SEISMIC CODE IMPROVEMENTS BASED ON RECORDED MOTIONS OF BUILDINGS DURING EARTHQUAKES

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ABSTRACT

Summarized in this paper are the results of two recent investigations utilizing recorded motions of buildings to develop improvements in two aspects of seismic code provisions for buildings: (1) fundamental vibration period formulas, and (2) accidental torsion.

INTRODUCTION

The recorded motions of buildings during earthquakes are the basic data against which methods of earthquake-resistant design and techniques for calculating earthquake response must be judged. Perhaps the most common research use of these data has been in refining and improving structural modelling and response analysis techniques to match the calculated responses with measured data. Our interest has been quite different in recent years. We have utilized measured responses to investigate issues in building design that are not amenable to traditional analytical approaches. In particular, we have investigated two aspects of seismic code provisions for buildings: (1) fundamental vibration period formulas, and (2) accidental torsion.

The code formulas for estimating building period must be related to the actual periods of buildings, not to calculated values. The actual periods of interest are those "measured" from recorded motions of buildings shaken strongly during earthquakes. We have developed a comprehensive database of "measured" periods of buildings and proposed new formulas suitable for code application to estimate building period.

The subject of accidental torsion is not amenable to investigations by traditional analytical approaches because standard dynamic analyses do not predict torsion in symmetric-plan buildings. Analysis of recorded motions of nominally-symmetric-plan buildings provides the most direct means of developing an understanding of the torsional response of such buildings and for evaluating building code provisions for accidental torsion. We have utilized recorded motions of seven nominally-symmetric-plan buildings to evaluate a recent procedure for considering accidental torsion in building design.

Comprehensive reports on these investigations have been published (Goel and Chopra, 1997a; De la Llera and Chopra, 1997). Summaries of relevant portions of these reports are presented in this paper.

PART I: FUNDAMENTAL VIBRATION PERIOD FORMULAS

Code Formulas

The fundamental vibration period of a building appears in the equations specified in building codes to calculate the design base shear and lateral forces. Because this building property can not be computed for a structure that is yet to be designed, building codes provide empirical formulas that depend on the building material (steel, RC, etc.), building type (frame, shear wall etc.), and overall dimensions. These formulas should be consistent with periods of buildings "measured" from their motions recorded during earthquakes.

The measured data that are most useful but hard to come by are from structures shaken strongly but not deformed into the inelastic range. Such data are slow to accumulate because relatively few structures are installed with permanent accelerographs and earthquakes causing strong motions of these instrumented buildings are infrequent. Thus, it is very important to investigate comprehensively the recorded motions when they do become available, as during the 1994 Northridge earthquake. Unfortunately, this obviously important goal is not always accomplished, as indicated by the fact that the vibration properties of only a few of the buildings whose motions were recorded during post-1971 earthquakes have been determined.

Period Database

Supported by an NSF project, we have "measured" the natural vibration periods of twenty-one buildings by system identification methods applied to building motions recorded during the 1994 Northridge earthquake (Goel and Chopra, 1997a). These data have been combined with similar data from the motions of buildings recorded during the 1971 San Fernando, 1984 Morgan Hill, 1986 Mt. Lewis and Palm Spring, 1987 Whittier, 1989 Loma Prieta, 1990 Upland, and 1991 Sierra Madre earthquakes reported by several investigators (an exhaustive list of references is available in Goel and Chopra, 1997a). The resulting database, which contains data for a total of 106 buildings, is described in Goel and Chopra (1997a). It includes thirty-seven data points for twenty-seven RC moment-resisting frame (MRF) buildings, fifty-three data points for forty-two steel MRF buildings, and twenty-seven data points for sixteen concrete shear wall (SW) buildings. The number of data points exceeds the number of buildings because the period of some buildings was determined from their motions recorded during more than one earthquake, or was reported by more than one investigator for the same earthquake.

Theoretical Formulas

With the use of Rayleigh's method, the following relationships for fundamental period of multistory moment-resisting frames with equal floor masses and story heights have been determined (Goel and Chopra, 1997a: Appendix E):

$$T = C_1\sqrt{H} \quad \text{or} \quad C_2 H \tag{1}$$

The exponent of H and the numerical values of C_1 and C_2 depends on the stiffness properties, including their heightwise variation.

Another formula for the fundamental period has been derived by Rayleigh's method under the following assumptions: (1) lateral forces are distributed linearly (triangular variation of forces) over the building height; (2) base shear is proportional to $1/T^{\gamma}$; (3) weight of the building is distributed uniformly over its height; and (4) deflected shape of the building, under application of the lateral forces, is linear over its height, which implies that the inter-story drift is the same for all stories. The result of this derivation (Goel and Chopra 1997a, Appendix D) is:

$$T = C_3 H^{1/(2-\gamma)} \tag{2}$$

If the base shear is proportional to $1/T^{2/3}$, as in U.S. codes of the recent past, $\gamma = 2/3$ and Eq. (2) gives

$$T = C_4 H^{3/4} \tag{3}$$

which is in the ATC3-06 report (1978) and appears in current U.S. codes.

The formulas presented in Eqs. (1) to (3) are of the form:

$$T = \alpha H^{\beta} \tag{4}$$

in which constants α and β depend on building properties, with β bounded between one-half and one. This form is adopted for moment-resisting frame buildings in the present investigation and constants α and β are determined by regression analysis of the measured period data.

To develop an appropriate theoretical formula for shear wall buildings we use Dunkerley's method. Based on this method the fundamental period of a cantilever, considering flexural and shear deformations, is:

$$T = \sqrt{T_F^2 + T_S^2} \tag{5}$$

in which T_F and T_S are the fundamental periods of pure-flexural and pure-shear cantilevers, respectively. For uniform cantilevers T_F and T_S are given by:

$$T_F = \frac{2\pi}{3.516}\sqrt{\frac{m}{EI}}\,H^2 \tag{6}$$

$$T_S = 4\sqrt{\frac{m}{\kappa G}}\,\frac{1}{\sqrt{A}}H \tag{7}$$

In Eqs. (6) and (7), m is the mass per unit height, E is the modulus of elasticity, G is the shear modulus, I is the section moment of inertia, A is the section area, and κ is the shape factor to account for nonuniform distribution of shear stresses (= 5/6 for rectangular sections). Combining Eqs. (5) to (7) and recognizing that $G = E \div 2(1+\mu)$, where the Poison's ratio $\mu = 0.2$ for concrete, leads to:

$$T = 4\sqrt{\frac{m}{\kappa G}}\,\frac{1}{\sqrt{A_e}}H \tag{8}$$

with

$$A_e = \frac{A}{\left[1 + 0.83\left(\frac{H}{D}\right)^2\right]} \tag{9}$$

where D is the plan dimension of the cantilever in the direction under consideration. Comparing Eqs. (8) and (9) with Eq. (7) reveals that the fundamental period of a cantilever considering flexural and shear deformations may be computed by replacing the area A in Eq. (7) with the equivalent shear area A_e given by Eq. (9).

