Aircraft Longitudinal Control Experiment

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This paper describes the design, fabrication, and analysis of an experiment that demonstrates the stability and control system characteristics of an aircraft constrained at the center of gravity in an air flow field. Given a set of basic requirements, the physical system (including the airframe, wings, tail, and mounted ball bearing) was designed, modeled, and manufactured. With the aircraft placed in front of a fan and allowed to rotate freely with the ball bearing, an angular rate sensor and servo motor to the deflect the elevator may be connected to any computer using an analog/digital Data Acquisition (DAQ) device to send and receive signals needed for the real-time control of the system. Using Simulink and Matlab[1] with the DAQ device, the user may take data on the response of the aircraft and design the control system. The basic open loop input/output responses, system identification, and comparison to a theoretical model are described, and future work will be used to identify the closed loop control characteristics of the experiment.

Nomenclature

C_L	=	Lift Coefficient
C_M	=	Moment Coefficient
c	=	chord, ft.
I_y	=	Moment of Inerita about the pitch axis, slugs-ft^2
MAC	=	Mean Aerodynamic Chord
q	=	pitching velocity, rad/s
V_H	=	Tail Volume Coefficient
x	=	distance from front of wing, ft.
y	=	output of output equation
α	=	Angle of Attack, rad.

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ε = downwash angle, rad.

η = tail efficiency

θ = Flight Path Angle, rad.

Subscripts

AC = aerodynamic center

CG = center of gravity

e = elevator

t = tail

I. Introduction

THE purpose of this senior project was to create a physical experiment that allows students in the Cal Poly Aerospace Control Systems classes to gain practical experience in applying theoretical stability and control system principles to real world applications. Applying these basic goals to an aircraft control system, the concept for this experiment was designed and fabricated, as shown in Figure 1. The experiment exhibits the longitudinal dynamics of an aircraft in a wind tunnel, where the vehicle is free to rotate about the Center of Gravity (CG) in response to commanded deflections of the elevator. The students will effectively run the experiment, design the control system, and analyze the response data in order to get a hands-on experience with physical systems and control system design.

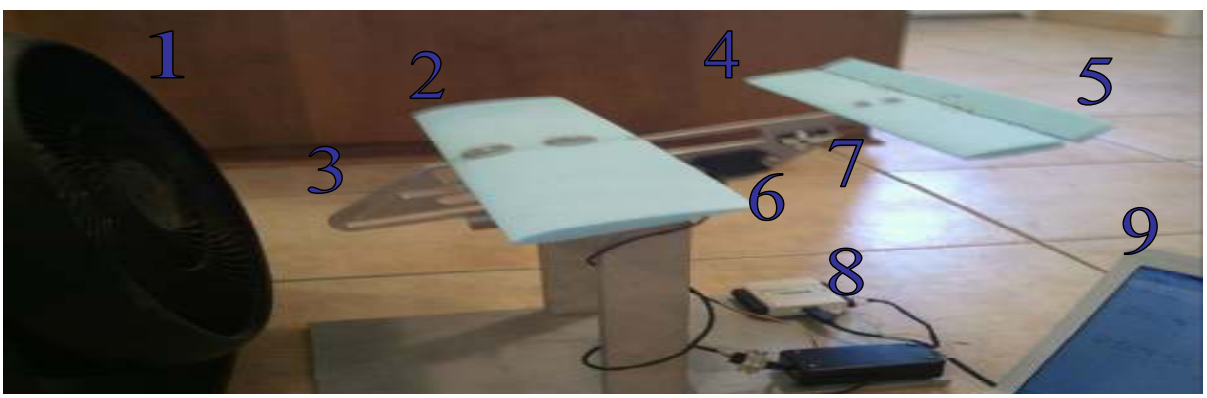

Figure 1. Experimental Set-up

Table 1. Experiment Components

	1	2	3	4	5	6	7	8	9
Item	Fan	Wing	Airframe	Tail	Elevator	Rate Gyro	Servo Motor	DAQ Device	Computer
Function	Air Flow	Lift	Structure	Stability	Control Moment	Angular Velocity	Elevator Deflections	I/O Signals	Data Analysis & Control

