Empirical models of debt-rescheduling with sovereign immunity

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INTRODUCTION

The purpose of this chapter is to review the literature on empirical models of debt-rescheduling in international financial markets. The discussion focuses primarily on the statistical techniques that have been developed. These fall into two areas: discriminant-analysis and probabilistic-choice models. We also present other methods that might prove useful in future empirical research in this area. In particular, we discuss debt-rescheduling from the point of view of an explicitly dynamic economic analysis.

This chapter is divided into three sections. In the first section, we review the applied literature on international debt-rescheduling. We focus primarily on empirical studies and give only cursory discussion of the theoretical models of debt with sovereign risk. We also describe the common characteristics of the data that are used in these studies and give some of their sources. In the second section, we present summaries of the statistical techniques that have been used to determine the creditworthiness of the debtor countries. In the third section, we present a method of estimating debt-reschedulings as a dynamic program where the relevant control variable is a debtor country’s decision to reschedule or not. This framework employs a forward-looking technique that has not yet been implemented in this literature.

REVIEW OF THE LITERATURE

There are two excellent surveys of the literature on international debt-rescheduling: McDonald (1982) and Solberg (1988). McDonald’s survey discusses both theoretical and empirical issues. Because McDonald wrote at a relatively early date, his work suffers from the disadvantage of not including the recent theoretical literature applying game theory and
information economics to debt-rescheduling. In his discussion of the empirical work, McDonald subdivides the work into studies employing discriminant analysis and logit analysis. Solberg employs a similar taxonomy but provides a more analytic discussion and a wider survey of the relevant empirical literature.

Most of the empirical analyses of the determinants of debt-rescheduling have been descriptive rather than derived from theoretical frameworks. This has not occurred because of a lack of good theoretical models. For example, Eaton and Gersovitz (1981) develop a particularly elegant theoretical model considered to be the seminal piece in this area. Kletzer (1984), Bulow and Rogoff (1987), Fernandez and Rosenthal (1990), and Hart and Moore (1989) also make noteworthy contributions. The theoretical literature focuses upon the fact that the decision to reschedule debt occurs in a dynamic framework. It also emphasizes the fact that debt contracts in situations in which sovereign immunity is a concern have to be self-enforcing. Because there are strategic elements in a debtor country's decision to reschedule, it is not true that a simple model of the supply and demand of loanable funds is an accurate one. Indeed, much of the development of the theoretical literature in the last decade has consisted of incorporating increasingly sophisticated concepts from game theory and the economics of information into the applied analysis of debt contracts.

The first systematic published empirical study of debt rescheduling was undertaken by Frank and Cline (1971). They use discriminant analysis to differentiate between countries that had rescheduled debt and those that had not. The fundamental unit of analysis was a country-year. They examined data from twenty-six countries over a period of nine years, but, because of problems with incompleteness of data, they were able to use only 145 country-years in their sample. In these data, there were thirteen reschedulings. Frank and Cline included eight different macro-economic variables in their analysis; they found that three of these had significant explanatory power in being able to discriminate between cases of rescheduling and cases of normal repayment. These three factors were the lagged ratio of the stock of debt to trend exports, the inverse of the maturity of a country's loans, and the ratio of a country's imports to its international reserves.

A second important early empirical analysis is that of Feder and Just (1976). They were the first authors to use a logistic model of debt-rescheduling. Again, the fundamental unit of analysis was the country-year. Their sample included 238 country-years spanning 41 countries and eight years. They too encountered problems with incomplete data. In their sample there were 21 cases of rescheduling. They found six
macro-economic variables that were statistically significant in explaining a country’s likelihood of rescheduling debt. These were the ratio of imports to foreign-exchange reserves, the ratio of amortization to the stock of total debt, the ratio of debt-service payment to total exports, the rate of growth of exports, per capita income, and the ratio of capital inflows to debt-service payments. Feder and Just were the first authors to point out that there are some difficulties in defining exactly when an episode of debt-rescheduling has occurred.