We next considered a class of a class of symmetric-plan buildings—symmetric in the lateral direction considered—with lateral-force resisting system comprised of a number of uncoupled (i.e., without coupling beams) shear walls connected through rigid floor diaphragms. Assuming that the stiffness properties of each wall are uniform over its height, the equivalent shear area, A_e, is given by a generalized version of Eq. (9) (details are available in Goel and Chopra 1997a, Appendix G):

$$A_e = \sum_{i=1}^{NW}\left(\frac{H}{H_i}\right)^2 \frac{A_i}{\left[1 + 0.83\left(\frac{H_i}{D_i}\right)^2\right]} \tag{10}$$

where A_i, H_i, and D_i are the area, height, and dimension in the direction under consideration of the ith shear wall, and NW is the number of shear walls. With A_e so defined, Eq. (8) is valid for a system of shear walls of different height.

Equation (8) was then expressed in a form convenient for buildings:

$$T = \bar{C}\frac{1}{\sqrt{\bar{A}_e}}H \tag{11}$$

where $\bar{C} = 40\sqrt{\rho/\kappa G}$, ρ is the average mass density, defined as the total building mass ($= mH$) divided by the total building volume ($= A_B H$ --A_B is the building plan area), i.e., $\rho = m/A_B$; and $\bar{A}_e$ is the equivalent shear area expressed as a percentage of A_B, i.e.,

$$\bar{A}_e = 100\frac{A_e}{A_B} \tag{12}$$

Regression Analysis Method

Regression analysis of the measured period data leads to values of α_R and β for Eq. (4) and $\bar{C}_R$ for Eq. (11) to represent the best fit, in the least-squared sense, to the data. However, for code applications the formula should provide lower values of the period in order not to underestimate the base shear, and this was obtained by defining α_L and $\bar{C}_L$ as the mean (α_R and $\bar{C}_R$) minus one standard deviation value; α_L and $\bar{C}_L$ are the 15.9 percentile values, implying that 15.9 percent of the measured periods would fall below the curves corresponding to α_L and $\bar{C}_L$ (subsequently referred to as the best-fit -1σ curve). If desired, α_L and $\bar{C}_L$ corresponding to other non-exceedance probabilities may be selected. Additional details of the regression analysis method and the procedure to estimate α_L and $\bar{C}_L$ are available elsewhere (Goel and Chopra 1997, Appendix F). Building codes also specify an upper limit on the period calculated by rational analysis. This limit is established in this investigation by defining α_U and $\bar{C}_U$ as the mean (α_R and $\bar{C}_R$) plus one standard deviation value to give the best-fit $+1\sigma$ curves.

Recommended Formulas

MRF Buildings. The formula for estimating the fundamental period of MRF buildings was obtained by calibrating the theoretical formula of Eq. (4) by regression analysis of the measured period data for twenty-seven RC MRF buildings (thirty-seven data points) and for forty-two steel MRF buildings (fifty-three data points); these buildings are listed in Tables 1 and 2 of Goel and Chopra (1997b).

Figures 1 and 2 give an impression of the scatter in the measured period data relative to the best-fit or T_R curve. The measured periods of a building in two orthogonal lateral directions are shown by circles connected by a vertical line. As expected, the data fall above and below the curve, more or less evenly, and most of the data are above the best-fit -1σ or T_L curve.

The best-fit -1σ and best-fit $+1\sigma$ curves for steel buildings are presented in Fig. 3. The fundamental vibration period of steel MRF buildings should be estimated from

$$T_L = 0.028H^{0.80} \tag{13}$$

The T_U curve is suitable for limiting the period of a building calculated by any rational analysis. Thus, the period from rational analysis should not be allowed to exceed $1.6T_L$; the factor 1.6 is determined as the ratio $0.045 \div 0.028$, rounded off to one digit after the decimal point.

In the best-fit -1σ and best-fit $+1\sigma$ curves for RC buildings presented in Fig. 4, observe that the coefficient 0.016 in T_L is slightly larger than the 0.015 value in Fig. 2a to recognize that the period of an RC building lengthens at motions large enough to cause cracking of concrete; $\alpha_L = 0.016$ was obtained from regression analysis of period data for buildings that experienced peak ground acceleration $\ddot{u}_{go} > 0.15\text{g}$. Thus, the fundamental vibration period of RC MRF buildings should be estimated from

$$T_L = 0.016H^{0.80} \tag{14}$$

and the building period calculated by any rational analysis should not be allowed to exceed $1.4T_L$; the factor 1.4 is determined as the ratio $0.023 \div 0.016$.

RC Shear Wall Buildings

The formula for estimating the fundamental period of concrete SW buildings was obtained by calibrating the theoretical formula of Eq. (11) by regression analysis of the measured period data for nine concrete SW buildings (17 data points) listed in Table 2 of Goel and Chopra (1998). For each building $\bar{A}_e$ was calculated from Eqs. (10) and (12) using dimensions from structural plans (Goel and Chopra 1997a, Appendix H); for shear walls with dimensions varying over height, A_i and D_i were taken as the values at the base. Regression analysis gives $\bar{C}_R = 0.0023$ and $\bar{C}_L = 0.0018$. Using these values for $\bar{C}$ in Eq. (11) give T_R and T_L, the best-fit and best-fit -1σ values of the period, respectively.

These period values are plotted against $H \div \sqrt{\bar{A}_e}$ in Fig. 5, together with the measured periods shown in circles; the measured periods of a building in the two orthogonal directions are not joined by a vertical line because the ratio $H \div \sqrt{\bar{A}_e}$ is different if the shear wall areas are not the same in the two directions. As expected, the measured period data falls above and below (more or less evenly) the best-fit curve.

The best-fit -1σ and best-fit $+1\sigma$ curves are presented in Fig. 6 wherein $\bar{C}_L = 0.0019$ is slightly larger than the 0.0018 value in Fig. 5 for reasons mentioned earlier in the context of the RC MRF buildings. Thus, the fundamental vibration period of RC shear wall buildings should be estimated from

$$T_L = 0.0019 \frac{1}{\sqrt{\bar{A}_e}} H \tag{15}$$

and the building period calculated by any rational analysis should not be allowed to exceed $1.4T_L$; the factor 1.4 is determined as the ratio 0.0026÷0.0019 rounded off to one digit after the decimal point.

