts of the triangle are outside the elevator deflection limits, the designer must increase the tail size or increase the elevator area. If the tail is already sufficiently large, the designer may ask the weights design engineers to limit the CG travel. The further the CG is aft of the wing's aerodynamic center, the larger the tail.

B. How the Code Works

The Longitudinal Static Stability code begins by prompting the user for the aircraft data file. The aircraft data file should be the same as previously selected. After reading in the aircraft file, the code will read in the data from the first two codes. It is of important that the first two codes have been run before this one using the correct aircraft. If another aircraft is run, this will overwrite the data stored. The code will then prompt the user which plots are to be displayed. The options are shown in the menu in Step 1.

Step 1)

If the user selects “Display All Plots”, seven plots will be displayed: C_M vs α and C_M vs C_L for the desired CG location, the most aft CG location, and the most forward position, followed by the trim triangle. If any of the other options are selected, it will only display the selected C_M vs α and C_M vs C_L plots and the trim triangle.

Before the code begins determining the pitching moment, it determines the optimal tail incidence angle to minimize trim drag. Using the optimal tail incidence angle, the pitching moments are found for various angle of attacks and C_L values for the three CG cases. The code will then make the trim triangle. The code starts at initial elevator deflections of -10 and + 30 degrees and determines how the moments fit inside the deflections. The code will then iterate and change the elevator deflections to move closer or further away from the triangle if necessary. The maximum and minimum elevator deflections that are needed to trim the aircraft during cruise are displayed on the trim triangle. The command window will display the optimal tail incidence angle as well as the static margins, $C_{M,\alpha}$’s, and $C_{M,o}$’s for the three CG cases.

C. Code Validation

The code was validated against the Navion aircraft from Nelson[4]. The command window and all the plots for the Navion are shown below:

Longitudinal Static Stability

Tail Incidence Angle -2.5 deg

Desired CG Location (Xcg = 1.682 ft)
Static Margin is 25%
CMo is 0.13735
CMalpha is -1.11

~~~~~~

AFT CG Location (Xcg = 1.9 ft)
Static Margin is 21%
CMo is 0.14217
CMalpha is -0.94019

~~~~~~

FWD CG Location (Xcg = 1.1 ft)
Static Margin is 35%
CMo is 0.087772
CMalpha is -1.5633