Fisk and Rimlinger (1979) conducted an analysis using precedent-based non-parametric methods, similar to ‘nearest neighbor’ techniques. Annual data on 49 countries from 1960 to 1975 were collected on ten factors believed to influence the choice to reschedule. They were: the ratio of international reserves to imports; the debt-service ratio; the ratio of the IMF reserve position to imports; the ratio of exports to gross domestic product; the ratio of the stock of external debt to exports; the inflation rate; the ratio of imports to exports; the ratio of the stock of ‘supplier-disbursed debt’ to the stock of external debt; the ratio of interest payments to the stock of external debt, and the ratio of the stock of ‘supplier-disbursed debt’ to imports. The model was tested by selecting 90 sample observations at random and then determining how accurately a decision to reschedule could be predicted on the basis of the historical performance of other countries with similar characteristics for the ten variables. Using a probability of one-half as a cutoff, the best Fisk-Rimlinger model had an error rate of 8 per cent versus nineteen per cent for a naïve model in which no countries were predicted to reschedule.

Although Eaton and Gersovitz (1981) do not analyze the probability of debt-rescheduling directly, they do conduct an extensive examination of the underlying supply and demand equations for international debt. They argue that the rates of return for international debt must be at least as great as that of alternative investments; that is, loans to ‘risky’ less developed countries must be larger than the market rate of interest on safe investments. Eaton and Gersovitz use a switching regression to distinguish between regimes of supply-constrained debt and demand-determined debt for a sample of 45 countries during the years 1970 and 1974. Their total sample included 82 country-years. Eaton and Gersovitz interpret variables that increase the quantity of debt in the supply-constrained regime as those that lower the likelihood of debt rescheduling. They show that increases in the variability in export revenues and increases in the ratio of imports to gross national product tend to increase the quantity of loans available to a debtor country precisely because these variables increase the effectiveness of a penalty
for default. They also show that an increase in the stock of debt a country owes increases the probability that it is in a supply-constrained regime.

Most analysts have approached the problem of debt-rescheduling from the perspective of the debtor country, although a few studies have examined it from the perspective of the creditor. Since most creditors are commercial banks in developed countries, this type of analysis has focused on the evaluation of such firms; in particular, how their market value is related to their holdings of debt in less developed countries. Two examples of this approach are Bruner and Simms (1987) and Musumeci and Sinkey (1990). Musumeci and Sinkey analyze the effects of the announcement of Brazil’s ‘open-ended’ debt moratorium, reported in The Wall Street Journal on 23 February 1987. They examined how the values of the equity of a sample of bank holding companies in the United States were affected by the announcement. They found that it had a significantly negative effect on the stock prices of these holding companies, and moreover, the size of the effect was significantly related to the size of their Brazilian exposures.

Although direct tests on bank equity value may seem appealing because of the wide availability of data, they may be very inefficient since many other factors influence equity value. Secondary-market price data for country debt offers an alternative data source from creditor countries with great potential value for the study of rescheduling. If rescheduling represents the only significant credit risk associated with country loans, such price data should be able to give strong inferences about the probability of future reschedulings. If time-series data are available, price changes can be related to measures of the economic and political environment in the debtor countries. Such data may be particularly useful in understanding the short-run dynamics associated with rescheduling. This is an open area of research.

DISCUSSION OF THE DATA

Surprisingly, it is difficult to get a complete list of all the debt-reschedulings that have taken place over the past three decades. Indeed, most of the studies cited above used their own idiosyncratic sources for reschedulings. The primary difficulty stems from a lack of agreement as to what constitutes a rescheduling. Fixing the precise timing of a rescheduling is even more problematic. Often a country misses a scheduled payment and then begins a process of renegotiation. The final agreement on rescheduling is typically reached many months after the first payment is missed, and this process may cover two calendar years.
The best single current source for debt-reschedulings is the International Bank for Reconstruction and Development’s *World Debt Tables*. These are published annually, and the recent issues contain exhaustive lists of the debt-reschedulings that occurred in the last decade. These publications also contain convenient macro-economic data relevant to research in this area; they are available at an annual frequency. The most important data that are presented are the stocks of foreign debt owed by the less developed countries. Another good source for a list of reschedulings between 1976 and 1987 is Keller and Weerasinghe (1988). They discuss the recent experiences with rescheduling with a primary focus on the negotiations within the Paris Club of the creditor countries.

The International Monetary Fund’s *International Financial Statistics* is another standard source for macro-economic data in a unified format for the member countries of the Fund. These data are available at both quarterly and annual frequencies. These data are available on tape at many universities and other research institutions, and they are relatively easy to retrieve.