ACCIDENTAL TORSION

Code Estimates of Accidental Torsion

Building codes require consideration of accidental torsion in one of two ways: (1) apply the equivalent static lateral forces at eccentricity e_d from the center of stiffness (CS), which includes the accidental eccentricity $e_a = \pm\beta b$; and (2) perform dynamic analyses with the center of mass (CM) of each floor shifted a distance equal to the accidental eccentricity $e_a = \pm\beta b$ from its nominal position, where b is the plan dimension of the building perpendicular to the direction of ground motion. For each structural element the algebraic sign in e_a that leads to the larger design force is to be used. Implementation of these code provisions requires two three-dimensional (3-D) static or dynamic analyses of the building for each lateral direction. The two types--static and dynamic--of analyses predict significantly different increases in design forces resulting from accidental eccentricity; the code-static analyses are not consistent with the analytical results (De la Llera and Chopra, 1994d).

Analytical Estimates of Accidental Torsion

Determined in earlier investigations (De la Llera and Chopra, 1994b,e) is the increase in response due to the following sources of accidental torsion: (1) rotational motion of the building foundation, (2) uncertainty in the stiffness of structural elements in both principal directions of analysis, (3) uncertainty in the location of the CM, and (4) uncertainty in stiffness and mass distributions in stories of a building other than the story analyzed.

Shown in Fig. 7 are the mean and mean-plus-one standard deviation of $\hat{u}_{b/2}$, the normalized edge ($x = b/2$) displacement, considering all sources of accidental torsion plotted against $\Omega = \omega_\theta / \omega_y$, the ratio between the natural vibration frequencies of the uncoupled torsional and lateral motions of the building (De la Llera and Chopra, 1995a). The normalized response in Fig. 7 and subsequent figures refers to the ratio of responses computed on two bases: considering accidental torsion and neglecting accidental torsion. The mean value of the increase in response, $\hat{u}_{b/2} - 1$, is usually less than 3%. Furthermore, with the exception of systems with $T_y < 0.5$ sec. and $\Omega < 1$, this mean increase in response is insensitive to Ω. The mean-plus-one standard deviation value of the response increase reaches a peak value of 45% for systems with $\Omega \approx 0.85$ or $\Omega \approx 1.1$; it decreases steadily

for values of Ω larger and smaller than these two values; and it varies rapidly between its peaks at $\Omega \approx 0.85$ and 1.1 to a minimum at $\Omega \approx 1$.

Design Procedure

Compared in Fig. 7 is $\hat{u}_{b/2}$ predicted by code-dynamic analysis with $e_a = \pm 0.05b$ and the analytical result. The code increase in edge displacements is much larger than the mean value of the analytical estimate, but is about one-half of the mean-plus-one standard deviation value. The code value corresponds to an exceedance probability of about 30%.

The discrepancy in the design forces due to accidental torsion, as predicted by code-specified static and dynamic analysis procedures, can be overcome by defining a unique design envelope for the edge displacements:

$$\hat{u}_{b/2} = \begin{cases} A & 0 \le \Omega \le 1 \\ A - \dfrac{A-1}{\Omega_c - 1}(\Omega - 1) & 1 < \Omega \le \Omega_c \\ 1 & \Omega > \Omega_c \end{cases} \tag{16}$$

where $\Omega_c = 1.8$ and

$$A = 1 + 0.0475(b/r)^2 \tag{17}$$

where r = radius of gyration of the floor diaphragm about the center of mass.

Equation (17) is a good approximation to the maximum value of $\hat{u}_{b/2}$ over all Ω (Fig. 8a) determined by code-specified dynamic analysis (Fig. 8b). Furthermore, Eqs. (16) and (17) have been intentionally calibrated to produce values that are conservative, especially for the range $0.9 \le \Omega \le 1.1$. There are three reasons for this. First, the estimation of Ω is obviously subject to error; therefore, taking advantage of the sharp dip in the analytical response curve near $\Omega = 1$ is not appropriate for design. Second, this conservatism proves to be useful in preventing resisting planes in the interior of the building plan to be underdesigned by the procedure developed. Third, the "recorded" increase in response for a system with $\Omega \approx 1$ can be larger than predicted by code-specified dynamic analysis for accidental torsion (De la Llera and Chopra, 1995).

In order to overcome the limitations of the present code procedures, the design envelope of Eqs. (16) and (17) forms the basis for a design procedure to include accidental torsion in the seismic design of buildings. This procedure is "exact" for single-story systems and for multistory buildings belonging to the special class defined in Hejal and Chopra, (1989); it is also a good approximation

for other multistory systems. It has several important advantages over the current seismic code procedures. First, it avoids the two additional 3-D static or dynamic analyses of the building in each lateral direction. Second, it includes the effects of all sources of accidental torsion whereas codes include only those that can be represented by a constant accidental eccentricity. Third, it gives a unique value for the increase in a design force due to accidental torsion, whereas current codes give very different results depending on whether the analysis is static or dynamic. Fourth, the procedure defines explicitly the expected increase in design forces due to accidental torsion, in contrast to the use of accidental eccentricity in codes implying an indirect increase in member forces. Fifth, the increase in design forces specified by the new procedure has a well-established probability of exceedance.

"Measured" Accidental Torsion

In this section a procedure is described to determine the torsion in nominally symmetric buildings from their motions recorded during earthquakes; three channels of horizontal acceleration are necessary at each instrumented floor of a building. First, the motions of uninstrumented floors are inferred from the motions of instrumented floors by a cubic spline interpolation procedure (Figure 9) without modelling or structural analysis of the building. In this procedure interpolation is performed on floor displacements (relative to the base) instead of the common choice of floor accelerations. Cubic spline functions satisfy conditions of continuity and differentiability of second-order at the interpolation points (i.e., instrumented floors) and, hence, provide smooth shapes for the heightwise distribution of displacements.

Next, floor accelerations are computed from the inferred or "measured" displacements at uninstrumented floors. The total displacement-time function for such a floor is obtained by adding the ground displacements to the floor displacement relative to the base determined by the cubic spline interpolation procedure. Floor velocity-time and acceleration-time functions are computed by time differentiation of the displacement-time function. Since differentiation emphasizes the high frequency components, these velocity and acceleration traces are low-pass filtered. The filter parameters are chosen so that the resulting accelerations match closely the recorded acceleration at an instrumented floor. A ninth-order Butterworth low-pass filter was selected with the cut-off frequency chosen to ensure a close match. Furthermore, the predicted velocity- and acceleration-time functions are filtered in both forward and reverse directions to eliminate any phase distortion (MATLAB, 1994).

Presented in Fig. 10 is an example of the low-pass filtering procedure and the results obtained by the interpolation procedure applied to the motions of a seven-story R/C-frame building recorded during the 1994 Northridge earthquake. The predicted motions of the building at the locations of the instruments on the sixth floor are compared with the actual recorded motions; the latter are not used in the interpolation process and hence provide a benchmark for evaluating the accuracy of the procedure. The predicted displacement-time and acceleration-time functions are seen to be accurate.

At the end of this interpolation and filtering procedure, the displacement-, velocity-, and acceleration-time functions are known for all floors; these will be referred to as the "recorded" motions. The "recorded" motions at the center of mass in the x-, y-, and θ-directions are then calculated assuming a rigid floor diaphragm.