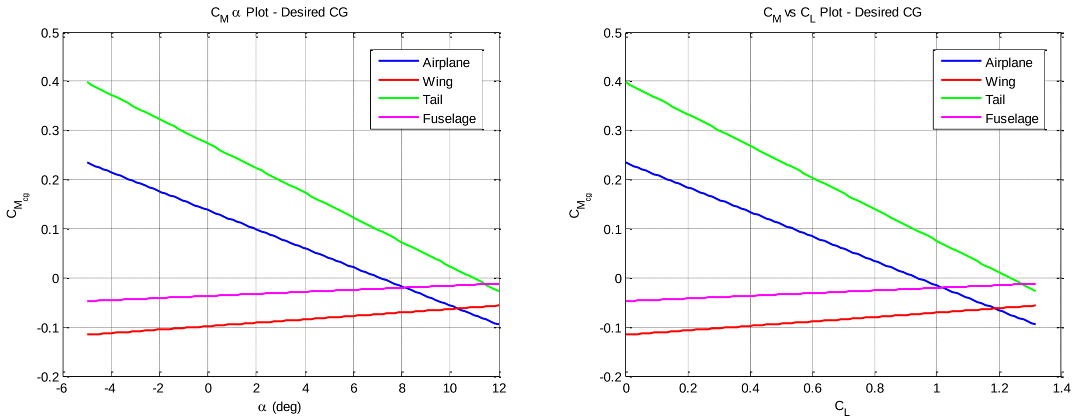


Figure 10. Longitudinal Static Stability for Desired CG Case - Navion

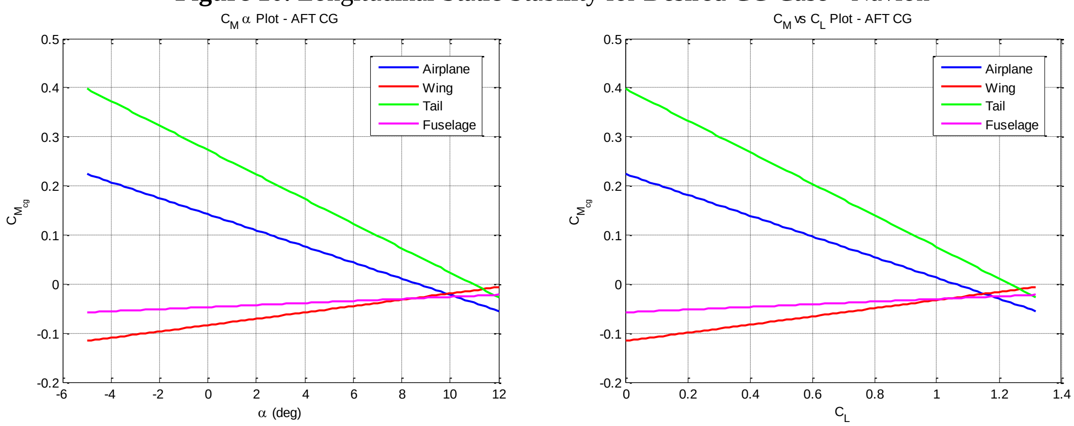


Figure 11. Longitudinal Static Stability for Aft CG Case - Navion

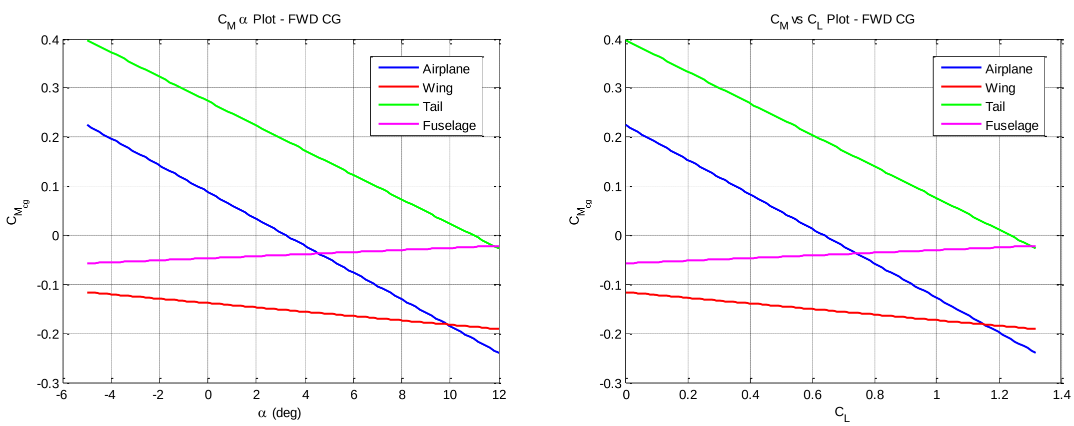


Figure 12. Longitudinal Static Stability for Forward CG Case - Navion

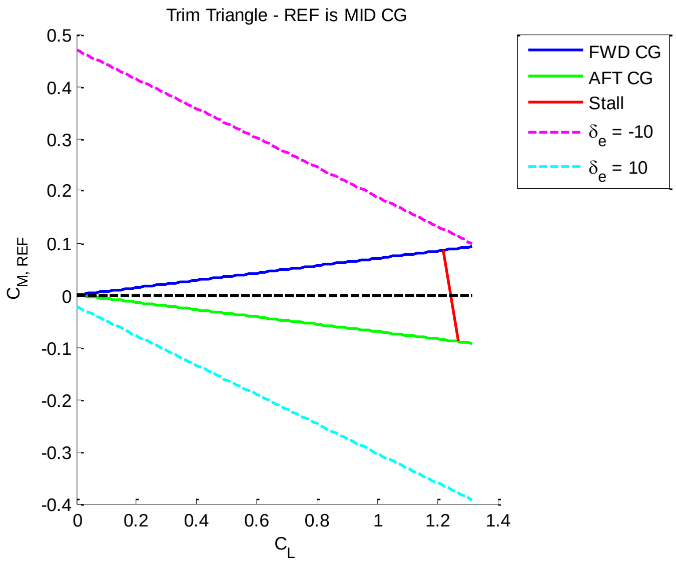


Figure 13. Cruise Trim Triangle - Navion

All of the CG locations are stable since the total aircraft $C_{M,\alpha}$ lines (blue lines) are all negative sloping. In most cases the wing and fuselage are destabilizing since they are positive sloping lines. From these charts, the trim condition angle of attack (or C_L) can be found. The trim triangle shows that the elevators easily control the entire range of pitching moments for the range of CG locations. To trim the aircraft at any CG location during cruise, the elevators would need to be deflected between -10 and +10 degrees.

To verify the code's accuracy, it was validated against the C_M vs α chart in Nelson[4] for the Navion Aircraft. The code's prediction is on the left in Fig. 14 and the actual C_M vs α curve from Nelson on the right.

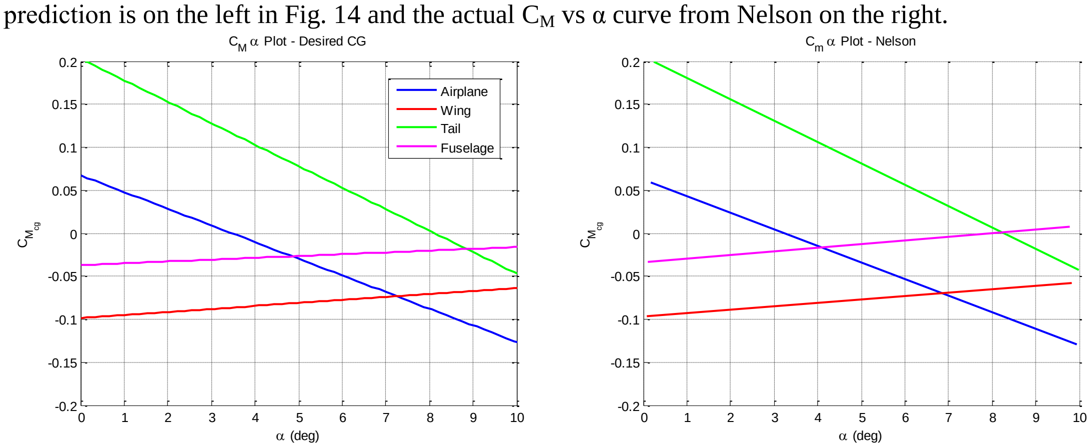


Figure 14. Longitudinal Static Stability Verification - Navion

The results are almost identical which validates the code's accuracy.

VI. Stability Augmentation and Autopilot

After the designer has finished sizing the empennage and checked the stability, the designer can then see how the aircraft will reject disturbances, how it can be better stabilized using feedback control, and what feedback gains are necessary for an autopilot system. The stability can be augmented by using feedback control to improve the handling and flying characteristics. At this point no further changes can be made to the empennage.

A. Design Process

Feedback control can augment a system by amplifying the output and comparing it to the reference input. The gains that the output is amplified by are known as the feedback gains, k. Correct selection of these gains allows the designer to have any desired system performance. The block diagram for a simple feedback control system is shown in Fig. 15.

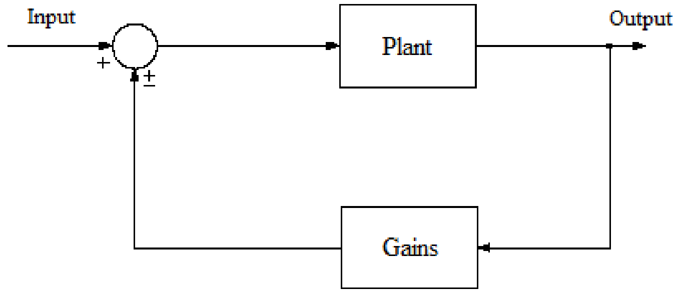


Figure 15. Feedback Control System with Feedback Gains

For this analysis, the plant is the linearized equations found in §IV. The state-space matrices in Eq. (16) and (17) are to be substituted into the plant. The input is the commanded or desired x vector. The output is the resulting x vector. There are many ways to determine the feedback gains just as there are many ways to control to a desired output.

To augment the longitudinal stability, the Bass-Gura technique was used to determine the gains for the desired performance. The gains, k, are calculated from

$$k = [(VW)^T]^{-1}[\bar{a} - a] \quad \text{Eq. (28)}$$

where V is the controllability matrix, W is the transformation matrix, and $\bar{a}$ and a are vectors made up of the coefficients of the characteristic equation of the augmented or closed loop system. By substituting in the desired eigen values or damping ratios and natural frequencies, the gains for augmentation can be found.