Most studies on debt-rescheduling have used country-years as the fundamental unit of analysis. Although most relevant variables are available on a quarterly basis, the crucial foreign-debt data will typically be reported with a lag which varies from country to country. It might be possible to obtain data with a better alignment from the creditor countries; however, such data have typically not been made public. Analysis with monthly or weekly data is even more problematic. Very few macro-economic statistics are available at a higher than quarterly frequency; this is especially true of data from the less developed countries. There are monthly series on industrial production, interest rates, exchange rates, prices, and the merchandise-trade balance for several less developed countries, but this is the exception rather than the rule. The fact that there is a lag between the shipment of exports from a foreign country and the month they are eventually reported makes the use of monthly trade statistics highly problematic.

**STATISTICAL TECHNIQUES**

A variety of statistical methods were employed to estimate models of debt-rescheduling in the studies cited above. Since most authors chose their dependent variable to be a discrete binary variable which took on the value one when a country ‘rescheduled’ within a given time-period and zero otherwise, the statistical methods used have been those designed for dichotomous dependent variables. These methods include
discriminant-analysis, linear-probability, probit, and logit models. In this section we briefly describe each of these methods and then discuss criteria to use when choosing among them. Perforce, our discussion will be brief. Those wanting more detail can refer to Altman et al. (1981), Maddala (1983), Amemiya (1985) or other similar sources.

**Linear-probability model**

Although the linear-probability model has generally not been used for the study of debt refinancing, it is one of the more popular methods of modeling dichotomous dependent variables. The model is defined as follows. Assume that observations are country-years, and consider the i-th observation. Then the dependent variable, $y_i$, is given by

$$ y_i = \begin{cases} 
1 & \text{if rescheduling occurs} \\
0 & \text{otherwise.}
\end{cases} $$

Furthermore, let the conditional probability that $y_i$ equal one be linear in $X_i$, a $K \times 1$ vector of independent variables. This implies that

$$ \text{Probability}(y_i = 1) = P_i = X_i' \beta, $$

where $\beta$ is a $K \times 1$ vector of coefficients. It can be shown that the assumption that the probability is linear implies that the expression

$$ y_i = X_i' \beta + \varepsilon_i, $$

where $\varepsilon_i$ is a random error term, meets all of the assumptions of the classical linear-regression model. Thus $y_i$ can be simply regressed against $X_i$ using a standard regression package, with the estimated coefficients being consistent and unbiased estimates of $\beta$ in the probability equation.

Although coefficients will be consistent and unbiased, several practical problems arise with the use of standard regression estimates of the linear-probability-model coefficients. First, because the dependent variable is dichotomous, the error terms, $\varepsilon_i$, will not satisfy the assumption of equal variance. This means that the standard errors and $t$-statistics reported from a standard regression program will be biased. The standard way of dealing with error terms with different variances is to use weighted least squares. It can be shown that if each observation is weighted by the term

$$ 1 / (X_i' \beta \ast (1 - X_i' \beta)), $$

then the standard errors reported from standard regression programs will be unbiased and the coefficient estimates will be asymptotically
It should be noted, though, that this adjustment requires that $\beta$ be known. In practice, estimates from an initial unweighted regression are used.

A second problem with regression estimates of the linear-probability model is the fact that probability estimates, $P_i$, can be less than zero or greater than one. Several methods have been proposed to deal with this, generally involving setting inadmissible probability estimates equal to bounds like .98 and .02.

**Logit model**

The logit model is very similar to the linear-probability model. Let $y_i$ be defined the same as for the linear probability model. If the conditional probability that $y_i$ equal one is

$$P_i = \frac{1}{1 + \exp(-X_i'\beta)},$$

then the model meets the assumptions of the logistic model.

Because of its functional form, the logistic model's predictions are constrained between zero and one. Moreover, the model shows diminishing returns. This means that the partial of the probability with respect to each variable in $X_i$ is proportional to $P_i(1 - P_i)$, whereas the partial is constant for the linear-probability model. Thus, changes in the independent variables will have less and less impact on the probability that $y_i$ is one as the probability moves away from one-half. In other words, the function's ability to discriminate is most sensitive near its midpoint.

Although the logistic model employs what many analysts believe are more realistic assumptions than the linear-probability model, one major cost is that the model cannot be estimated using a standard regression package. Coefficient estimates for models such as those of individual-country debt reschedulings must be computed using iterative techniques, generally maximum-likelihood methods. Although many good software programs are available to do this, they can be expensive to operate and may require some knowledge of non-linear estimation to use.