The system features an aluminum airframe, which houses the angular rate sensor, servo motor, mounted ball bearing, and connections to the foam wing and horizontal tail. Using the movable weight at the nose, the Center of Gravity can be moved to forward and aft of the wing in order to demonstrate the effect of the Static Margin on the static and dynamic stability of the vehicle. A Pololu LPY510AL Dual-Axis Gyro is used to measure angular velocity, and a Hiltec HS-65HB Mighty Feather Servo motor is used to rotate and deflect the elevator for commanded inputs. These signals are received and sent, respectively, by the NI USB-6009 DAQ device, which interfaces with Simulink and Matlab to process and send data from the control system.

The project consisted of a design process, a theoretical analysis, and an experimental analysis before it was released to the students for use in the classroom. The majority of the project was in the design, which involved the difficult task of turning basic design requirements into a complete system ready for fabrication. In order to make the experiment applicable and easy to use for any student at any time, the first two requirements in Table 2 were derived and applied to the design process. The last requirement was derived to add greater capability to the experiment and exhibit one of the major concepts of the experiment, which is the effect of CG location of the stability of the vehicle. With these requirements in mind, the experiment could be systematically developed from conceptual design to testing and analysis, which results in the final project goal of providing the students a practical and educational experiment in stability and control systems.

Table 2. Experiment Design Requirements

Requirement	Application
Portable	Wingspan limited to basic house fan (20” diameter)
	Easy to assemble, disassemble
Compatible with Matlab and Simulink	DAQ Device compatible with Matlab and Simulink functions
Variable Center of Gravity location	Slot for mounted ball bearing

II. Experimental Design

The project and its objectives required some distinct skills of the aircraft design process. First, the basic design philosophy was to use the most basic hardware and manufacturing techniques in order to maintain simplicity and low cost while accomplishing the goals of the experiment. This philosophy emphasizes the ability to create practical control systems with basic tools while remaining reasonable in complexity and cost. With these basic principles in

mind, the design can develop from the steps of basic requirements (as seen in Table 2), conceptual design, fabrication, and finally analysis.

Using the given and derived design requirements developed in the introduction, the initial concept was developed and modeled from the basic components used to the actual drawings needed for manufacturing. The first step was to choose the components of the system so that the airframe could be designed to house the components. First, the rate gyro that would measure the vehicle's angular velocity needed to be chosen. After research for basic sensors on the market, the Pololu LPY510AL Dual-Axis Gyro was chosen for its very low price, small size, and filtered analog voltage output. Next, a servo motor had to be chosen with the same basic characteristics, and thus the Hiltec HS-65HB Mighty Feather was chosen for its capable torque output and small size. In order to interface these devices with the computer software, the National Instruments USB-6009 DAQ device was chosen for its capability and Matlab compatibility, with ample analog and digital inputs and outputs. Finally, the ball bearing had to be chosen that would create a minimal amount of rotational friction and allow a variable point of rotation. A mounted ball bearing was the best choice, with slots that allow the bearing to move along the pitch axis.

The main element of the initial design is the airframe, which houses the wings, tail, and components, as well as maintains similarity with the configuration and properties of an actual aircraft. First, the slots for the mounted ball bearing needed to be placed so that the range of CG locations could vary from in front of the wing to behind the wing, allowing a range of practical CG configurations to test. The distance between the wing and the tail was chosen such that the Tail Volume Coefficient, V_H, was similar to a small general aviation aircraft, which demonstrates the same basic configuration as this vehicle. The wing mounting method was chosen to be a set of screws in the both the wing and tail into the top of the airframe, with a large washer on the wing to distribute the load into the wing. The servo and rate sensor, with a case to protect from airflow vibrations, were placed behind the slots. Finally, a general added requirement for the airframe design was to keep the moment of inertia as low as possible in order to counteract the added damping caused by friction in the ball bearing, as well as to make the system more responsive to control inputs. As a result, material was removed just enough to maintain structural integrity, and the thickness of the airframe could be chosen for any desired value of I_y. The system could then be modeled, and the drawings could be made directly for machining.. The result of the airframe design is a simple, cost-effective, and easy part for machining, and a system that can be assembled and disassembled in less than a minute.

