THE IMPORTANCE OF CONSIDERING SIZE EFFECT ALONG THE CUTTING EDGE IN PREDICTING THE EFFECTIVE LEAD ANGLE FOR TURNING

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Abstract
The concept of an effective orthogonal cutting edge in turning is considered. The orientation of this edge in the radial-longitudinal plane, as commonly modeled through an effective lead angle, is studied. The methods of effective lead angle prediction used in numerous previously developed force models are plagued with large errors over ranges of process inputs, in particular feed rate and depth of cut. Four previously developed methods of effective lead angle prediction are reviewed and compared to a new method presented here. This new method accounts for the size effect as introduced through the variation in chip thickness along the cutting edge, especially along the tool nose region. The difference in the new method is that the effect of continuous chip thickness variation along the cutting edge is included when evaluating the specific machining energies rather than using an average chip thickness, which has been used in the other methods. Therefore, the differential normal and friction force components acting on the rake face are functions of chip thickness through both the elemental chip load and the specific energies. Their directions are characterized by the orientations of the rake face and edge. By numerically integrating the differential force components modeled in this fashion, a significant improvement in effective lead angle prediction accuracy is realized. This improved accuracy is verified using experimental data obtained for 1018 steel and 304 stainless steel at varying levels of feed rate, depth of cut, cutting speed, nose radius and tool lead angle.
Introduction

The demand for improved quality and reduced cost of manufactured products has lead to a need for better means to predict the outputs of machining operations. Surface error, a strong determinant of quality, and machine dynamic response, an indicator of the most cost-effective machining conditions available, are process outputs directly related to the magnitude and direction of cutting forces. Accurate force prediction is therefore a key step towards achieving high quality, low cost designs. When modeling the turning process, as well as boring and face milling for which the fundamental tool edge and rake face geometry are the same, the forces acting on the tool are three dimensional. One of the force components is the tangential force which lies in the direction of the cutting velocity and hence is analogous to the cutting force in orthogonal cutting. The other two force components in the longitudinal and radial directions are usually modeled with a single thrust force through considering an extension of orthogonal process modeling theory. Under this approach, the magnitude of the resultant force in the radial-longitudinal (chip load) plane is predicted as a single thrust force. The thrust direction is in this plane and normal to the effective orthogonal cutting edge, the orientation of which is defined by the effective lead angle.

To demonstrate this effective cutting edge concept, Figure 1(a) shows the cutting edge profile and the profile of the previous tooth pass. Between these profiles lies the chip load. Therefore, the previous tooth pass generated a surface that is analogous to the un-machined surface in orthogonal machining. Figure 1(b) shows the effective straight edge equivalent to the turning situation as based on the effective cutting edge concept. The turning analogy to the orthogonal machining width of cut is generally the length of the cutting edge measured from the un-machined diametral surface to the intersection of the two profiles (Figure 1(a)), which will be assumed here to occur on the nose of the cutting edge as is generally the case. The analogous chip thickness must be such that the chip area of the orthogonal equivalent is equal to that of the actual turning situation. The orthogonal equivalent forces are $f_C$ in the cutting direction and $f_T$ in the thrust direction. As shown, the orientation of the effective edge in the radial-longitudinal plane is defined by the effective lead angle, $\gamma_{el}$. The effective lead angle can be defined as

$$\gamma_{el} = \tan^{-1}\left(\frac{f_{Rad}}{f_{Lon}}\right),$$

so that the longitudinal and radial forces are

$$f_{Lon} = f_T \cos(\gamma_{el}) \quad \text{and} \quad f_{Rad} = f_T \sin(\gamma_{el}),$$

where the predicted value of $f_T$ is calculated from the total chip load and the specific machining energies.

The accuracy with which the total thrust force is proportioned into its longitudinal and radial components directly depends on the accuracy with which the effective lead angle is predicted. In the simulation of precision machining processes, the accuracy of this proportioning is essential since (i) the radial component strongly influences surface error as it tends to separate the tool and workpiece in the direction normal to the final machined surface, and (ii) the longitudinal component causes displacements in the dominant chip thickness direction and therefore is closely related to the prediction of dynamic response, in particular chatter. Numerous investigators have studied and modeled the turning process including both static modeling (Kuhl, 1987; Young, et al., 1989) and dynamic modeling (Endres, et al., 1990; Minis, et al., 1990). A common deficiency of these models is the inaccurate prediction of effective lead angle over ranges of process inputs, in particular feed rate and depth of cut, and to some degree tool lead angle. Models of geometrically equivalent processes, such as cylinder boring (Subramani, et al., 1987) and face milling (Fu, et...
al., 1984), have also suffered from this problem. More accurate prediction of effective lead angle will also improve the usefulness of the more recently developed Dual Mechanism model (Endres, et. al., 1993) for orthogonal cutting as it is extended to three-dimensional processes.