This procedure was implemented for the nominally-symmetric buildings listed in Table 1. The influence of accidental torsion on response of selected buildings is shown in Fig. 11 by superimposing the time variation of the deformation (displacement relative to the base) at the edge of the roof plan due to building translation only (dashed line) and due to building rotation and translation simultaneously (solid line). The difference between these two functions is due to accidental torsion induced in these nominally-symmetric buildings. The normalized edge displacement, defined as the peak (maximum absolute) value of the displacement including torsion divided by the corresponding value excluding torsion is computed for each building considered. This is the "measured" value of normalized edge displacement considering accidental torsion.

Table 1. Nominally Symmetric Buildings Considered

Buildings	CSMIP	PGA	Material	$(b/r)_x$	$(b/r)_y$	Ω_x	Ω_y
A: Richmond	58506	0.11g	Steel	3.12	1.49	1.36	1.52
B: Pomona	23511	0.13g	RC	2.22	2.67	1.42	1.34
C: San Jose	57562	0.20g	Steel	3.22	1.28	1.00	1.03
D: Sylmar	24514	0.67g	Steel	3.06	3.04	0.75	0.82
E: Burbank	24370	0.30g	Steel	2.45	2.45	1.72	1.72
F: Burbank	24385	0.29g	RC	1.11	3.28	1.14	1.10
G: Warehouse	24463	0.26g	RC	2.13	2.74	1.54	1.40

Evaluation of Design Procedure

The normalized edge displacement is plotted as a function of Ω in Fig. 12. The "measured" values are plotted for each building at its Ω value in Table 1. For buildings D, E, F and G the vertical bar gives the range of values for the different floors and the star denotes the mean value. For buildings A, B and C the open circles denote the value for the roof only from an earlier investigation (De la Llera and Chopra, 1994a). Superimposed on these data are the design curves (Eqs. 16 and 17) for $b/r = 1, 2$, and 3.

Figure 12 shows that the increase in response due to accidental torsion of buildings with Ω close to one varies from essentially zero for building F to about 40% for building C. The implied sensitivity of this increase to small changes in Ω is consistent with theoretical predictions (mean-plus-one standard deviation curve in Fig. 7). It is for this reason that the dip in the theoretical curve at $\Omega = 1$ has been ignored in the design envelopes (Fig. 8). Obviously, Fig. 12 must be interpreted carefully since for each value of Ω, the displacement increase due to accidental torsion is a random variable, and the data points are just a very few outcomes of this random variable.

Figure 12 also indicates that the measured accidental torsion is smaller for buildings with larger Ω, i.e., torsionally-stiff buildings. The design curves are consistent with this trend and neglect accidental torsion for buildings with $\Omega > 1.8$.

CONCLUDING COMMENTS

Earthquake-resistant design must be based upon recorded motions of buildings during earthquakes. In this paper we have utilized these records to develop improvements in two aspects of seismic code provisions for buildings—fundamental vibration period formulas and accidental torsion—that are not amenable to traditional analytical approaches. We hope that these proposals would receive the attention of code-writing committees.

ACKNOWLEDGEMENTS

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REFERENCES

Applied Technological Council (1978). *Tentative provisions for the development of seismic-regulations for buildings*, Report No. ATC3-06, Applied Technological Council, Palo Alto, California.

De la Llera, J.C. and A.K. Chopra (1994a). Evaluation of code accidental-torsion provisions from building records. *Journal of Structural Engineering*, ASCE, **120**, 597-616.

De la Llera, J.C. and A.K. Chopra (1994b). Accidental torsion in buildings due to stiffness uncertainty. *Earthquake Engineering and Structural Dynamics*, **23**, 117-136.

De la Llera, J.C. and A.K. Chopra (1994c). *Accidental and Natural Torsion in Earthquake Response and Design of Buildings*, Report No. UCB/EERC-94/07, Earthquake Engineering Research Center, University of California, Berkeley.

De la Llera, J.C. and A.K. Chopra (1994d). Using accidental eccentricity in code-specified static and dynamic analyses of buildings. *Earthquake Engineering and Structural Dynamics*, **23**, 947-967.

De la Llera, J.C. and A.K. Chopra (1994e). Accidental torsion in buildings due to base rotational excitation. *Earthquake Engineering and Structural Dynamics*, **23**, 1003-1021.

De la Llera, J.C. and A.K. Chopra (1995). Estimation of accidental torsion effects for seismic design of buildings. *Journal of Structural Engineering*, ASCE, **121**, 102-114.

De la Llera, J. C. and A. K. Chopra (1997). *Evaluation of seismic code provisions using strong-motion building records from the 1994 Northridge earthquake*, Report No. UCB/EERC-97/16, Earthquake Engineering Research Center, December, 1997, 308 pp.

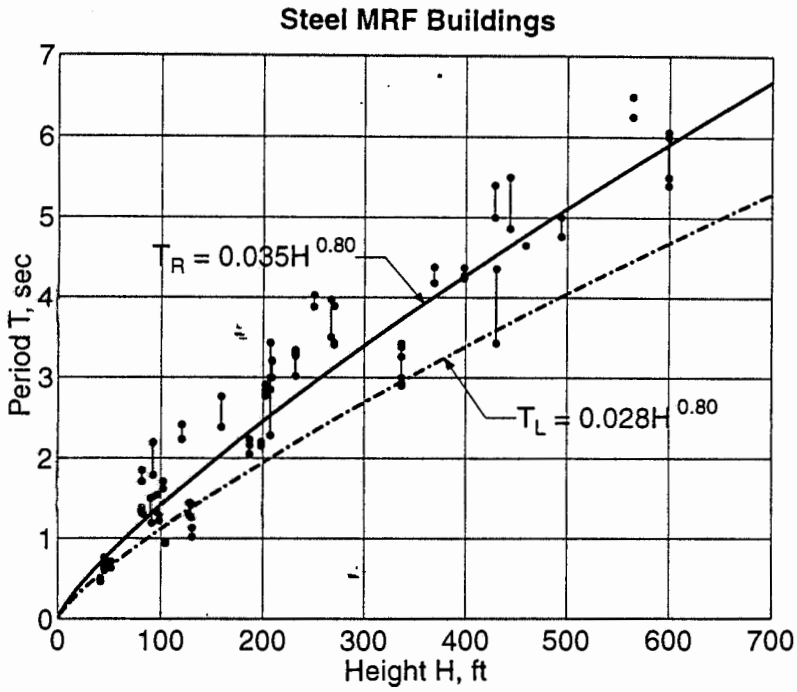
Goel, R. K. and A. K. Chopra (1997a). *Vibration Properties of Buildings Determined from Recorded Earthquake Motions*, Report No. UCB/EERC-97/14, Earthquake Engineering Research Center, December, 1997, 271 pp.