Finally the system, including the wings, airframe, and electrical components, could be manufactured and assembled. The airframe was manufactured using a CNC mill with the provided drawings, using an aluminum 2011

alloy. The wings were manufactured using the Cal Poly flight lab's hot wire foam wing cutter, where a NACA 2412 airfoil was chosen for the wing and a symmetric NACA 0012 airfoil for the tail. Next, the electrical components had to be powered, grounded, and connected to the DAQ device to send and receive signals. Both the rate sensor and the servo motor require a 5 Volt power source, so a basic wall plug 5 Volt power supply was connected to each unit. Finally the base-plate, rotation shaft, basic hardware, and housing for the wiring were manufactured and purchased, and the experiment could be fully assembled and tested. The complete list of required parts highlights the valuable benefit of a cheap and easy reproduction of the experiment for multiple design iterations. The overall design resulted in the fabrication of an effective experiment, and it utilizes a complete and basic process which turns a concept into reality for improving or reproducing the experiment.

III. Theoretical Analysis

An important aspect of physical experiments, especially in control systems, is the development of a theoretical model of the system for prediction and validation of the experimental results. In actual aircraft control systems and design, theoretical models are used to determine the control system parameters (gains, architecture, etc.) for the control of the first prototype vehicle. Because of the capability to easily build and test the experiment at low cost, the control system parameters can be determined empirically with plant identification, as will be discussed in the next section. As a result, a basic theoretical model will be used for comparison with actual results and a basis for determining error in both the model and the experiment. The theoretical analysis includes derivation of the equations of motion, state-space representation, and control system modeling and analysis.

The equations of motion for the model were derived from a conventional method for linearized aircraft equations using small disturbance theory. A detailed description may be found in Robert Nelson's text[2], but the general method will be described for the vehicle modeled in this experiment. The basic assumptions of the method are thin airfoil theory, uniform and steady airflow, negligible drag forces, and a Linear Time-Invariant system. Using Newton's second law for the moments and angular acceleration about the center of gravity, small disturbance theory linearizes the moments by determining the derivatives due to changes in each of the states. In other words, terms are derived for how the vehicle's moments change due to perturbations from the equilibrium state. Given that the rotation is in only one axis, the constant airspeed, and the special case where the body axes are aligned with the inertial axes, the only moment derivatives are due the change in pitch angle and pitch rate (since the rotation is about

the constrained CG, the pitch angle and angle of attack are the same). The following equation shows the simplified linear equation of motion for the moments about the CG in the pitch axis.

$$\sum M = \frac{\partial M}{\partial \alpha}\alpha + \frac{\partial M}{\partial q}q + \frac{\partial M}{\partial \dot{\alpha}}\dot{\alpha} + \frac{\partial M}{\partial \delta_e}\delta_e = I_y\ddot{\theta} \tag{1}$$

The terms due to the pitch rate are basic equations dependant on the characteristics of the wing and tail, but the derivative due to the pitch angle, M_α, is the most complicated and influential on the stability of the system. Summing the moments due to the wing and the tail respectively, and shown in non-dimensional parameters, this term may be expressed by the following equation.

$$C_{M_\alpha} = C_{L_\alpha}\left(\frac{x_{CG}}{\bar{c}} - \frac{x_{AC}}{\bar{c}}\right) - \eta V_H C_{L_{\alpha,T}}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{2}$$

The main determination of static stability of the system is whether this term is negative, which means that the vehicle will return to equilibrium with a counteracting moment when perturbed. Assuming constant values for the wings and airflow parameters, the sign of this equation depends on the location of the CG, where a short enough tail arm (or further back CG locations) will cause the system to be statically unstable. Solving this equation for zero, the location of CG may be determined for which the system is neutrally stable, called the Neutral Point (NP). Any CG location behind this point requires use of active control to remain stable, and the actual NP on this experiment can be determined experimentally.