**Probit model**

The probit model is virtually identical to the logit model; indeed, the logistic model was developed historically as an approximation to the probit model. Again, defining $y_i$ as in the linear-probability model, if the conditional probability that $y_i$ equal one is
$P_i = F(X'_i; \beta),$

where $F(\cdot)$ is the cumulative standard normal distribution function, then the model meets the assumptions of the probit model.

Like the logit model, the probit model shows diminishing returns with partials proportional to $f(X'_i; \beta)$, where $f(\cdot)$ is the standard normal density function. Similarly, coefficients must be computed using non-linear iterative methods. However, the probit model is scaled somewhat differently than the logit. Typically, the logistic-model coefficients will be 1.8 times as large as those of the probit model. However, t-statistics of the coefficients and probability predictions for specific observations are likely to be very similar.

**Discriminant analysis**

The linear-probability, logit, and probit models evolved from the traditional regression model. The most popular method used for modeling debt-rescheduling, discriminant analysis, evolved from a different tradition, that of analysis of variance. Instead of a dependent variable, $y_i$, caused by $X_i$'s, two groups of country-years are assumed: years in which a country reschedules its debt; and years in which a country does not reschedule. Each country-year observation, $i$, is assumed to be characterized by measurements on a set of independent variables, $X_i$. The crucial additional assumption is that, within each group, the $X$ variables are distributed according to a *Multivariate Normal Distribution*:

$X_i \sim N(\mu_1, \Sigma_1)$ if observation $i$ is in the rescheduled group,

$X'_i \sim N(\mu_2, \Sigma_2)$ if observation $i'$ is in the non-rescheduled group,

where $\mu_1$ and $\mu_2$ are $K \times 1$ group mean vectors, and $\Sigma_1$ and $\Sigma_2$ are $K \times K$ group covariance matrices.

Unlike the early probability models, the causal flow is assumed to be from group membership to the $X_i$'s. Thus membership is determined first, and this determines the values of the $X_i$'s. The concept of prediction is also different from those of the techniques presented earlier. We do not try to predict rescheduling on the basis of the values of the $X_i$'s, but rather we try to infer to which group a country-year observation belongs on the basis of its $X_i$ values. This is akin to forming a posterior probability in classical Bayesian analysis.

Another difference between discriminant analysis and the techniques presented earlier is that there are no real parameters to estimate in discriminant analysis. Instead, analysis generally consists of two
procedures: first, testing whether the two groups have the same mean vectors, i.e. $\mu_1 = \mu_2$; and second, constructing an expression for the posterior probability for a random country-year observation. Each of these procedures requires knowledge of the mean vector and covariance matrix for each group. Generally these are estimated using the sample means and covariances.

Tests for the difference in the means depend upon whether or not the two group covariances are assumed equal. If the covariance matrices are equal, then, under the null hypothesis of group mean vector equality, the expression

$$\left( X_1 - X_2 \right)' S^{-1} \left( X_1 - X_2 \right) \times \frac{N_1 + N_2 - K - 1}{N_1 + N_2 - 2} \times \frac{1}{K} \times \frac{N_1 * N_2}{N_1 + N_2}$$

is distributed as an F statistic with $K$ and $N_1 + N_2 - K - 1$ degrees of freedom. Here $N_1$ and $N_2$ are the number of observations in groups one and two respectively, $X_1$ and $X_2$ are the respective group-sample mean vectors, and $S$ is the sample within-group covariance matrix. There are similar, but more complicated tests when group covariances are not assumed to be equal (see Altman et al. 1981).

The posterior probabilities for a random country-year $i$ are derived from the likelihood expressions for each group. Define

$$f_{i_1} = \frac{1}{(2\pi |\Sigma_1|)^{1/2}} \exp\left[ -\frac{(X_i - \mu_1)'\Sigma_1^{-1}(X_i - \mu_1)}{2} \right]$$

and

$$f_{i_2} = \frac{1}{(2\pi |\Sigma_2|)^{1/2}} \exp\left[ -\frac{(X_i - \mu_2)'\Sigma_2^{-1}(X_i - \mu_2)}{2} \right].$$