Goel, R. K. and A. K. Chopra (1997b). Period formulas for moment-resisting frame buildings. *Journal of Structural Engineering*, ASCE, **123**, 1454-1461.

Goel, R. K. and A. K. Chopra (1998). Period Formulas for Concrete Shear Wall Buildings. *Journal of Structural Engineering*, ASCE, **124**, 426-433.

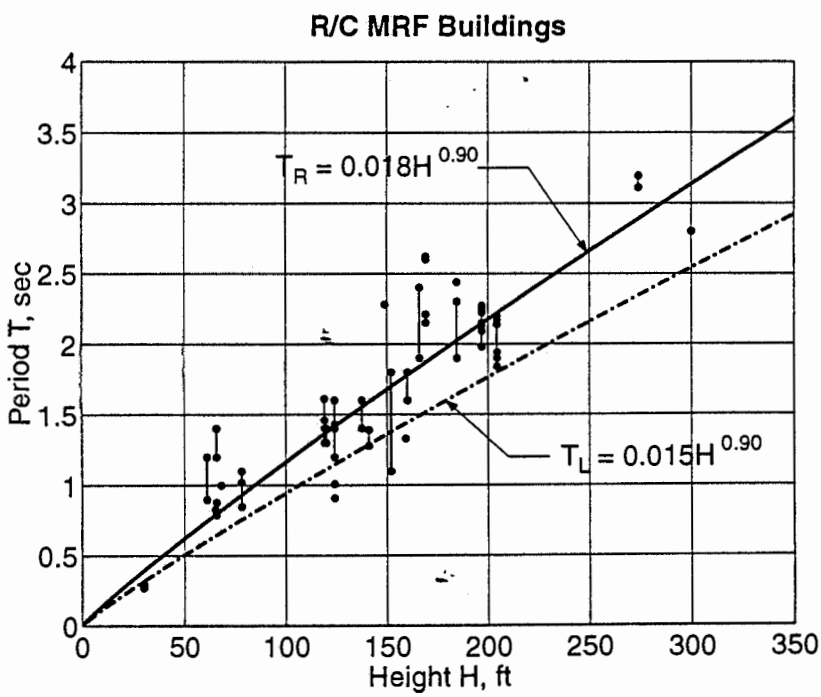
Hejal, R. and A. K. Chopra (1989). Response spectrum analysis of a class of torsionally coupled buildings, *Journal of Engineering Mechanics*, ASCE, **115**, 1761-1781.

MATLAB (1994). *Reference Guide*, the Mathworks, Inc., Natick, Massachusetts.



(a)

Figure 1: Regression analysis of measured period data for steel MRF buildings.



(a)

Figure 2: Regression analysis of measured period data for RC MRF buildings.

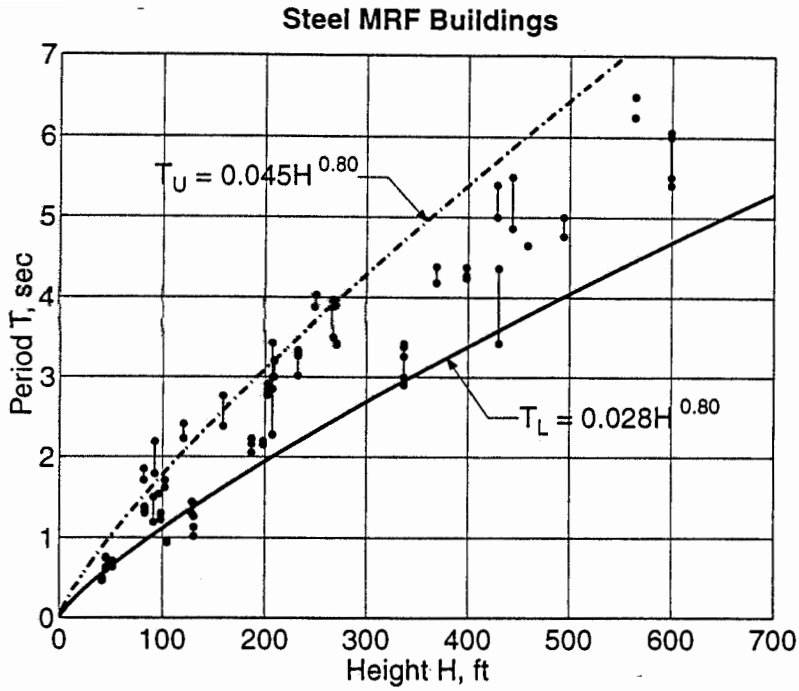


Figure 3: Recommended period formula and upper limit for fundamental period of steel MRF buildings.

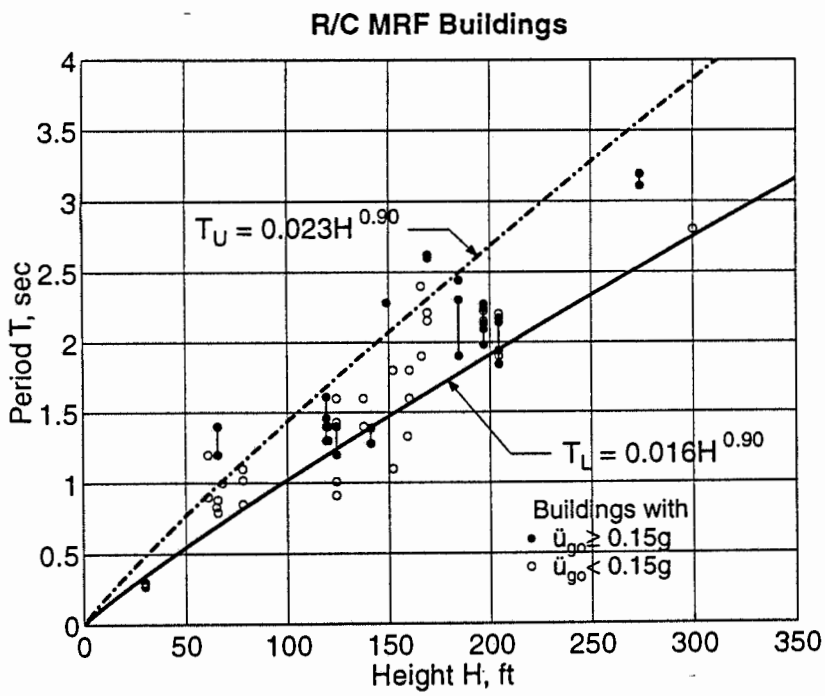


Figure 4: Recommended period formula and upper limit for fundamental period of RC MRF buildings.