The longitudinal equation of motion can next be implemented in state-space form for a dynamic stability analysis and simulation in Simulink. State-space form puts the equation into a set of first order matrix equations, as shown in the following equations.

$$\begin{bmatrix} \ddot{\theta} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} (M_q + M_{\dot{\alpha}}) / I_y & M_\alpha / I_y \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \theta \end{bmatrix} + \begin{bmatrix} M_{\delta_e} / I_y \\ 0 \end{bmatrix} \delta_e \tag{3}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \theta \end{bmatrix} + [0]\delta_e \tag{4}$$

The parameters can be easily entered and changed in order to build the state matrix, and the stability characteristics can be calculated for each location of the CG. The basic stability parameters of interest are the eigenvalues of the A matrix, the rank of the observability matrix, and the rank of the controllability matrix. Next, the state-space model can be implemented into Simulink block diagrams to simulate input responses and design a control system using the program's design tools. Figure 2 shows the simulated open loop impulse responses for various CG locations, which

demonstrates the basic stability, as well as the speed and oscillations for convergence back to equilibrium. Typically aircraft become unstable in front of 100% MAC, but the large tail used here pushes the aft CG limit further back.

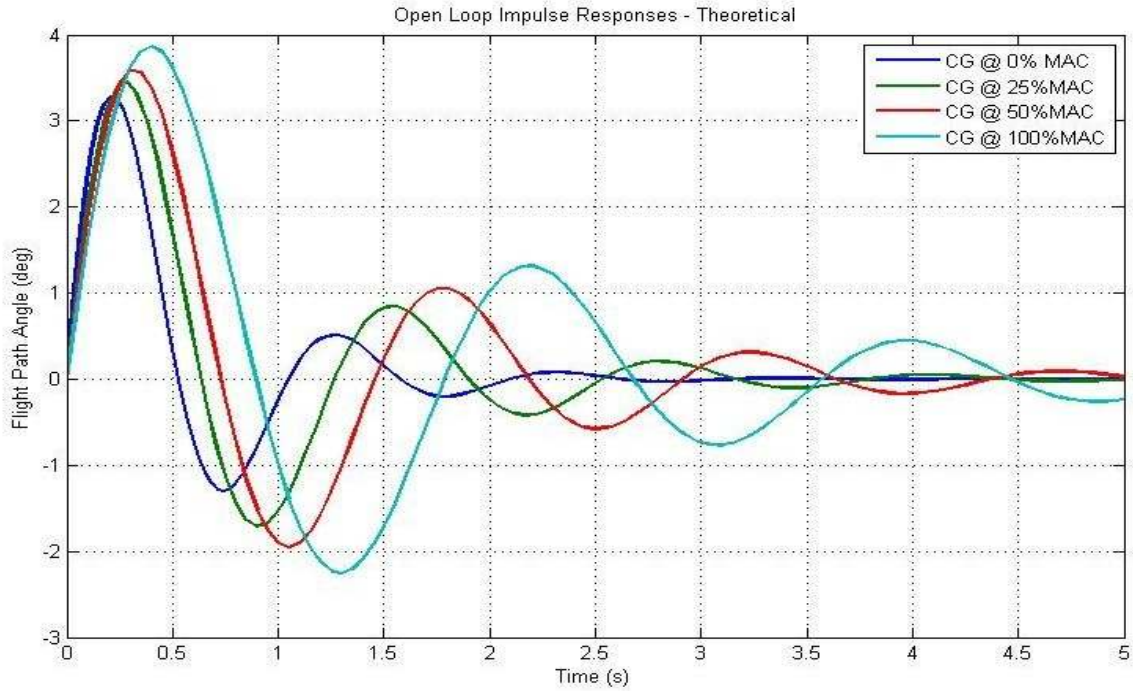

Figure 2. Impulse Responses with Varying CG

Finally, the closed-loop control system could be designed and implemented using Simulink and its design tools. In order to use a common control technique and keep the system simple, a basic PID controller was chosen for initial control of the system in the forward path of a closed loop. Using a PID tuner, the gains are determined for a given response time, bandwidth, or phase margin and applied for the desired response. Figure 3 shows the Simulink PID tuner, which demonstrates the interactive design parameters along with the controlled and uncontrolled step response. The controller also limits the commanded input to elevator deflection between +/- 25 degrees, as with an actual aircraft controller, due to constraints on the hinge moment of the elevator. The same tools and expected experimental responses can be used with the experimental control system, where the PID tuner may be used to specify certain closed loop characteristics, such as phase margin and bandwidth. The theoretical analysis develops from governing equations to a modeled and designed dynamic control system, and this provides an example for students for improving and generating a comparison model.