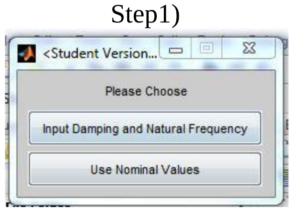
To augment the lateral stability, the same technique was used. However, the problem is simplified into a 2X2 matrix with sideslip (β) and yaw rate (r). The same Bass-Gura technique was used to determine the gains needed given a desired damping ratio and the undamped natural frequency for the dutch roll mode.

Finally, an altitude hold autopilot system was created using the same feedback control system as used above. The plant to the autopilot system is now the entire closed loop (augmented) system from above. The inputs to this outer loop are different than the inputs to the inner loop plant. In the outer loop, the angle of attack (α), the pitch rate (q), the pitch angle (θ), and the altitude (h) is the x vector. This will change both the A and B state-space matrices for the outer loop. Although the Bass-Gura technique can be used to determine the gains, this method does not determine the optimal gains. To determine the optimal gains, the Linear Quadratic Regression (LQR) method was used. This method solves the algebraic Ricatti equations and uses these to determine the optimal gains. To solve for the LQR gains, the lqr function in Matlab was used.

B. How the Code Works

Before the code is run, it is important that Detailed Empennage Sizing be run first. This code relies on the state-space matrices for stability augmentation. It is extremely important that the state-space matrices outputted from the Detailed Empennage Sizing code are the correct matrices. The code begins by asking the user to input the desired damping and natural frequencies or to use the nominal values that are already imbedded in the code (Step 1).

Step1)



The nominal values are given in Table 11.

Table 11. Imbedded Damping Ratios and Natural Frequencies

Longitudinal Augmentation	**Lateral Augmentation**
$\zeta_{SP} = 0.6$	$\zeta_{DR} = 0.3$
$\zeta_{P} = 0.05$	$\omega_{n,SP} = 1$
$\omega_{n,SP} = 3$	
$\omega_{n,SP} = 0.1$	

If the user wishes to input their own damping and natural frequencies, the following menu will appear (Step 2):

Step 2)

After entering the desired damping ratios and natural frequencies, the code will then determine the gains for the longitudinal augmentation, lateral augmentation, and the autopilot methods as outlined above. The Matlab command window will then display these gains. The gains will then be entered into a Simulink block diagram that will determine the time response for all three cases. The time responses will then be plotted to show the user the stability augmentation.

C. Validation

To validate the methods, three examples were used from Nelson[4]. These examples gave state space matrices and the desired aircraft performance. The resulting gains from the code were then compared to the gains stated in the example as shown in Table 12.

Table 12. Stability Augmentation and Autopilot Gains Validation

	Code Prediction	**Nelson**	**% Diff**
Longitudinal Augmentation	-0.00549	-0.0055	+3.3%
	-0.01202	-0.0120	-0.1%
	-0.77848	-0.7785	0%
	-0.06557	-0.0656	0%
Lateral Augmentation	-15.9563	-16.1	+0.9%
	-13.6445	-13.7	+0.4%
Autopilot	0.09705	0.098	-1.0%
	-0.3045	-0.304	-0.1%
	-1.7198	-1.715	-0.3%
	-0.00175	-0.0017	-2.9%

The code's predicted gains were extremely accurate when compared to the actual gains listed in Nelson which verifies the code's accuracy. Next, the time responses were compared to verify that the Simulink block diagram accurately calculated the responses. The lateral stability augmentation and the autopilot were compared to the plots given in Nelson for the examples used to predict the gains as shown in Fig. 16.

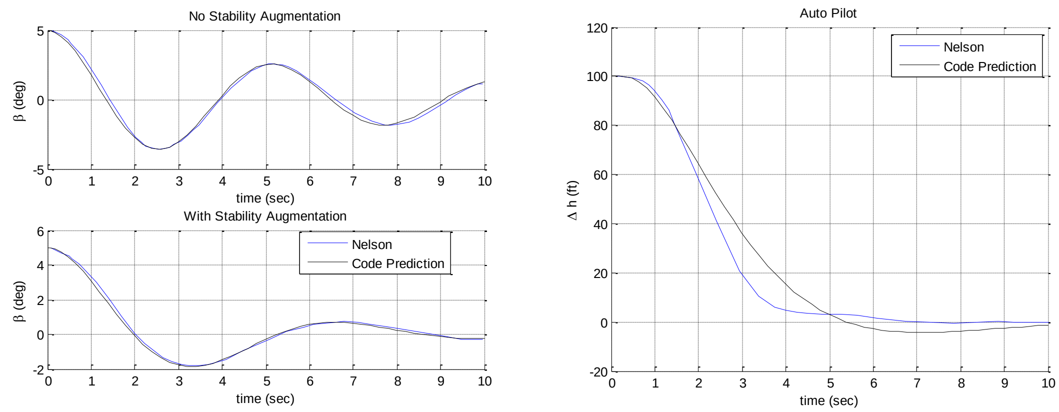


Figure 16. Lateral Augmentation and Autopilot Time Response Validation

The lateral stability was extremely accurate while the autopilot was not as accurate but still very close. These plots validate the methods ability as well as the accuracy of the code.

VII. OTG-3 Test Case

To show how the designer would use this code on a conceptual aircraft design, a test aircraft was run through all the codes. OTG-3 is a conceptual aircraft in response to the AIAA Undergraduate Team Design Competition RFP which calls for an unmanned air vehicle that must takeoff and land in 500 ft. This aircraft was designed by a group of seven students in the Aerospace Engineering senior aircraft design class at California Polytechnic State University, San Luis Obispo in 2011-2012. The empennage was designed using this process and the four codes.

Figure 17. OTG-3

A. Basic Empennage Sizing

The empennage sizing began by filling out the first 9 variables in the Empennage Sizing text file. (OTG-3's file can be found in the Appendix, Empennage Sizing OTG-3.txt) The inputs in Table 13 were then used for OTG-3.