Four previously developed methods of predicting the effective lead angle will be compared to the improved method presented here. The first pair of other methods approximate the orientation of the effective edge strictly based on an analytical geometry expression and hence will be termed here as 'geometry-based'. The second pair of other methods base the calculation of effective lead angle on an integration of a distributed force in the radial-longitudinal plane and hence will be termed 'distribution-based'. The model presented here is also distribution-based, however it includes the effect on the specific machining energies of size (chip thickness) variation along the cutting edge.

Review of Previously Developed Methods

The 'geometry-based' methods considered here are those of Colwell (1954) and Endres, et al. (1990), both of which were applied to turning situations. The 'distribution-based' methods include that used in the face milling model of Fu, et al. (1984) and another used in the cylinder boring model of Subramani, et al. (1987). The same general force modeling approach employing an equivalent orthogonal force system as discussed above was used by all these investigators. The two total machining force components, either as cutting and thrust components or as normal and friction rake face components, are modeled as being proportional to the total chip load through the specific machining energies. The specific energies are empirically modeled to include the effects of chip thickness (size) (Backer, et al., 1952), as well as cutting speed and in some instances normal rake angle. When evaluating the specific energies, in addition to accounting for the obvious effects of feed rate and lead angle on chip thickness along the lead edge \( (t_c=f \cos(\gamma_L)) \), the effects of depth of cut and nose radius are also accommodated through the use of an average chip thickness

\[
\bar{t}_c = A_c/w_c,
\]

where \( A_c \) is the total chip load and the chip width \( w_c \) is some quantification of the chip dimension measured along the cutting edge, such as total length as demonstrated in Figure 1(a). However, despite including the size effect term in the empirical specific energy models, the specific energy values are assumed constant across the entire chip. In other words, the two total machining force components are modeled as being proportional to the chip load through the specific energies, which are constant along the cutting edge for a given set of tool geometry and cutting conditions. The modeling method presented here will demonstrate that by including the effect of size variation along the edge, rather than the concept of average chip thickness, the effective lead angle is much more accurately predicted\(^2\). Following is a brief description of the four effective lead angle prediction methods to which this new method is compared.

The Colwell Method

Colwell (1954) was one of the first to study chip flow in double-edged cutting, the geometry of which is shown in Figure 2 for turning. The effective edge is a line defined by the following two points: (point 1) the intersection of the cutting edge profile with the previous edge profile, and (point 2) the intersection of the cutting edge profile with the un-machined diametral surface. A solution for the effective lead angle is determined by calculating the slope of the effective cutting edge in the x-y system shown in Figure 2. This slope is calculated from the relative positions of points 1 and 2, the coordinates of which are readily determined based on profile geometry. The

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1 The four effective lead angle prediction methods compared here will be referred to by the first author's name, e.g the 'Colwell', 'Endres', 'Fu', and 'Subramani' methods.

2 The effective lead angle prediction method presented here will be referred to as the 'Size Effect' method.
result depends on whether or not cutting occurs on the lead edge. This is expressed through the condition on depth of cut given for Point 2. Hence, the effective lead angle is

\[ \gamma_{eL} = \tan^{-1}\left( \frac{y_2 - y_1}{x_1 - x_2} \right), \]  

where

\[ x_1 = +\left( r_n^2 - f^2/4 \right)^{1/2}, \quad y_1 = -f/2 \]

and

\[ x_2 = r_n - d, \quad y_2 = \begin{cases} +\left[ r_n^2 - (r_n - d)^2 \right]^{1/2}, & d < r_n(1 - \sin(\gamma_L)) \\ \frac{r_n}{\cos(\gamma_L)} + \left[ d - r_n(1 - \sin(\gamma_L)) \right] \tan(\gamma_L), & d \geq r_n(1 - \sin(\gamma_L)) \end{cases} \]