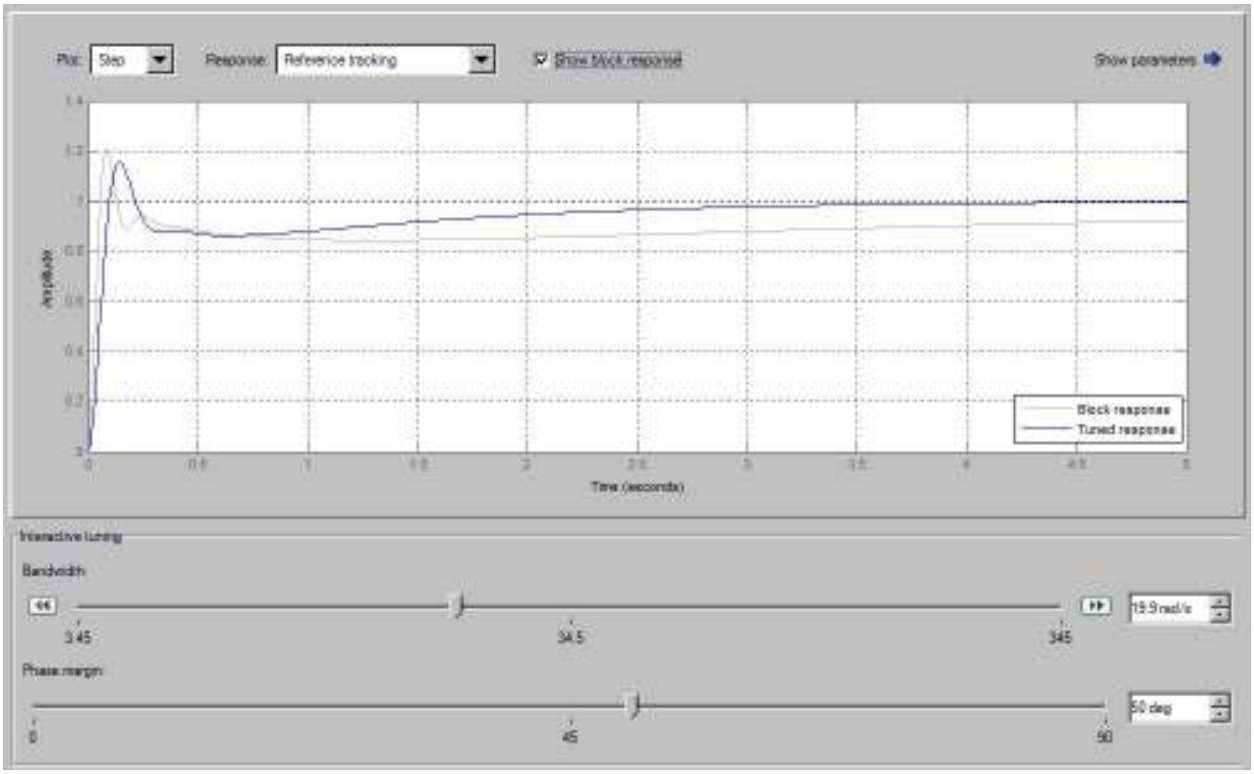

Figure 3. PID Tuner

IV. Experimental Analysis

With the experiment built and dynamics predicted with the theoretical model, the system can be tested and analyzed for several useful results. The experiment is left for future analysis and improvements by students, so only a basic initial analysis is needed to allow students to begin use of the system. First, the basic response from an impulse input can be obtained for comparison with the theoretical model as well as empirically determine the basic stability characteristics of the system. Next, system identification can be performed in order to determine the actual input-output response of the system, which is used for designing the control system more effectively than the theoretical model.

The testing of the response of the actual system yields several important conclusions about the experiment. Figure 4 shows the impulse response of the vehicle, with the input increased for a more clear picture of the response, for the CG location at 95% of the MAC. The response shows a convergence back to the equilibrium position, which is zero degrees flight path angle, but it clearly does not follow the 2nd order response predicted by the theoretical model. This result suggests an expected result, as well as the real world problem of controlling an actual plant rather than an expected model. Because of the friction caused by the ball bearing, a significant amount of added damping is

added to the dynamics of the system. Similarly, the very asymmetric and unsteady flow created by a basic fan greatly affects the aerodynamic performance, which causes unexpected results. Unfortunately, the effect is so large that it is difficult to compare the results to the model, but it is an inevitable result of constraining the vehicle to a ball bearing. However, it does add the practical problem of modeling the plant with system identification, as well as controlling the system regardless of the dynamics. Using the System Identification toolbox in Matlab, more empirical testing can develop an accurate model of the plant, which can be used in design of the control system and expected response of the system. From this point, the same analyses and tools used in the theoretical model can be applied to the system, and experiment can be successfully controlled. The result of the experimental analysis is a significant deviation from the expected results, but it also provides a large, practical platform for students to analyze the plant and control unknown dynamics.