If $Q_1$ and $Q_2$ are the relative sizes of the rescheduled group and non-rescheduled group respectively, then the posterior probability that a random country-year with values $X_i$ was drawn from the rescheduled group is

$$\text{Probability (} X_i \text{is in group one)} = \frac{Q_1 * f_{i_1}}{Q_1 * f_{i_1} + Q_2 * f_{i_2}}.$$ 

The probability that $X_i$ is in group two is defined similarly. This is often referred to as ‘quadratic classification’ since it does not assume that the two groups have the same covariance matrix. If we assume that the two groups have the same covariance matrix, then the probability that a random country-year comes from the rescheduled group reduces to

$$\text{Probability (} X_i \text{is in group one)} = 1 / \left[ 1 + \frac{Q_2}{Q_1} \exp\left( -X_i'\beta + \alpha \right) \right],$$

where $\beta = \Sigma^{-1}(\mu_1 - \mu_2)$, $\alpha = (\mu_1 + \mu_2)'\beta / 2$, and $\Sigma$ is the population within-group covariance matrix. The vector $\beta$ is often referred to as the
'linear discriminant function', and classification using this formula is referred to as 'linear classification'. In practice, the function is formed using sample group mean vectors $X_1$ and $X_2$ and the sample within-group covariance matrix $S$.

**MODEL SELECTION**

The similarity of the regression, logit, probit, and discriminant-analysis models we have presented in this section raises the question as to how the choice of model should be made. Although some authors have argued otherwise, there is nothing that should categorically exclude any of the models from consideration. A case could be made for each of the models we have presented on the grounds of computational ease, theoretical structure, or functional flexibility. Indeed, there are conditions where data can be consistently described by more than one model.

Although the choice of model will often not greatly affect the implications of a study, there are a number of considerations that can be used in making this choice. These range from the researcher's beliefs as to the theoretical causal structure of the process being modeled to the 'fit' of each potential model with actual data. Moreover, there are several different ways to measure fit. Model fit can be judged by how well the model correctly classifies historical country-year observations. The criterion of fit is measured by how often the predicted 'most likely' group or choice actually occurs. Alternatively, model fit can be measured by how accurately predicted probabilities reflect observed group frequencies.

If the first method is used, the misclassification rates of models can be compared and used to select the best model. Thus, for example, if a discriminant-analysis model predicted better than a logit model, then the former model would be chosen. Although this is an attractive mechanism for model selection, several words of caution should accompany its use.

First, there is a question of which sample to use. If the original sample used to estimate parameters is used, misclassification rates will be biased in small samples. Alternatively, another or 'holdout' sample could be used. This yields unbiased estimates of misclassification rates; however, it has the disadvantage of requiring large samples and not using all the data to estimate the model. Note that misclassification estimates constructed from either original or holdout samples may be poor indicators of how the model would work prospectively, particularly if structural changes occur.

A second concern with using misclassification rates as a measure of goodness of fit is that it weighs both misclassifications equally. Clearly,
saying that a country will reschedule, when it does not, may not be as serious an error as saying it will not, when it does. Finally, perhaps the most serious flaw with using misclassification rates to choose among models is that it is sensitive only to observations with probabilities near the one-half threshold. Since rescheduling is a rare event, the evidence of a good model will not be that it predicts rescheduling with probabilities of one-half or more; rather, a good model predicts rescheduling with higher probabilities for countries in the years that they do reschedule than in years they do not.

The inadequacies of the misclassification-rate criterion have led to alternative measures of model fit that take into account the predicted group or choice probabilities, not just 'most likely' predictions. One suggested approach is to compare the average predicted probabilities for each group. For example, the mean predicted failure probability for known reschedulings could be compared to the mean probability for non-rescheduled observations. The wider the difference, the better the model. Another similar approach is to rank observations by predicted probabilities and compare the actual rescheduling rates of, say, the lowest decile to the next lowest, and so on. Both of these approaches are primarily descriptive. Other, more objective criteria have been proposed that are variations of the regression multiple-correlation coefficient $R^2$ (see McFadden 1976).

An attractive feature of these $R^2$ measures of goodness of fit is that they can be used to compare the performance of different model forms on the same data. If, for example, the logit model appeared to have a significantly better fit than the discriminant-analysis model, it would offer a persuasive argument to adopt the logit-model form. However, these statistics should not be used blindly. It is quite possible for 'wrong' models to perform better in particular small samples, even though in an infinite-sized sample they would not. The predictions of a particular model are quite sensitive to the distribution of the independent variables. Thus the policy analyst should be wary of changing models simply in order to fit better a new sample of data.