Table 13. Basic Empennage Sizing Inputs – OTG-3

Static Margin	5%
Horizontal Taper (λ_H)	1
Aircraft Type	Enter Desired AR_H, V_H, and V_V
AR_H	3
V_H	0.4
V_V	0.04
AR_V	1.44
Vertical Taper (λ_V)	0.5625

With these inputs, the follow tail size was determined:

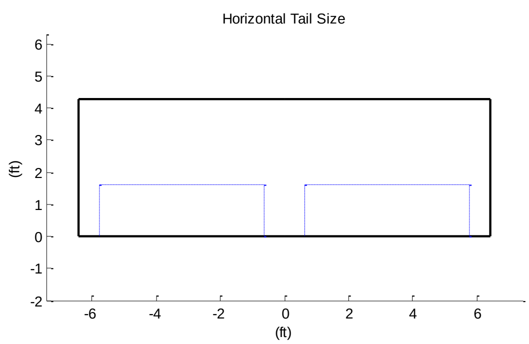
Horizontal Tail Sizing

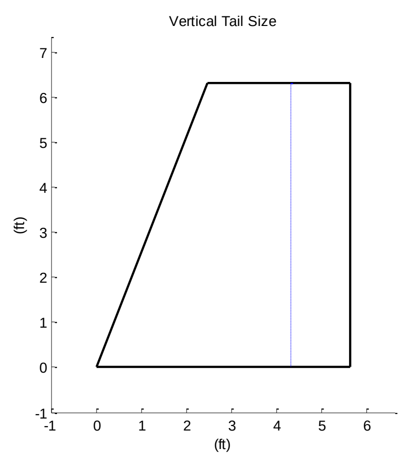
Static Margin is 5%
Vh is 0.71254
ARh is 3
Horizontal Tail Area is 54.9363 ft^2
Horizontal Tail Span is 12.8378 ft
Horizontal Tail Root Chord is 4.2793 ft
Horizontal Tail Tip Chord is 4.2793 ft
Horizontal Tail Taper Ratio is 1
Horizontal Tail 1/4 Chord Sweep is 0 deg
Horizontal Tail Max Thickness is 0.51351 ft
Elevator Area is 16.4809 ft^2 (Add Margin of Safety)
Elevator Span is 5.777 ft (per side)
Elevator Root Chord is 1.6047 ft
Elevator Tip Chord is 1.6047 ft

~~~~~~

Vertical Tail Sizing

Vv is 0.036063
ARv is 1.44
Vertical Tail Area is 27.8036 ft^2
Vertical Tail Span is 6.3275 ft
Vertical Tail Root Chord is 5.6244 ft
Vertical Tail Tip Chord is 3.1637 ft
Vertical Tail Taper Ratio is 0.5625
Vertical Tail 1/4 Chord Sweep is 16.2602 deg
Vertical Tail Max Thickness is 0.67493 ft
Rudder Area is 8.3411 ft^2 (Add Margin of Safety)
Rudder Span is 6.3275 ft
Rudder Chord is 1.3182 ft

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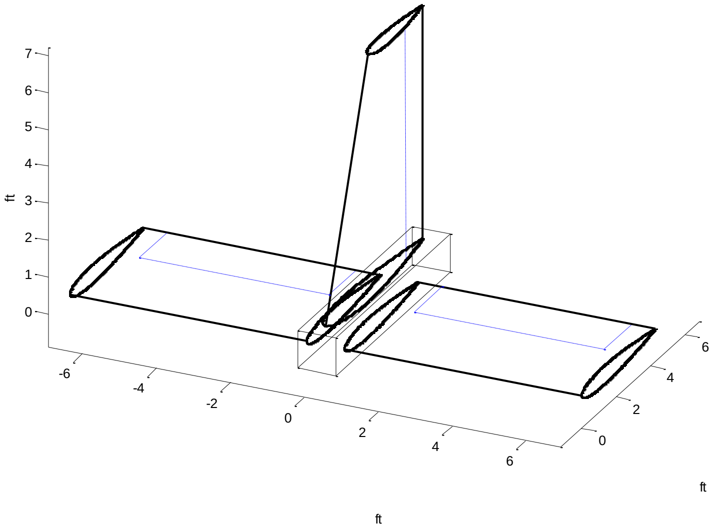

Figure 18. Basic Empennage Sizing Plots for OTG-3

Since this aircraft is unmanned, the static margin can be reduced significantly. The best static margin would be 0% or neutrally stable. An unmanned aircraft is more capable to reject disturbances and remain neutrally stable because of the fly-by-wire control system as well as the on-board flight computer system. However, for conceptual design, it is best to leave a positive static margin to account for shifting of the CG location as the design progresses into preliminary and detailed design. As more details of the aircraft are determined, the CG location will change which could lead to an unstable aircraft if not taken into account.

B. Detailed Empennage Sizing

Using the tail size found in the in the Basic Empennage Sizing code, the dynamic stability of OTG-3 was determined. The resulting Matlab command window output and the root locus plots are shown below:

Detailed Empennage Sizing

Longitudinal Flying Qualities

Phugoid Damping is Level 1
Phugoid Damping = 0.071391

Short Damping is Level 1
Short Period Damping = 0.35601