**The Endres Method**

The effective lead angle prediction used in the dynamic turning model of Endres, et al. (1990) considers an average orientation of the cutting edge, profile \( k \), and un-machined surface (previous edge profile), profile \( k-1 \) as shown in Figure 3. The chip load, and hence these profile curves, are divided into two elements. The result is four curve segments, each of which has an average orientation angle \( \gamma \). Along the lead edge, this angle is simply equal to the tool lead angle \( \gamma_{L_k} = \gamma_{L_{k-1}} = \gamma_L \). Along the nose, the average orientation angle is that of the tangent at the midway point along the arc segment as shown in Figure 3. The effective lead angle is modeled as a segment-length weighted average of the average segment orientation angles as

\[ \gamma_{eL} = \frac{\gamma_{i_k} w_{i_k} + \gamma_{i_{k-1}} w_{i_{k-1}} + \gamma_{2_k} w_{2_k} + \gamma_{2_{k-1}} w_{2_{k-1}}}{w_{i_k} + w_{i_{k-1}} + w_{2_k} + w_{2_{k-1}}}. \]  

Substituting for the segment lengths and orientation angles yields

\[ \gamma_{eL} = \frac{r_n}{2} \left[ \left( \sin^{-1}(f/2r_n) \right)^2 + \pi^2/2 - \varphi^2 - \pi \sin^{-1}(f/2r_n) \right] + \gamma_L w_{2_k}, \]  

where

\[ \varphi = \sin^{-1}(1 - d/r_n) \quad \text{and} \quad w_{2_k} = 0 \quad \text{for} \quad d < r_n(1 - \sin(\gamma_L)), \]

and

\[ \varphi = \gamma_L \quad \text{and} \quad w_{2_k} = \frac{d - r_n(1 - \sin(\gamma_L))}{\cos(\gamma_L)} \quad \text{for} \quad d \geq r_n(1 - \sin(\gamma_L)). \]

**The Fu Method**

The effective lead angle prediction used in the static face milling model of Fu, et al. (1984) considers the total thrust force in the radial-longitudinal plane, \( f_r \), to be the result of a uniform force distribution along the cutting edge as shown in Figure 4. The force per unit length of cutting
edge can be represented by a constant $P$. Note that it is assumed that this distribution extends only to the tip of the nose and not to the intersection of the cutting edge with the previous edge profile. The total longitudinal and radial force components can be determined by integrating, along the region over which the distribution is applied, $P \cdot \cos(\gamma) dw$ and $P \cdot \sin(\gamma) dw$, respectively, where $\gamma$ is a function of edge location and $dw$ is the width of the differential element. The value of $P$ is unknown. However in the ratio of Equation (1), the constant $P$ cancels, so that analytically evaluating the integrals of $\cos(\gamma) dw$ and $\sin(\gamma) dw$ and simplifying yields

$$
\tan^{-1} \left( \frac{2r_n}{d(1-\sin(\gamma_L))} \right), \\
\tan^{-1} \left( \frac{1-\sin(\gamma_L)}{\cos(\gamma_L)} \right) \quad d \geq r_n (1-\sin(\gamma_L))
$$

(7)

The Subramani Method

The effective lead angle prediction used in the static cylinder boring model of Subramani, et al. (1987), as also used by Kuhl (1987) and Young, et al. (1989) in their static turning models, considers the total force in the radial-longitudinal plane to be the result of a non-uniform force distribution along the cutting edge as shown in Figure 5. This force distribution is assumed to be proportional to chip thickness, hence being non-uniform. Subramani, et al. employed numerical integration while Kuhl and Young, et al. performed a piece-wise analytical integration. These analyses differ from that of Fu, et al. in that $P$ is replaced with $P' t_c'$ where $P'$ is a constant. Note that this still implies the assumption, common to all four of these models, that the specific energies ($P'$) are constant with chip thickness. Upon cancellation of $P'$ in the ratio of Equation (1) and integrating, a value for effective lead angle results.

Implementation to Account for Size Effect Variation Along the Edge

This discussion can begin with an evaluation of the non-uniform distribution method of Subramani, et al. The distributed force acting normal the cutting edge is shown in Figure 5 and can be integrated by considering a differential element at some edge location $w$. As shown in Figure 5, this distribution is more heavily weighted on the lead edge due to the larger chip thickness and hence differential chip load. The weighting employed in this new method will not quite be proportional to chip thickness due to the increase in specific energy along the nose due to the size effect, as shown in Figure 6(a). Due to the nonlinearity of the size effect, the influence of the increased force distribution along the nose can cause a noticeable increase in effective lead angle as shown in Figure 6(b).