Thus far we have focused on measures of how different models fit actual data. Often, however, researchers may have to make model decisions before examining data. It may also be desirable to have the model decision guided by theoretical rather than empirical considerations and to choose the model form most consistent with the structure of the problem being modeled. We now focus on the problem of model selection on theoretical grounds. We will consider arguments for and against the linear probability, probit, logit, and discriminant-analysis models.
To begin with, there is nothing inherently wrong with any of the models we have presented. Each rests on sound statistical grounds and under appropriate assumptions can be properly used to model any categorical dependent variable problem. Statements such as 'it is improper to use a linear-regression model with a dichotomous dependent variable' or 'discriminant analysis cannot be used if groups are ranked' are dangerous and inaccurate generalizations. However, each of the models rests on different distributional and, to some extent, structural assumptions. Thus, for a given problem it may be that the assumptions required for one model are more appropriate than those of other models and therefore argue for the model's use. The researcher's goal is to match the assumptions to the problem. Running the risk of violating our own caveat about generalizations, we can use several general guides in matching problems and models.

If the researcher's problem involves measuring the association between rescheduling and a group of independent variables, where the only goal is to estimate parameters of a forecasting function for reschedulings, then theoretical considerations should not preclude any model. The regression, probit, logit, and discriminant-analysis models merely represent different prediction functions. Model selection in these circumstances should be based primarily on empirical fit and statistical considerations. Robustness, computing costs and sampling concerns may also be important. For example, most analysts having access to a personal computer will be able to estimate linear-probability models because regression software is so readily available. Software designed to implement logistic models is not as widely available. We caution that selecting on the basis of fit limits the ability to draw causal inferences from the estimated coefficients and parameters. It would be a mistake to choose a model because it 'fits well' and then interpret its parameters as supportive of a particular hypothesis.

If the researcher, however, is interested in estimating and perhaps testing a causal model, it may appear that there are strong theoretical reasons for choosing one of the three probabilistic-choice models. It appears that a particularly good argument can be made for probit and logit models in this case, since a number of authors have shown that both models can be derived from utility-maximizing behavior. However, it can be shown that the linear-probability model can also be derived from utility-maximizing behavior with a slightly different assumption about the error terms. Moreover, McFadden (1976) shows that a case can be made for the discriminant-analysis model, even if the independent variables are assumed to cause rescheduling. He shows that, if appropriate distributional assumptions are made, then discriminant
analysis will provide consistent estimates of the parameters of an underlying causal process running from the independent variables to the rescheduling decision. He does voice concern, though that this justification of the discriminant analysis is not very robust with respect to assumptions.

THE DECISION TO RESCHEDULE AS A DYNAMIC PROGRAM

The decision to reschedule a country's debt occurs in time. It is also a decision that is taken under uncertainty. Both of these facts make it attractive to model the phenomenon of debt-rescheduling as a stochastic dynamic program. This is an avenue of research that has not yet been pursued very far in the empirical literature. In this section, we will build upon the seminal work of Rust (1987) in describing how one might estimate a dynamic program describing a country's decision to reschedule debt.

Consider a debtor country making the decision to reschedule its stream of debt service. It must decide whether to service its debt this year or to seek rescheduling. This entails deferring some payment now for the possibility of a stream of higher payments in the future. The decision depends upon the trade-off between the current benefits of maintaining a payments schedule versus the potentially uncertain future costs of repayment entailed by a rescheduling agreement.

Consider a given debtor country. Let $y_t$ be the real gross national product of this country in year $t$, and let $D_t$ be the real stock of outstanding sovereign debt in year $t$. Then we can impute the real debt-service burden at time $t$ as

$$d_t = r_t D_t,$$

where $d_t$ is the flow payment for debt service and $r_t$ is the real interest rate facing the country in year $t$. Even though inflation expectations are not observable, it is convenient here to assume that the real interest rate is observable; this point will become clearer below. Now we can define the state of the system at time $t$ as the $2 \times 1$ vector

$$x_t = (y_t, d_t)^\prime.$$

The policy-maker's decision is whether to reschedule foreign debt, conditional upon this year's realization of real gross national product and the real debt service. We shall assume that the policy-maker is concerned about the total consumption available to the economy. In particular, we write:
\[ c_t = y_t + b_t - d_t, \]