~~~~~~

Lateral Flying Qualities

Spiral Mode is Level 1
Spiral Mode time to double (or half) = 54.1013 seconds

Roll Mode is Level 1
Roll Mode time to double (or half) = 0.024653 seconds

Dutch Roll Mode is Level 1
Dutch Roll damping = 0.1182
~~~~~~

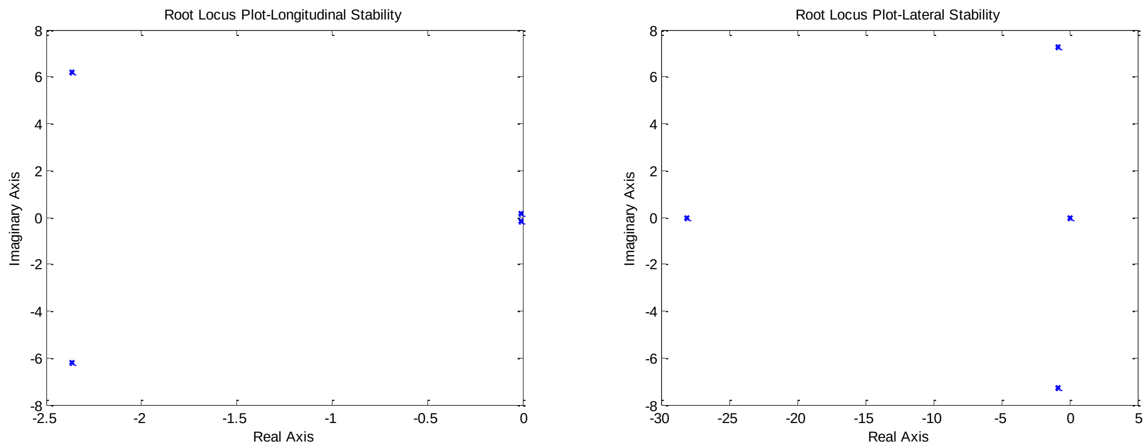

Figure 19. Longitudinal and Lateral Root-Locus Plots for OTG-3

All of the damping ratios and time constants satisfy Level 1 flying qualities. OTG-3 is also dynamically stable since all the poles are in the left half plane. The roll mode pole (the one furthest away from the origin), is extremely far away. Usually this pole is located between -7 and -10 on the real axis. The far away pole may explain why the roll mode is extremely small. To optimize this tail, stability augmentation should be used to move the roll pole to the desired location closer to the origin. The outputted stability derivatives of OTG-3 are given in the Appendix.

C. Longitudinal Static Stability

The C_M vs α and C_M vs C_L plots were then created for OTG-3. For OTG-3, the 5% static margin was chosen for the furthest aft CG location. This was to ensure that a shifting CG would only further stabilize the aircraft and not lead to instability since the CG can only shift forward. Since the desired CG is the aft CG, the desired CG plots are not shown.

Longitudinal Static Stability

Tail Incidence Angle -3.5 deg

Desired CG Location (Xcg = 1.7228 ft)
Static Margin is 5%
CMo is 0.021979
CMalpha is -0.24561