The force model implemented here considers the specific energies as nonlinear functions of chip thickness, where the specific thrust energy has the form

$$
K_T = e^{a_0 t_c a_1 V a_2},
$$

(8)

where $V$ is the cutting speed. As a result, the force in the radial-longitudinal plane acting in direction $\gamma(w)$ on some differential edge element is

$$
df_T = K_T(w) \cdot t_c(w) \cdot dw = \left( e^{a_0 t_c(w) a_1 V a_2} \right) \cdot t_c(w) \cdot dw.
$$

(9)

Now, the specific energy is not constant as was $P'$ in the approach of Subramani, et al. Therefore, it cannot be removed from the integrals and hence does not cancel in the ratio of Equation (1) used
to determine the effective lead angle. By combining the chip thickness terms, this force expression could be analytically integrated.

The friction force rather than a thrust force is considered here, as in Kuhl's model, because it tends to better physically represent at least one of the two primary energy consumption mechanisms, rake face friction. Unlike Kuhl, it is also desired to account for the dependence on edge location of not just the edge orientation angle \( \gamma \), but also the inclination and normal rake angles as they also influence the direction of chip flow. To transform a differential friction force to longitudinal and radial components at any point along the edge, a complex, edge location dependent, three-dimensional rotational transformation is required. Therefore, so as not to detract from the point being made here by a complex analytical evaluation of these integrals, a numerical integration will be employed, as was used by Subramani, et al.

The Curved Turning Edge as a Series of Differential Oblique Edges

The machining forces acting on the tool, as measured in the external tangential-longitudinal-radial basis coordinate system, are determined by considering the turning tool's curved cutting edge as a series of many differently-oriented differential oblique cutting edges. Each element is approximated as a straight-edged oblique tool. The obliquity of the edge depends on the rake face orientation as defined by the side and back rake angles, and the edge orientation in the radial-longitudinal plane at the edge location of interest \( w \). Therefore, the inclination angle takes the same form as for the main (lead) cutting edge of a typical turning tool,

\[
\tan(i(w)) = \tan(\alpha_b) \cos(\gamma(w)) - \tan(\alpha_s) \sin(\gamma(w)),
\]

(10)

where the lead angle has been replaced with its edge location dependent equivalent \( \gamma(w) \). Along the main cutting edge, \( \gamma(w) \) is equal to the nominal lead angle \( \gamma_L \). If cutting occurs along the end cutting edge, \( \gamma(w) \) is equal to \( \pi/2 + \gamma_E \), where \( \gamma_E \) is the nominal end cutting edge angle. Along the nose, \( \gamma(w) \) varies linearly from \( \gamma_L \) to \( \pi/2 + \gamma_E \).

The machining force is modeled as normal and friction force components on the rake face. The normal force component is modeled as being proportional to the chip load and the friction force component is proportional to the normal force component as

\[
dN(w) = K(w) \cdot dA_c(w) \quad \text{and} \quad dF(w) = \mu(w) \cdot dN(w),
\]

(11)

where \( K(w) \) is the normal rake face force coefficient, \( \mu(w) \) is the rake face coefficient of friction, and

\[
dA_c(w) = t_c(w) \cdot dw
\]

(12)

is the differential chip load. As explicitly written in Equations (11) and (12), the differential chip load \( dA_c(w) \) is dependent on edge location through the chip thickness. The coefficients \( K \) and \( \mu \) are the empirically modeled quantities that represent the specific machining energies. These coefficients are modeled using the empirical form of Equation (8). Therefore, due to the chip thickness term in Equation (8), these terms must also be dependent on edge location as explicitly written in Equations (11).

The role of the chip thickness in Equations (11) and (12), and how it is dealt with, provide the key difference between the Size Effect method presented here and the other four methods to which it is compared. Besides directly influencing the chip load as noted in Equation (11), the chip thickness takes on its elemental value hence changing along the cutting edge and introducing the size effect, through the empirical coefficients, as a continuous effect along the edge. The inclusion of this chip thickness or size effect as a continuous one along the cutting edge provides a more
accurate representation of the actual situation, and will be shown to have a significant impact on the prediction accuracy of the effective lead angle when implemented through the vectoral integration of the differential force components of Equation (11). Although normal rake angle, which also varies along the cutting edge, has been shown to affect the specific energies and has been included in the form of Equation (8), its effect is assumed to be small in comparison to the size effect and is neglected in this analysis.