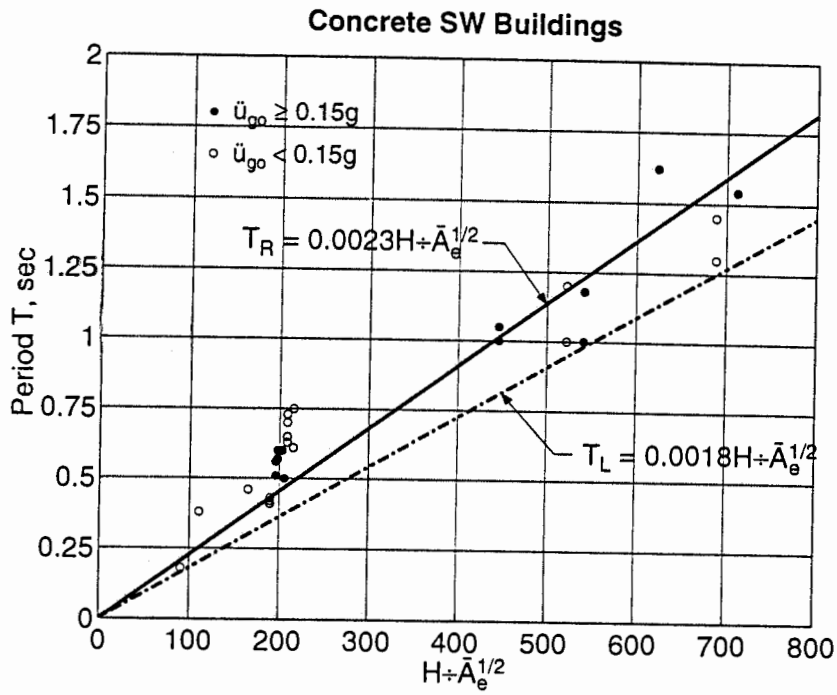


Figure 5: Regression analysis of measured period data for shear wall buildings.

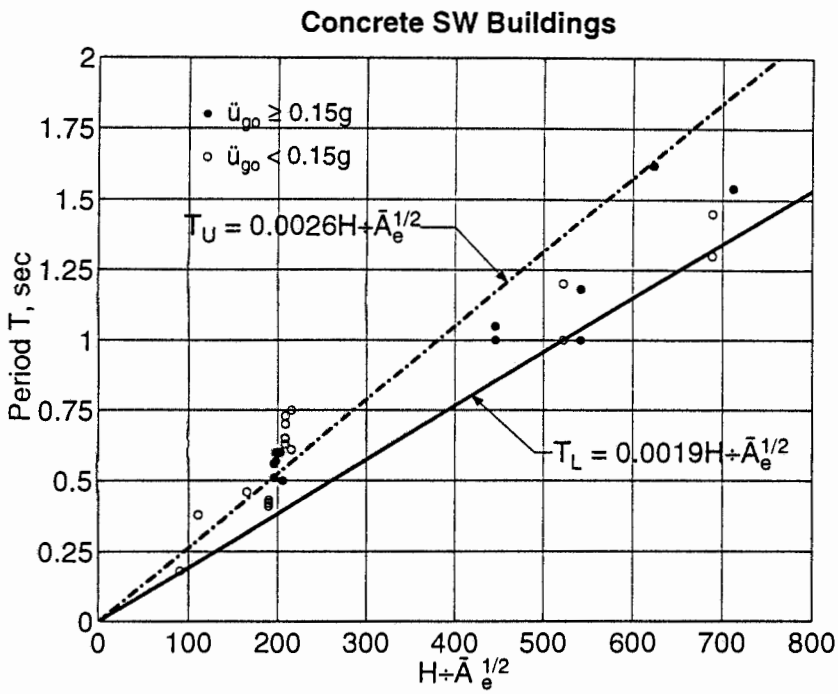


Figure 6: Recommended period formula and upper limit for fundamental period of shear wall buildings.

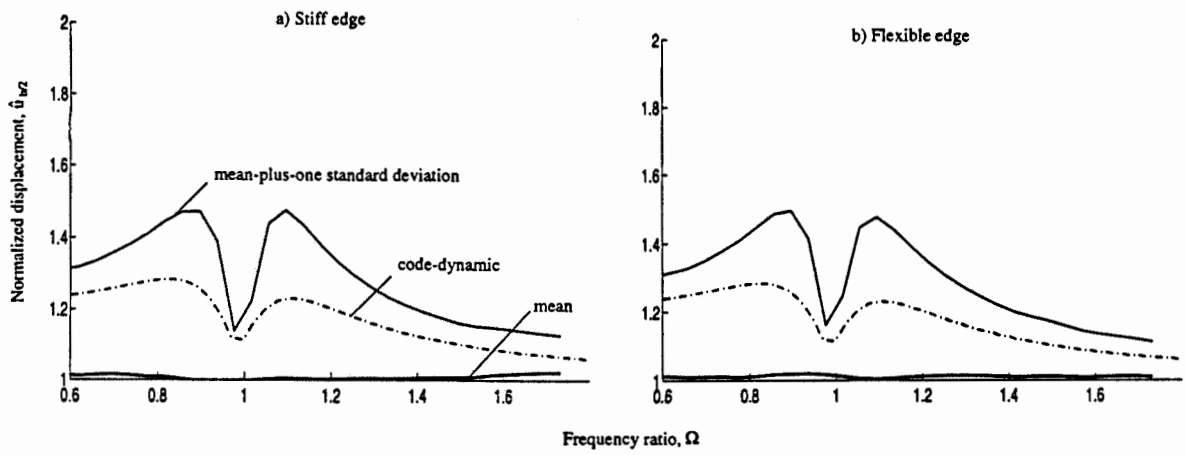


Figure 7: Comparison between $\hat{u}_{b/2}$ computed from code-dynamic analysis and from statistical analysis of different sources of accidental torsion.

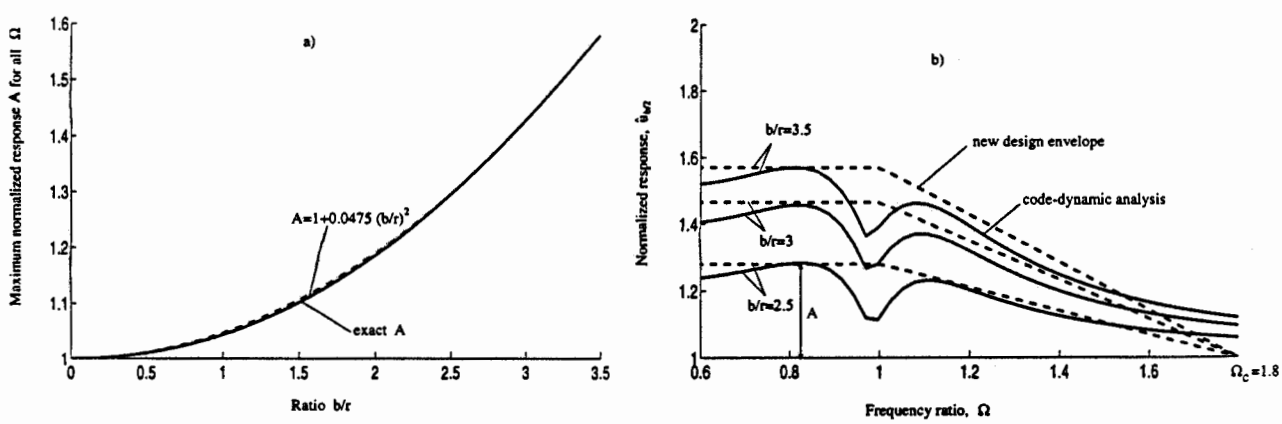


Figure 8: Design envelopes for normalized edge displacement $\hat{u}_{b/2}$: (a) variation of A as function of b/r; and (b) design envelopes for b/r = 2.5, 3, and 3.5.