where \( c_t \) is national consumption and \( b_t \) is new borrowing at time \( t \). In year \( t \), the policy-maker must choose one of two options. We shall model this as a choice \( i_t \in \{0, 1\} \), where 0 is the decision to maintain a payment schedule and 1 is the decision to reschedule. The set \( \{0, 1\} \) is the set of controls available to the policy-maker at time \( t \). Notice that this set is independent of the state; this is a convenient simplification and it suits our problem well. If the policy-maker chooses to reschedule, national consumption is \( y_t - P \), where \( P \) is a penalty. Otherwise, consumption is as above. If the policy-maker has constant relative risk aversion, the reward function is

\[
 u(x_t, i_t, \tau_t) + \varepsilon_i(i_t) = \begin{cases} 
 f(y_t - P) + \varepsilon_i(1) & \text{if } i_t = 1 \\
 f(y_t + b_t - d_t) + \varepsilon_i(0) & \text{if } i_t = 0
\end{cases}
\]

where \( f(c_t) = c_t^{-\tau}/(1 - \tau) \), and \( \varepsilon_i(i_t) \) is the error term associated with choice \( i_t \).

The term \( \varepsilon_i(i_t) \) is known to the policy-maker, but it is not observable to the econometrician. A large realization of \( \varepsilon_i(1) \) might be interpreted as the policy-maker’s perception that the penalty from rescheduling is less burdensome than \( P \), and a small realization of \( \varepsilon_i(1) \) reflects the policy-maker’s belief that the penalty from rescheduling is actually more onerous than \( P \). We may state analogously that a large value of \( \varepsilon_i(0) \) is the perception that continued unencumbered access to international credit markets is quite valuable, whereas a small realization of \( \varepsilon_i(0) \) reflects the notion that the policy-maker places little value on free trade.

These error terms make the problem of debt-rescheduling a truly stochastic one. Without the errors, the solution to a control problem of this type with only two variables would be simple and consist only of finding the threshold level of gross national product above which the country would not seek rescheduling. Such a simple rule is belied by the data. It is traditional to assume that \( \{\varepsilon_i(0), \varepsilon_i(1)\} \) are independently and identically distributed and that they follow a multivariate extreme-value process. This implies that the choice of whether to reschedule in state \( x_t \) can be described by a logistic function; such a function is practical in the estimation of the model.

It is necessary to specify the transition function in order to complete the description of the dynamic program. This function describes how the state evolves from year to year. We can write

\[
 h(x_{t+1} | x_t, i_t, \tau_t) = \begin{cases} 
 (g(y_t, \tau_2), d_t) & \text{if } i_t = 1 \\
 (g(y_t, \tau_2), d_t + b_t \tau_t) & \text{if } i_t = 0
\end{cases}
\]
where the function \( g(Y_t, \tau_2) \) describes the distribution of next year's gross national product conditional upon this year's \( Y_t \). The parameter \( \tau_2 \) captures the natural rate of growth of the economy. Although \( y_{t+1} \) is a realization from the continuous distribution \( g(y_t, \tau_2) \), it is typical in practical problems to make the state space discrete. We are assuming here that the debtor country's debt service next year does not decrease if it seeks rescheduling this year, and we have allowed next year's debt service of a country in compliance with its agreements to increase by the debt service on new borrowings.

We are now in a position to describe the policy-maker's dynamic program fully. An optimal policy for rescheduling is one that maximizes

\[
V(x_t | \tau) = u(x_t, i_t, \tau) + \beta E\{V(x_{t+1}, i | \tau)\}
\]

where \( \tau = (\tau_1, \tau_2) \) is the vector of parameters to be estimated and the expectation of \( V(x_{t+1} | \tau) \) is taken with respect to the joint distribution of \( x_{t+1} \) and \( \varepsilon_{t+1} \). Knowing the current value of gross national product and the current realizations of \( \varepsilon_t \), the policy-maker forecasts the future path of national product and then decides whether to seek rescheduling in this period.