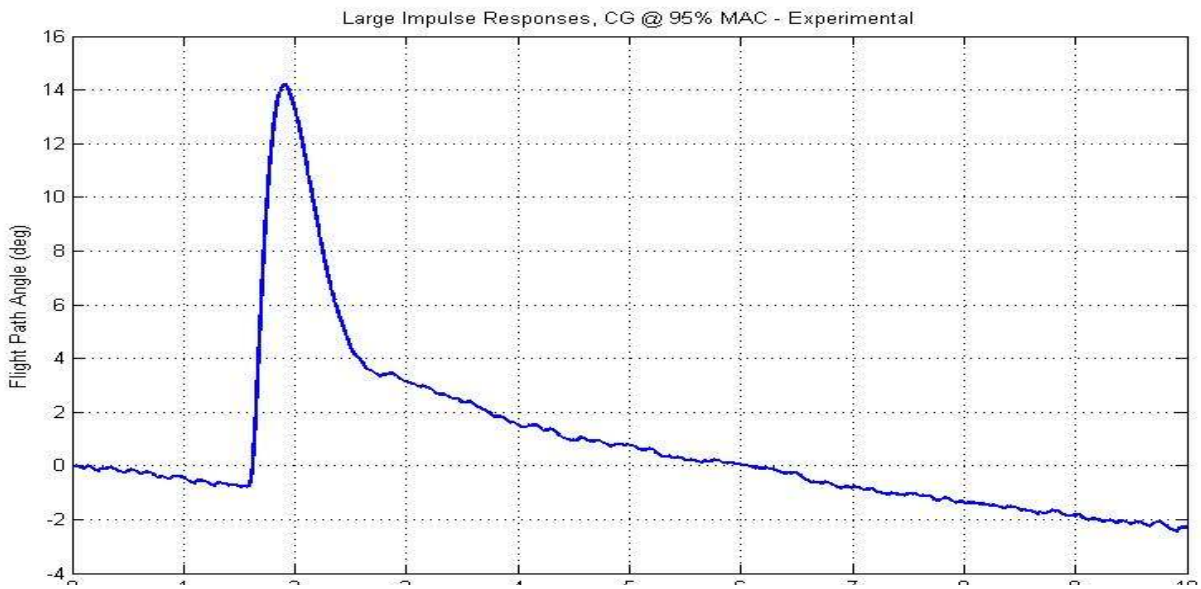

Figure 4. Experimental Impulse Response

V. Conclusion

The design, theoretical analysis, and experimental analysis each produced a valuable result for the project, as well as steps for improvement of the experiment. The experimental design yielded a very effective and simple design, where the students can easily transport, assemble, and use the experiment. As a result of the initial prototype, improvements would include a longer airframe for better balance, more similar Moment of Inertia to an actual aircraft, more durable material for the wings, and a less capable DAQ device to drive the cost down significantly. A

microcontroller will likely be added to the control loop in order to provide easier pulse generation, as well as allow a DAQ device without a clocked output capability for simplicity. The theoretical model allows students to simulate vehicle dynamics, improve the assumptions in order to approach the experimental results, and test control systems before being implemented into the actual experiment. The experimental results provided useful characteristics of the plant, as well as a platform for a complete real-time control system. The significant difference between the experimental and theoretical results suggests making improvements to the theoretical model with better assumptions, more complex terms, and more accurate measurements of aerodynamic parameters.

The most significant result of the experimental design is the wide range of types of stability and controls analysis capable by a simple and user-friendly system. The overall goal of the experiment is to teach students basic principles of stability and controls in a practical context, so the multiple types of analysis capable by this experiment extends the educational value. First, the main type of analysis is the effect of the location of the CG on static and dynamic stability, which can be analyzed on any type of vehicle configuration. Next, the basic wing mounting method allows any type of wing, with holes in the correct location, to be used on the airframe. As such, the effect of wing sweep, camber, Tail Volume Coefficient, and tail configuration can be analyzed and controlled. Next, the compact size and structural