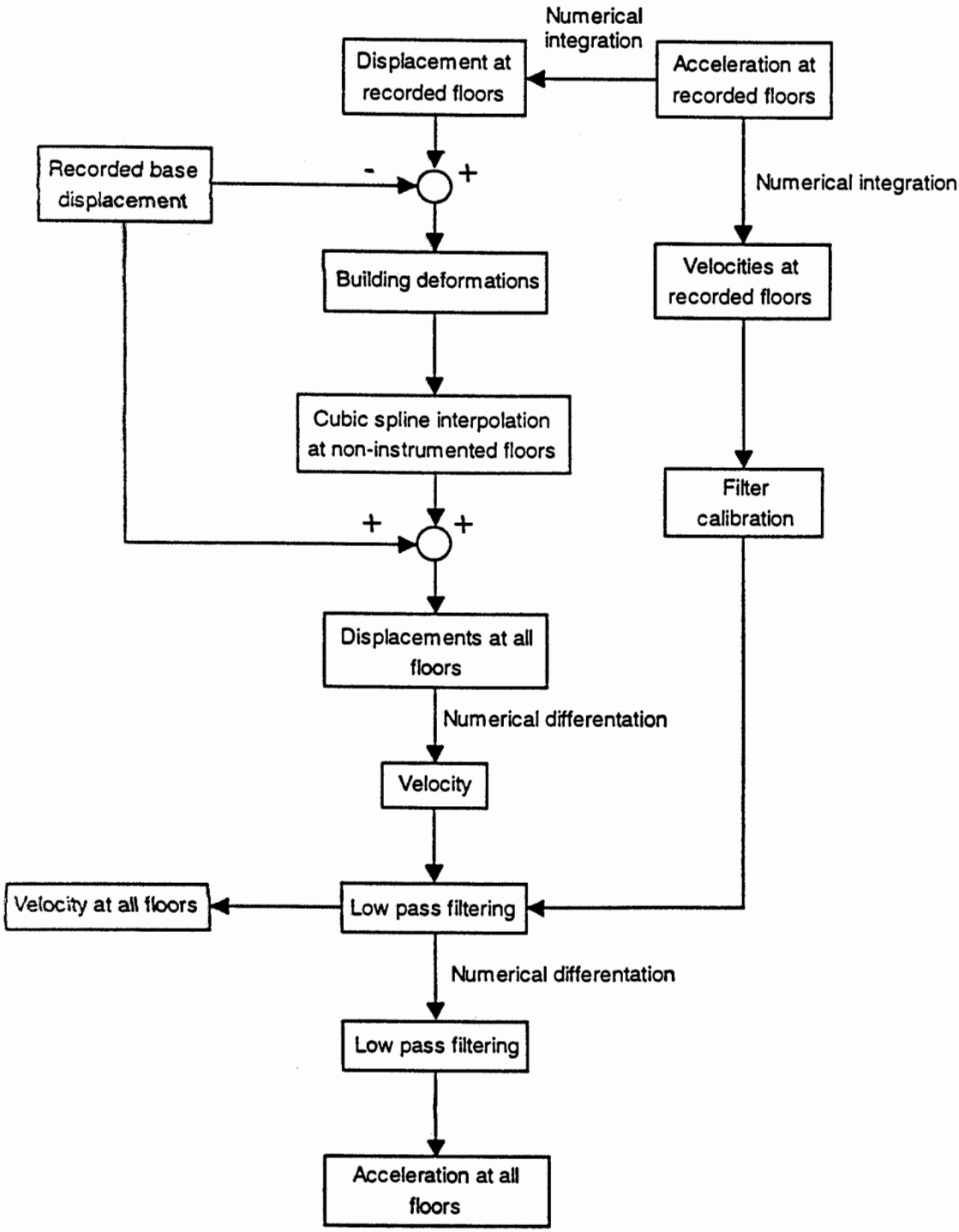


Figure 9: Heightwise interpolation of recorded floor motions of buildings.

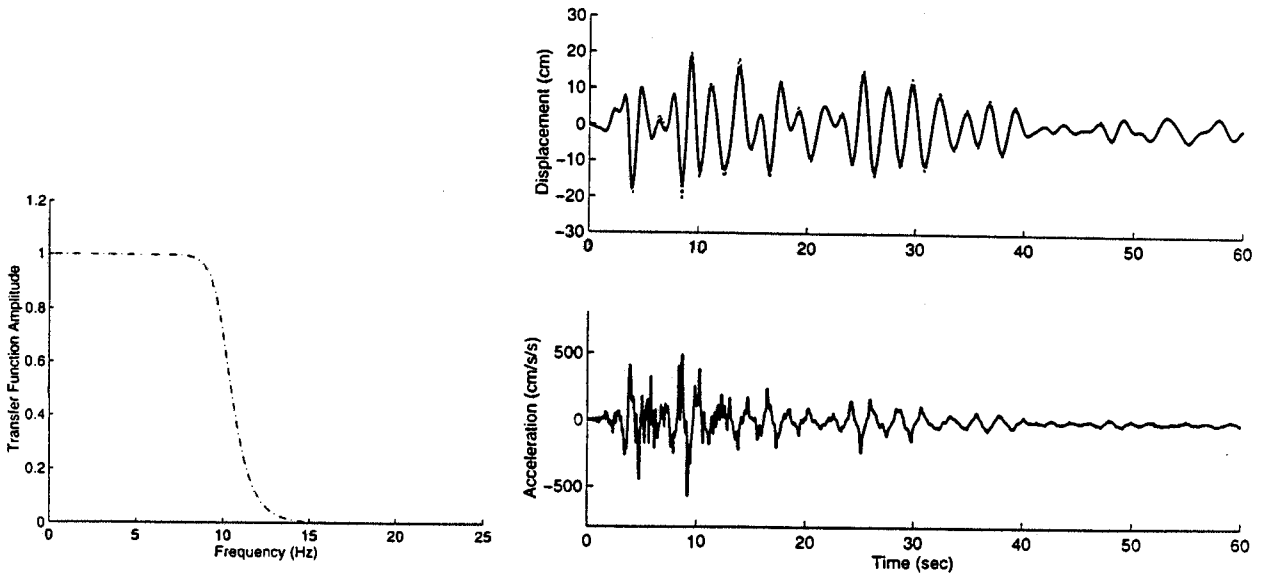


Figure 10: Example of heightwise interpolation of floor motions; recorded motions are shown by a solid curve and predicted motions by a dashed curve.