Vectoral Integration of the Differential Force Components

The total forces must be calculated through vectoral summation of the differential forces. This can be done by transforming the differential normal and friction force components into differential components in the orthogonal tangential-longitudinal-radial basis coordinate system. Such a transformation requires knowledge of normal and friction component directions. The elemental and hence total normal force direction is readily determined from the orientation of the rake face. The friction force direction is not so obvious.

Oblique cutting analysis shows that the chip tends to flow up the tool rake face at an angle $\eta_c$ relative to the normal to the cutting edge. This chip flow angle is frequently approximated by Stabler's Rule, which states that $\eta_c$ is equal to the inclination angle. Because the inclination angle and even more so the cutting edge orientation varies with edge location, the differential chip elements will tend to flow in many different directions. This is not physically possible on a large scale. Therefore, to account for the flow of the chip, as a whole, being in one unified direction, an average chip flow direction is determined through a vectoral integration, along the edge, of the differential friction force $dF(w)$. In other words, the numerical integration of this force is equivalent to a vector summation yielding a friction force direction. The approach is similar to that used by Young, et al. (1989), except that the effect of size variation with edge location is considered in computing the differential friction force.

The transformation of $dF(w)$ into tangential, longitudinal, and radial components is

$$
\begin{bmatrix}
    dF_{Tan} \\
    dF_{Lon} \\
    dF_{Rad}
\end{bmatrix} = [B] \cdot dF,
$$

(13)

where

$$
[B] = \begin{bmatrix}
    \sin(\eta_c)\sin(i) + \cos(\eta_c)\sin(\alpha_n)\cos(i) \\
    \cos(\eta_c)\cos(\alpha_n)\cos(\gamma) - [\sin(\eta_c)\cos(i) - \cos(\eta_c)\sin(\alpha_n)\sin(i)]\sin(\gamma) \\
    [\sin(\eta_c)\cos(i) - \cos(\eta_c)\sin(\alpha_n)\sin(i)]\cos(\gamma) + \cos(\eta_c)\cos(\alpha_n)\sin(i)
\end{bmatrix}
$$

As stated above, the chip flow angle is approximated by the inclination angle, which is determined from Equation (10). The normal rake angle, $\alpha_n(w)$, is related to the tool and elemental geometry as

$$
\tan(\alpha_n(w)) = [\tan(\alpha_n)\cos(\gamma(w)) + \tan(\alpha_b)\sin(\gamma(w))]\cos(i(w)).
$$

(14)

The numerical integration of $dF_{Tan}$, $dF_{Lon}$, and $dF_{Rad}$ yields the force components $F_{Tan}$, $F_{Lon}$, and $F_{Rad}$. The direction of the resultant friction force vector can then be defined by its direction cosines, which are

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3 Unless otherwise stated in this discussion, all terms in the differential relations are functions of edge location $w$ though not explicitly written as such for the sake of simplicity of the equations.
\[
\begin{bmatrix}
\cos(\eta_{\text{Tan}}) \\
\cos(\eta_{\text{Lon}}) \\
\cos(\eta_{\text{Rad}})
\end{bmatrix} = \begin{bmatrix}
F_{\text{Tan}} \\
F_{\text{Lon}} \\
F_{\text{Rad}}
\end{bmatrix} \cdot \frac{1}{\sqrt{F_{\text{Tan}}^2 + F_{\text{Lon}}^2 + F_{\text{Rad}}^2}}.
\] (15)

Now, given the flow direction of the entire chip, and hence all the differential friction force components, the vectorial integration for the total forces acting on the tool can be performed. The transformation of the differential rake face force components into tangential, longitudinal, and radial components is

\[
\begin{bmatrix}
df_{\text{Tan}} \\
df_{\text{Lon}} \\
df_{\text{Rad}}
\end{bmatrix} = [A] \cdot \begin{bmatrix}
\cos(\eta_{\text{Tan}}) \\
\cos(\eta_{\text{Lon}}) \\
\cos(\eta_{\text{Rad}})
\end{bmatrix} \cdot dF,
\] (16)

where

\[
[A] = \begin{bmatrix}
\cos(\alpha_n) \cos(i) \\
\cos(\alpha_n) \sin(i) \sin(\gamma) - \sin(\alpha_n) \cos(\gamma) \\
-\cos(\alpha_n) \sin(i) \cos(\gamma) - \sin(\alpha_n) \sin(\gamma)
\end{bmatrix}
\]