The assumption that the policy-maker's private information follows an extreme-value process allows us to write the probability of rescheduling (\( i = 1 \)) or not rescheduling (\( i = 0 \)) as

\[
\text{Prob}(i | x, \tau) = \frac{\exp\{u(x, i, \tau_1) + \beta E V (x, i, \tau)\}}{\exp\{u(x, 0, \tau_1) + \beta E V (x, 0, \tau)\} + \exp\{u(x, 1, \tau_1) + \beta E V (x, 1, \tau)\}}
\]

which is identical to Rust's (1987) formula (4.13). This states that the policy-maker's probability of choosing to reschedule can be represented as a non-linear function of his degree of risk aversion, given the expected costs of rescheduling.

In order to make the estimation of \( \tau \) feasible, it is necessary to assume that \( x_t \) and \( \varepsilon_t \) are conditionally independent. First, the econometrician assumes that the distribution of \( x_{t+1} \) depends only on \( x_t \), not on \( \varepsilon_t \); this states that the distribution of next year's gross national product is independent of the policy-maker's private information. Second, the econometrician assumes that any dependence between \( \varepsilon_{t+1} \) and \( \varepsilon_t \) is transmitted through the state variable \( x_t \); this implies that next year's gross national product is a sufficient statistic for next year's realization of the policy-maker's private information.

The estimation of \( \tau \) can be accomplished in two steps. The first consists of determining the probability distribution of \( x_t \) conditional upon \( x_{t-1} \). Although both gross national product and debt service are
continuous variables, it is necessary to use discrete approximations of them. For a given country, the econometrician chooses levels of gross national product that correspond to relevant stages in the growth process. Then the estimate of \( \tau \) is a Markovian transition probability corresponding to the likelihood of moving from one level of growth to another. \(^2\) This transition probability is conditional upon the observed level of debt service.

The second step consists of estimating the parameter \( \tau \), representing the policy-maker's degree of risk aversion. This involves estimating the choice probabilities described in the logistic formula given above. This step requires the use of a nested fixed-point algorithm. For a given value of \( \tau \), it is necessary to calculate the entire value function defined on a discrete state space. Then the nested fixed-point algorithm\(^3\) searches for the value of \( \tau \) that maximizes the product

\[
\prod_{t=1}^{T} \text{Prob}(i_t | x_t, \tau)
\]

where these probabilities are defined above. Rust (1985) has developed an efficient algorithm for implementing this step on a personal computer.

This technique can be implemented for a given country or for a set of different countries. It will estimate jointly a country's natural rate of growth and the degree of risk aversion of its policy-makers — information which would be of tremendous use to lending institutions in creditor countries. The primary advantage of using the technique of dynamic programming is that it captures the essence of the decision a country makes in deciding to reschedule and puts it in its proper intertemporal setting.

**CONCLUSIONS**

Several points of conclusion can be drawn from this chapter. First, despite the fact that a number of good theoretical models of country debt-rescheduling have appeared in the literature, virtually all the empirical studies have been primarily descriptive. These studies have focused on macro-economic variables related to a country's ability to sustain debt-service payments. Some of the most important of these variables are the openness of the debtor country's economy, the ratio of debt-service payments relative to export revenues, and measures of economic growth. These data typically appear at an annual frequency. Further, several authors have noted that the definition of an episode of rescheduling can be problematic. Hence, the unit of analysis has almost uniformly been a country-year.
Second, although several authors have advocated particular statistical techniques, there appears to be little justification for choosing one technique over another. Both discriminant-analysis and logistic models have been used in the literature, and, because of the dichotomous nature of the debt-rescheduling variable, probit and linear-probability models could also be used. Little guidance has emerged from the theoretical literature on debt-rescheduling on the error-distributional assumptions needed to select among these techniques. Thus a strong case could be made for selecting a model form on the basis of sample fit. However, because the probability of a rescheduling is low, the predictions from all of these models are likely to be very similar. Therefore, model selection could very well be made on the basis of technical concerns such as the availability of software.

Third, new econometric techniques based upon dynamic programming have a ready application to issues of debt-rescheduling. These techniques have been used before in studying patent renewals and bus-engine replacement, and they are beginning to be used in many other applied fields in economics. New software has been developed to implement solution algorithms for these models on the personal computer. Data on reschedulings and the economies of the debtor countries are rich enough so that it is practicable to these kinds of models. Indeed, the extension of applications of dynamic programming to forecasting debt-rescheduling seems quite promising.

NOTES

1 A notable exception is Solberg (1988).
2 See Feller (1950) for a discussion of Markov models.
3 See Rust (1987) for a description of this procedure.

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