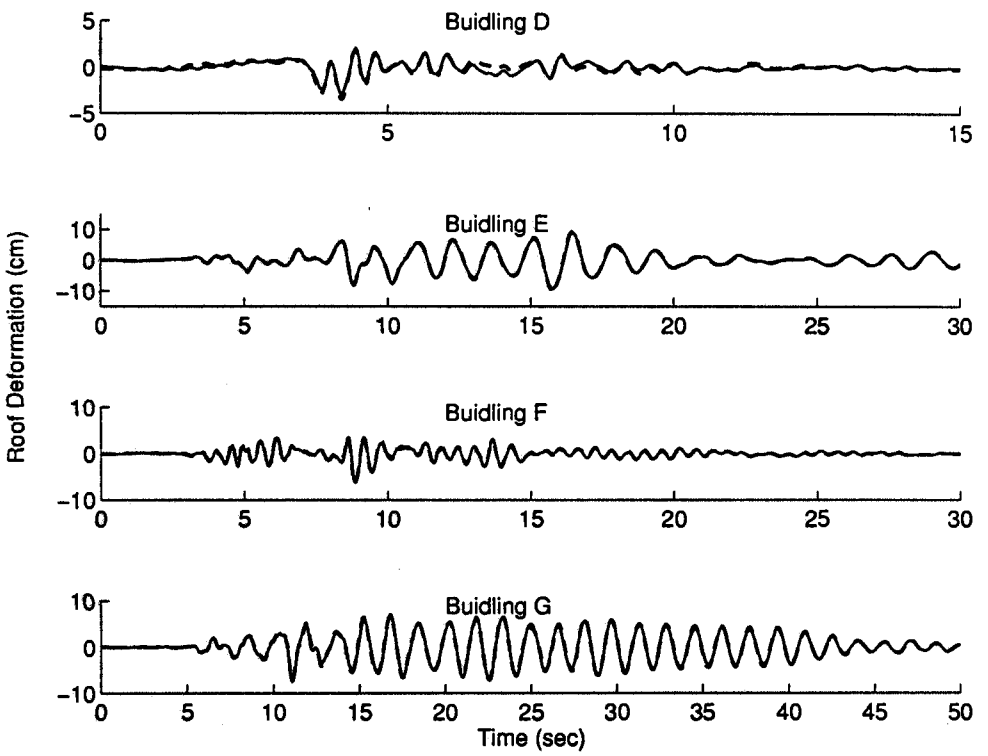


Figure 11: Accidental torsion effects in roof displacements of buildings D, E, F, and G.

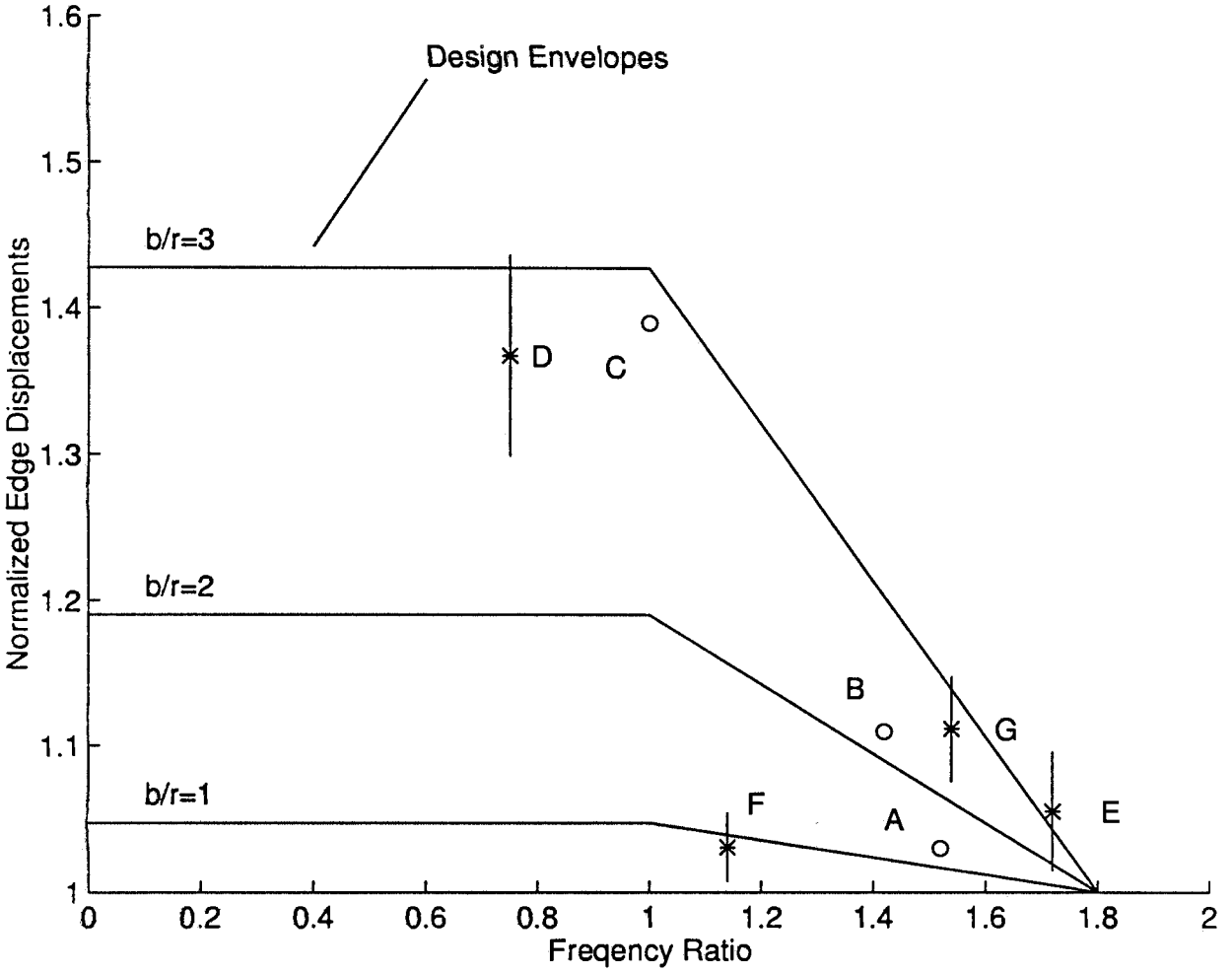


Figure 12: Comparison of design envelopes with "measured" increase in edge displacements of seven buildings due to accidental torsion.