It should be noted that, although the terms \(\alpha_n, i,\) and \(\gamma\) in \([A]\) depend on edge location, \([A]\) will turn out to be constant along the edge since for a flat rake face the direction of the rake face normal must be constant.

At this point, the curved cutting edge has been accommodated through considering the force system on a differential element of the cutting edge and the differential force relations have been transformed into the external coordinate system for vectoral summation through numerical integration. The final step is to calibrate the empirical models for \(K\) and \(\mu\) from experimental data.

**Force Model Calibration**

Two materials will be investigated, 1018 steel and 304 stainless steel. The same calibration procedure is followed for each, and is similar to those discussed by other investigators, such as Subramani, et al. (1987) and Endres, et al. (1990). The conditions used for model calibration encompass two levels of cutting speed and two levels of feed rate for a single lead angle of 0\(^\circ\) and a single rake angle geometry of \(\alpha_r = 5^\circ\) and \(\alpha_b = 0^\circ\). The result is a \(2^2\) full factorial design.

The chip thickness term used in calibrating these models is the average chip thickness of Equation (3), which is influenced by the feed rate, depth of cut, nose radius and lead angle. Therefore, assuming appropriate empirical model forms, the models fit to these data should be valid anywhere within these ranges of cutting speed and average chip thickness. For the 304 stainless steel, different depth of cut levels were used at the low and high feed rate levels to obtain a larger range, especially at the low end, of (average) chip thickness levels. However, because the actual chip thickness approaches zero along the nose while the calibration cannot actually include a zero chip thickness level, a portion along the nose will require extrapolation of the empirical models. The implications of this extrapolation will be discussed later.

The design matrices are shown in Tables 1 and 2 along with the average measured forces. The resulting specific energy models are obtained through linear regression to be

\[
K = e^{19.8573 - 0.2506V - 0.1041t_c}, \quad R^2 = 0.992 \quad \text{and} \quad \mu = e^{-1.2596 - 0.1182V - 0.0349t_c}, \quad R^2 = 0.791
\]

for the 1018 steel, and
\[ K = e^{20.2339} t_c^{-0.2233} V^{-0.1076}, \ R^2 = 0.999 \quad \text{and} \quad \mu = e^{-4.4781} t_c^{-0.4010} V^{-0.1300}, \ R^2 = 0.951 \]

for the 304 stainless steel.

### Table 1 Calibration Data for 1018 Steel, \( r_n = 1.191 \text{ mm} \).

<table>
<thead>
<tr>
<th>Feed Rate (mm/r)</th>
<th>Speed (m/min)</th>
<th>Depth of Cut (mm)</th>
<th>Average ( t_c ) (mm)</th>
<th>( f_{\text{tan}} ) (N)</th>
<th>( f_{\text{lon}} ) (N)</th>
<th>( f_{\text{rad}} ) (N)</th>
</tr>
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<tbody>
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<td>0.0188</td>
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<td>393</td>
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<tr>
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<td>298</td>
</tr>
</tbody>
</table>

### Table 2 Calibration Data for 304 Stainless Steel, \( r_n = 0.794 \text{ mm} \).

<table>
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<th>Feed Rate (mm/r)</th>
<th>Speed (m/min)</th>
<th>Depth of Cut (mm)</th>
<th>Average ( t_c ) (mm)</th>
<th>( f_{\text{tan}} ) (N)</th>
<th>( f_{\text{lon}} ) (N)</th>
<th>( f_{\text{rad}} ) (N)</th>
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<td>301</td>
</tr>
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</table>

### Discussion of Results

Predicted effective lead angles using the model calibrations of the previous section are compared here to experimental values determined using Equation (1). The experimental values correspond to measured forces collected from tests at numerous different sets of process conditions: 34 tests encompassing ranges of feed rate from 0.025 mm/r to 0.300 mm/r and depth of cut from 0.500 mm to 2.800 mm at a 1.191 mm nose radius and a 0° lead angle for the 1018 steel (each replicated two or four times for a total of 96 data points), and 64 tests encompassing ranges of feed rate from 0.102 mm/r to 0.305 mm/r and depth of cut from 0.635 mm to 1.905 mm at 0.794 and 1.191 mm nose radii and 0° and 15° lead angles for the 304 stainless steel. The predicted effective lead angle values are compared to the measured values through relative magnitude prediction accuracy and percentage prediction error in Figures 7 through 10. The comparisons of the Size Effect predictions to those of the other four methods are grouped into the two categories of 'geometry-based' and 'distribution-based'.