~~~~~~

AFT CG Location (Xcg = 1.7228 ft)
Static Margin is 5%
CMo is 0.011979
CMalpha is -0.24561

~~~~~~

FWD CG Location (Xcg = 1.266 ft)
Static Margin is 14%
CMo is -0.019991
CMalpha is -0.70363

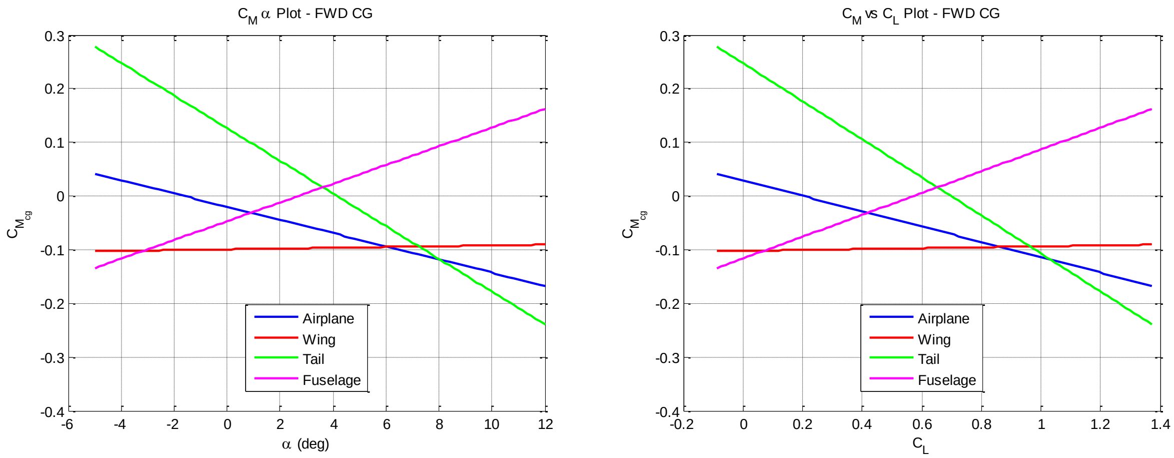

Figure 20. Longitudinal Static Stability for Forward CG Case - OTG-3

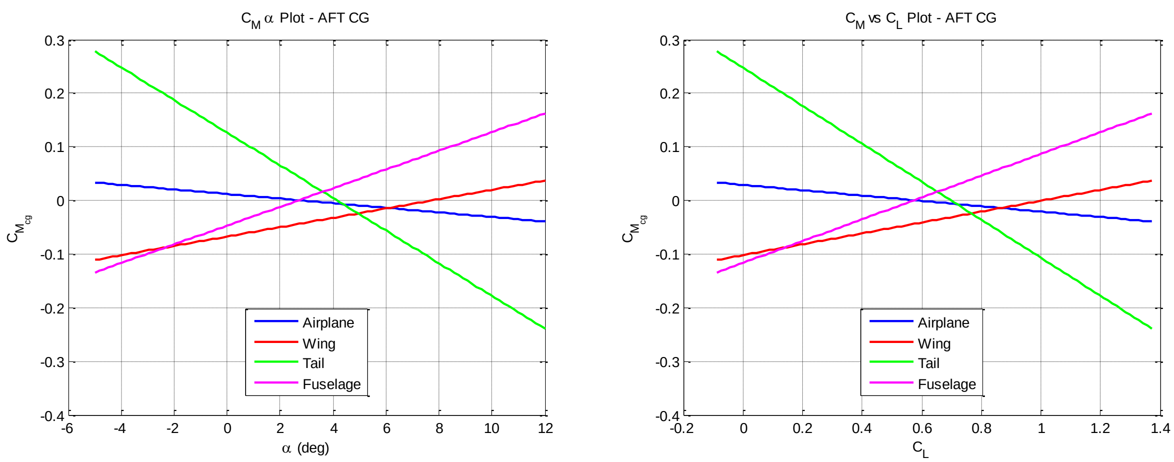

Figure 21. Longitudinal Static Stability for Aft CG Case – OTG-3

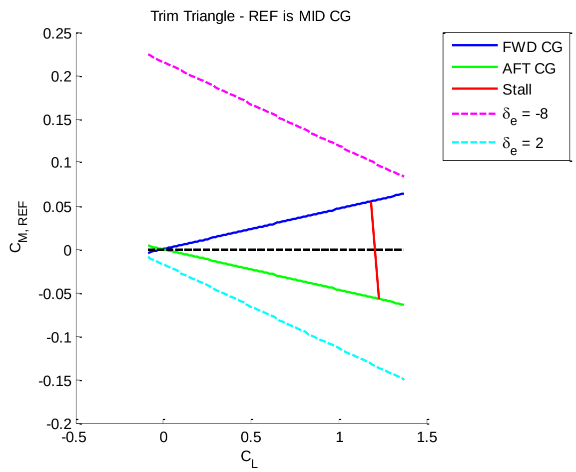

Figure 22. Cruise Trim Triangle – OTG-3

OTG-3 is statically stable at all CG locations and is easily trimmable at the cruise condition. However, it may be more important for the designer to check stability on takeoff, landing, and approach. In these conditions, many of the variables change as lift is increased due to flaps. Flaps also create a large increase in pitching moment which must be offset by the tail. By replacing the variables in the text file with the variables for the different conditions, the tail can be accurately sized for all the cases.

D. Stability Augmentation and Autopilot

To see how the stability of OTG-3 can be augmented and what gains are needed for an effective autopilot, the final code was run using the nominal stability values embedded in the code. The results are shown below.

Longitudinal Stability Augmentation Gains
-7.3843e-005   0.0044384   0.038649   0.0026165

Lateral Stability Augmentation Gains
7.1355   -0.15144

Altitude Hold Autopilot Gains
0.53908   -0.17891   -1.4659   -0.00175

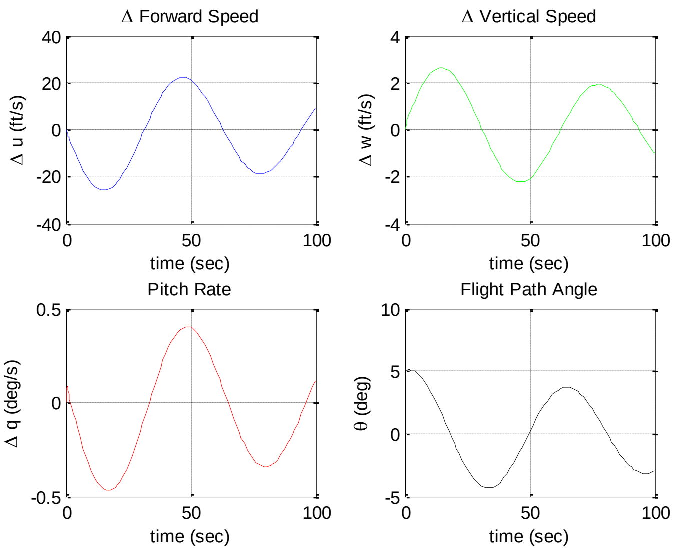

Figure 23. Longitudinal Stability Augmentation – OTG-3

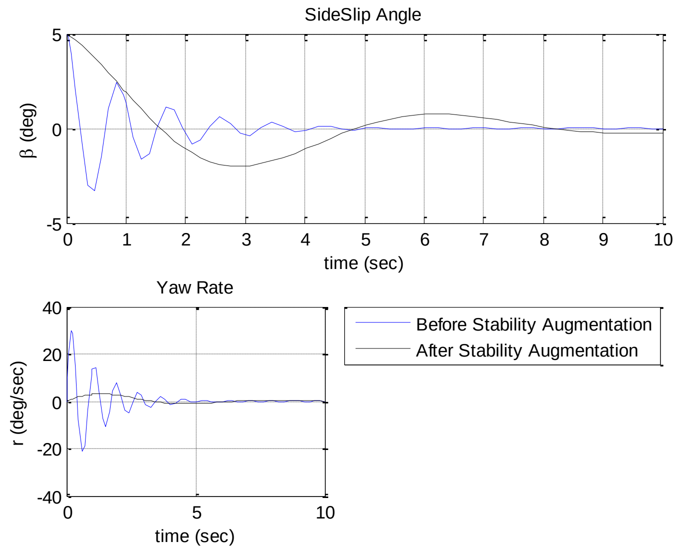

Figure 24. Lateral Stability Augmentation – OTG-3

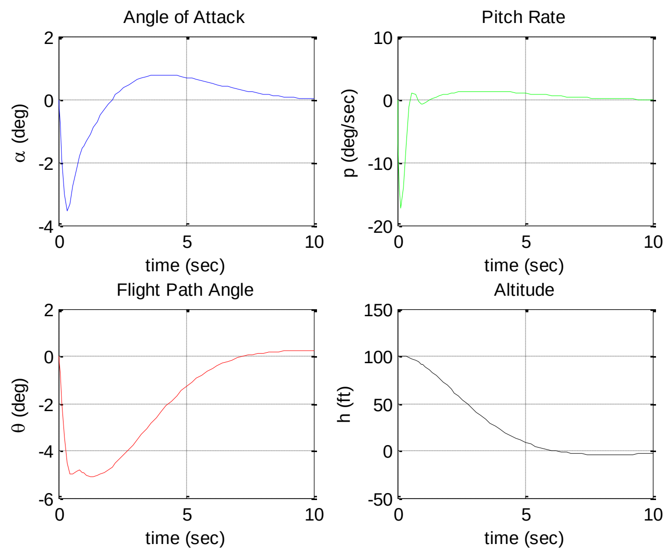

Figure 25. Altitude Hold Autopilot – OTG-3

OTG-3 has an extremely long phugoid oscillation (Fig. 23, Flight Path Angle). However, a long phugoid oscillation is desirable as compared to a short oscillation. The lateral stability augmentation does not seem to benefit OTG-3 since the augmented case takes longer to go to zero. However, there are fewer oscillations. Fewer oscillations is desirable because too many oscillations could lead to damaging the aircraft as they are not designed to oscillate quickly. Quick oscillations will also create extreme discomfort if this were a passenger aircraft. Long, shallow oscillations are desired over short and quick oscillations. The altitude hold autopilot also performed very well, requiring only 10 seconds to descend 100 feet and remain level after the descent.

VIII. Conclusion

It has been shown how important the empennage is to the stability of an aircraft. Proper design of the empennage is of upmost importance. However, the design process is iterative so the tail is always changing. By creating a simple computer program, constant changes in the design will not result in hours of work to create a new tail. Although there are many good empennage design programs, the designer should be weary when using an off-the-shelf program for their empennage design. It is always beneficial to know what the code is doing and as well as the background knowledge on how to stabilize an aircraft. More important than the code is the correct process and understanding what each design change will mean for the empennage. The methods presented above are just one way to effectively size the empennage as they just scratch the surface into aircraft stability and control.

Acknowledgments

The author would like to thank the Aerospace Engineering professors at California Polytechnic State University, San Luis Obispo for all the knowledge and support they have given in the four years of “learn by doing”.

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[2]Purser, P., and Campbell, J., “*Experimental Verification of a Simplified Vee-Tail Theory and Analysis for Available Data on Complete Models with Vee Tails*.” NAVA 823, 1945.

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[6]Drela, Mark and Youngren, Harold. Athena Vortex Lattice (AVL) [Windows Executive File] Cambridge, Massachusetts MIT, 2008.

Appendix

Empennage Sizing Blank.txt

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Empennage Sizing