strength allow the vehicle to be placed in a variety of types of fans, including wind tunnels and variable speed fans. This allows the accuracy of the aerodynamic model to improve with better air flow, as well as accurate testing of parameters such as the vehicle's lift-curve slope. Also, an airspeed sensor, such as a pitot tube or anemometer, and airspeed controller can be used to add velocity as both a state and control input, respectively. Lastly, the integration of the hardware with the Matlab and Simulink platforms allows the student to implement and analyze any type of control technique or architecture for many more experimental procedures with a large library of Matlab function and design tools. While fabrication of the experiment was educational as an engineering student, it more importantly provided a platform for many more students to understand the principles of aircraft stability, aircraft control, data manipulation of real-time control data, and interactive control systems.

Appendix

Appendix A: Basic Vehicle Characteristics

Wingspan	1.5 ft.
Chord length	5 in.
V_H (CG at 0% MAC)	0.65
Weight	4.8 lb.
I_y (from solid model)	0.073 slugs/ft^2
V_{Fan}, Velocity of fan air	25 ft/s

Acknowledgments

First, I would like to thank Dr. Eric Mehiel, my project advisor and control systems professor, for his guidance through the entire project. He helped formulate the concept of the experiment, as well as help in each step of the process to implement modern control principles in an educational way.

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Next, I would like to thank Mason Borda, an Electrical Engineering student at Cal Poly, for help designing and building the electrical components required for use of the rate sensor and servo motor.

Finally, I would like to thank Kyle Johnson for the help in using practical tools in Solidworks[3] for building the solid model, as well as modeling accurate mass properties for a reasonable Moment of Inertia value.

References

[1] Matlab and Simulink, v. 2011b. The Math Works Inc., Mattick, MA, 2011.

[2] Nelson, R., C., *Flight Stability and Automatic Control*, 2nd ed., McGraw Hill, Boston, 1998, Chaps. 2-4.

[3] Solidworks, v. 2011. Dassault Systèmes SolidWorks Corp., Waltham, MA, 2011.