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Only Change Numbers - Do not change any text

Basic Sizing - Fill out first 9 variables
Advanced Sizing - Fill out all variables

Aircraft Inputs

Value	Variable	Description	Unit
	S	Wing Area	ft sq
	c_bar	Wing Chord	ft
	AR	Aspect Ratio	-
	lt	Distance from CG to Tail AC	ft
	Xcg	CG Location from wing LE	ft
	Xac	Wing AC location from LE	ft
	CLalpha_w	Wing Lift Curve Slope	1/rad
	CMalpha_f	CM alpha fuselage	1/rad
	CNbeta_wf	CNbeta Wing and Fuselage	1/rad
	u	Velocity	kts
	h	Altitude	ft
	W	Aircraft Weight	lbs
	Ix	Mass Moment of Inertia	slug/ft sq
	Iy	Mass Moment of Inertia	slug/ft sq
	Iz	Mass Moment of Inertia	slug/ft sq
	CL	Aircraft Lift Coefficient	
	CDo	Reference Drag Coefficient	
	CLo	Reference Lift Coefficient	
	alpha_o	Zero Lift Angle of Attack	rad
	CMac_w	Coefficient of Moment about AC	
	iw	Wing Incidence Angle	deg
	e	Oswald Efficiency	
	zw	Distance- wing chord to fuselage centerline	ft
	d	Maximum fuselage depth	ft
	Zv	Distance- Cp of vert tail to fuselage centerline	ft
	lambda	Wing Taper Ratio	-
	XcgAFT	AFT CG Location from wing LE	ft
	XcgFWD	FWD CG Location from wing LE	ft

Empennage Sizing Navion.txt

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Empennage Sizing

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Only Change Numbers - Do not change any text

Basic Sizing - Fill out first 9 variables
Advanced Sizing - Fill out all variables

Aircraft Inputs

Value	Variable	Description	Unit
184	S	Wing Area	ft sq
5.7	c_bar	Wing Chord	ft
6.06	AR	Aspect Ratio	-
14	lt	Distance from CG to Tail AC	ft
1.682	Xcg	CG Location from wing LE	ft
1.425	Xac	Wing AC location from LE	ft
4.44	CLalpha_w	Wing Lift Curve Slope	1/rad
.12	CMalpha_f	CM alpha fuselage	1/rad
-.0516	CNbeta_wf	CNbeta Wing and Fuselage	1/rad
104	u	Velocity	kts
0	h	Altitude	ft
2750	W	Aircraft Weight	lbs
1048	Ix	Mass Moment of Inertia	slug/ft sq
3000	Iy	Mass Moment of Inertia	slug/ft sq
3530	Iz	Mass Moment of Inertia	slug/ft sq
.41	CL	Aircraft Lift Coefficient	
.05	CDo	Reference Drag Coefficient	
.41	CLo	Reference Lift Coefficient	
-.0873	alpha_o	Zero Lift Angle of Attack	rad
-.116	CMac_w	Coefficient of Moment about AC	
1	iw	Wing Incidence Angle	deg
.75	e	Oswald Efficiency	
1.9	zw	Distance- wing chord to fuselage centerline	ft
1	d	Maximum fuselage depth	ft
1	Zv	Distance- Cp of vert tail to fuselage centerline	ft
.54	lambda	Wing Taper Ratio	-
1.9	XcgAFT	AFT CG Location from wing LE	ft
1.1	XcgFWD	FWD CG Location from wing LE	ft

Longitudinal Stability Derivatives Navion.txt

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Longitudinal Stability Derivatives
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Value	Stability Derivative	Units
-0.0674	X u	1/s
+0.0353	X w	1/s
+12.2294	X alpha	ft/s^2
-0.3685	Z u	1/s
-2.0180	Z w	1/s
-354.2231	Z alpha	ft/s^2
-3.2029	Z alpha dot	ft/s
-6.8668	Z q	ft/s
-39.6112	Z delta e	ft/s^2
+0.0000	M u	1/ft s
-0.0985	M w	1/ft s
-0.0083	M w dot	1/ft
-17.2820	M alpha	1/s^2
-1.4589	M alpha dot	1/s
-3.1278	M q	1/s
-18.0424	M delta e	1/s^2

X - X Direction (Forward)
Z - Z Direction (Downward)
M - Pitching Moment
u - Forward Velocity
w - Vertical Velocity
q - Pitch Rate
alpha - Angle of Attack
e – elevator

Lateral Stability Derivatives Navion.txt

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Lateral Stability Derivatives

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Value	Stability Derivative	Units
+0.5395	Y p	ft/s
+2.3799	Y r	ft/s
-26.1087	Y beta	ft/s^2
+0.0000	Y delta a	ft/s^2
+8.2201	Y delta rud	ft/s^2
-0.2904	N p	1/s
-0.5516	N r	1/s
+6.9274	N beta	1/s^2
+1.5574	N delta a	1/s^2
-3.1820	N delta rud	1/s^2
-11.7679	L p	1/s
+2.3439	L r	1/s
+0.0000	L beta	1/s^2
-47.0084	L delta a	1/s^2
+1.3397	L delta rud	1/s^2

Y - Y Direction (Out the Right Wing, facing FWD)
N - Yawing Moment
L - Rolling Moment
p - Roll Rate
r - Yawing Rate
beta - Sideslip Angle
a - aileron
rud - rudder

Empennage Sizing OTG-3.txt

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Empennage Sizing

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Only Change Numbers - Do not change any text

Basic Sizing - Fill out first 9 variables
Advanced Sizing - Fill out all variables

Aircraft Inputs

Value	Variable	Description	Unit
240	S	Wing Area	ft sq
4.899	c_bar	Wing Chord	ft
10	AR	Aspect Ratio	-
15.25	lt	Distance from CG to Tail AC	ft
1.7228	Xcg	CG Location from wing LE	ft
1.2247	Xac	Wing AC location from LE	ft
4.9121	CLalpha_w	Wing Lift Curve Slope	1/rad
1	CMalpha_f	CM alpha fuselage	1/rad
-.0516	CNbeta_wf	CNbeta Wing and Fuselage	1/rad
185	u	Velocity	kts
8000	h	Altitude	ft
6100	W	Aircraft Weight	lbs
2096	Ix	Mass Moment of Inertia	slug/ft sq
6000	Iy	Mass Moment of Inertia	slug/ft sq
3530	Iz	Mass Moment of Inertia	slug/ft sq
.2731	CL	Aircraft Lift Coefficient	
.0212	CDo	Reference Drag Coefficient	
.3457	CLo	Reference Lift Coefficient	
-.0698	alpha_o	Zero Lift Angle of Attack	rad
-.1022	CMac_w	Coefficient of Moment about AC	
-.9	iw	Wing Incidence Angle	deg
.7468	e	Oswald Efficiency	
1.9	zw	Distance- wing chord to fuselage centerline	ft
.5	d	Maximum fuselage depth	ft
2	Zv	Distance- Cp of vert tail to fuselage centerline	ft
1	lambda	Wing Taper Ratio	-
1.7228	XcgAFT	AFT CG Location from wing LE	ft
1.266	XcgFWD	FWD CG Location from wing LE	ft

Longitudinal Stability Derivatives OTG-3.txt

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Longitudinal Stability Derivatives

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Value	Stability Derivative	Units
-0.0235	X u	1/s
+0.0582	X w	1/s
+23.1773	X alpha	ft/s^2
-0.2554	Z u	1/s
-1.8224	Z w	1/s
-569.0232	Z alpha	ft/s^2
-1.4369	Z alpha dot	ft/s
-4.5949	Z q	ft/s
-49.4680	Z delta e	ft/s^2
+0.0000	M u	1/ft s
-0.1283	M w	1/ft s
-0.0022	M w dot	1/ft
-40.0642	M alpha	1/s^2
-0.6919	M alpha dot	1/s
-2.2124	M q	1/s
-23.8186	M delta e	1/s^2

X - X Direction (Forward)
Z - Z Direction (Downward)
M - Pitching Moment
u - Forward Velocity
w - Vertical Velocity
q - Pitch Rate
alpha - Angle of Attack
e – elevator

Lateral Stability Derivatives OTG-3.txt

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Lateral Stability Derivatives

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Value	Stability Derivative	Units
+0.3789	Y p	ft/s
+4.1490	Y r	ft/s
-84.9496	Y beta	ft/s^2
+0.0000	Y delta a	ft/s^2
+17.7612	Y delta rud	ft/s^2
-0.8121	N p	1/s
-1.3502	N r	1/s
+53.8758	N beta	1/s^2
+11.0541	N delta a	1/s^2
-14.5359	N delta rud	1/s^2
-28.2081	L p	1/s
+3.4854	L r	1/s
+0.0000	L beta	1/s^2
-197.8629	L delta a	1/s^2
+3.2106	L delta rud	1/s^2

Y - Y Direction (Out the Right Wing, facing FWD)
N - Yawing Moment
L - Rolling Moment
p - Roll Rate
r - Yawing Rate
beta - Sideslip Angle
a - aileron
rud - rudder

Summary of Longitudinal and Lateral Stability Derivatives[4]

Summary of longitudinal derivatives

$X_u = \frac{-(C_{D_u} + 2C_{D0})QS}{mu_0}$	$X_w = \frac{-(C_{D_\alpha} - C_{L0})QS}{mu_0}$
$Z_u = \frac{-(C_{L_u} + 2C_{L0})QS}{mu_0}$	
$Z_w = \frac{-(C_{L_\alpha} + C_{D0})QS}{mu_0}$	$Z_{\dot{w}} = C_{Z_{\dot{\alpha}}} \frac{\bar{c}}{2u_0} QS/(u_0 m)$
$Z_\alpha = u_0 Z_w$	$Z_{\dot{\alpha}} = u_0 Z_{\dot{w}}$
$Z_q = C_{Z_q} \frac{\bar{c}}{2u_0} QS/m$	$Z_{\delta e} = C_{Z_{\delta e}} QS/m$
$M_u = C_{m_u} \frac{(QS\bar{c})}{u_0 I_y}$	
$M_w = C_{m_\alpha} \frac{(QS\bar{c})}{u_0 I_y}$	$M_{\dot{w}} = C_{m_{\dot{\alpha}}} \frac{\bar{c}}{2u_0} \frac{QS\bar{c}}{u_0 I_y}$
$M_\alpha = u_0 M_w$	$M_{\dot{\alpha}} = u_0 M_{\dot{w}}$
$M_q = C_{m_q} \frac{\bar{c}}{2u_0} (QS\bar{c})/I_y$	$M_{\delta e} = C_{m_{\delta e}} (QS\bar{c})/I_y$

Summary of lateral directional derivatives

$Y_\beta = \frac{QSC_{y\beta}}{m}$	(ft s^{-2})	$N_\beta = \frac{QSbC_{n\beta}}{I_z}$	(s^{-2})	$L_\beta = \frac{QSbC_{l\beta}}{I_x}$	(s^{-2})
$Y_p = \frac{QSbC_{y_p}}{2mu_0}$	(ft/s) or (m/s)	$N_p = \frac{QSb^2 C_{n_p}}{2I_z u_0}$	(s^{-1})		
$L_p = \frac{QSb^2 C_{l_p}}{2I_x u_0}$	(s^{-1})				
$Y_r = \frac{QSbC_{y_r}}{2mu_0}$	(ft/s) or (m/s)	$N_r = \frac{QSb^2 C_{n_r}}{2I_z u_0}$	(s^{-1})		
$L_r = \frac{QSb^2 C_{l_r}}{2I_x u_0}$	(s^{-1})				
$Y_{\delta a} = \frac{QSC_{y\delta a}}{m}$	(ft/s^2) or (m/s^2)	$Y_{\delta r} = \frac{QSC_{y\delta r}}{m}$	(ft/s^2) or (m/s^2)		
$N_{\delta a} = \frac{QSbC_{n\delta a}}{I_z}$	(s^{-2})	$N_{\delta r} = \frac{QSbC_{n\delta r}}{I_z}$	(s^{-2})		
$L_{\delta a} = \frac{QSbC_{l\delta a}}{I_x}$	(s^{-2})	$L_{\delta r} = \frac{QSbC_{l\delta r}}{I_x}$	(s^{-2})		