Figures 7 and 8 show the results for the 1018 steel. A comparison of the prediction accuracy among the methods is presented in the graphs of Figure 7 showing the Size Effect method to provide a significant improvement over the other methods in the general average prediction accuracy. Although each method provides the best prediction for at least one of the tests, on average the Size Effect method is clearly superior. The Colwell, Fu, and Subramani methods consistently either over- or under-predict the effective lead angle, while the Endres method over-predicts for small values and under-predicts for large values.

An evaluation of the prediction error trends relative to changes in conditions can be observed from Figure 8. As shown by the near-zero slopes of the average trends over a range of feed rate, only the Size Effect and Subramani methods accurately predict the effect of feed rate on effective lead angle. Note that these are the two models that most accurately account for the chip thickness variation along the nose, the region that is most strongly affected by changing the feed rate. Additionally, it can be seen that the Subramani method under-predicts on the average, most likely the result of not weighting the smaller chip thicknesses near the tip of the nose more heavily as does the Size Effect method. As a result, the predicted total radial force component is not as large as it should be, hence causing the direction of the predicted total thrust force to be more in the longitudinal direction than it should be. The other methods show a definite average trend in prediction error with feed rate.

The effect of depth of cut on prediction error can also be seen in Figure 8. Each of the five
models shows about the same level of dependence on depth of cut. The Size Effect method appears to under-predict effective lead angle for the cases of larger depths of cut. Such a trend is not unexpected since the effect of chip thickness on the specific energies along the nose is relatively uncertain due to the considerable extrapolation that is required below the range of chip thickness used in the calibration. The error, though small, will likely depend on the depth of cut since this quantity strongly governs the proportion of the total cutting that takes place on the nose. Furthermore, the variation of normal rake angle along the edge is likely to have a small effect on force prediction. Since the variation takes place mainly on the nose, this effect will also depend on depth of cut.

Figures 9 and 10 indicate that the results for 304 stainless steel are similar to those for 1018 steel, though the additional effects of varying nose radius and tool lead angle are included. A direct comparison of the five methods in Figure 9 confirms that the Size Effect method again most accurately predicts the effective lead angle. Although Fu's method appears to be of comparable accuracy, it proves to be inconsistent when compared to the results for 1018 steel. Varying nose radius appears to have very little effect on the comparisons since the top (large nose) and bottom (small nose) graphs are nearly identical.

Finally, the effects on prediction error of nose radius, nominal lead angle, and again feed rate and depth of cut can be considered as shown in Figure 10. The Size Effect method shows a slight dependence on both feed rate and depth of cut. This can be attributed to the extrapolation issue discussed for the 1018 steel results, which is enhanced here due to the larger low level of chip thickness (see Tables 1 and 2) used in the calibration of the 304 stainless steel models. Each of the other methods shows a strong dependence of prediction error on feed rate and/or depth of cut. The Fu method, which is closest to the Size Effect method in accuracy, shows a very strong dependence on feed rate in Figure 10(b). Such a dependence limits the usefulness of the Fu approach and is likely caused by the uniform rather than non-uniform weighting along the nose, again the region most strongly affected by changes in feed rate.

Conclusions

A method for predicting the effective lead angle in turning was developed in which the effect of varying chip thickness along the length of the cutting edge is considered by numerically integrating the differential oblique cutting elements along the edge. This method differs from four previously developed methods, considered for comparison purposes, in that it accounts for the variation in size effect along the nose of the turning tool. From the comparison of these five methods over ranges of feed rate, depth of cut, nose radius and tool lead angle for 1018 steel and 304 stainless steel, it is concluded that the 'Size Effect' method presented here provides a significant improvement in prediction accuracy. Some specific observations are summarized as follows:

- both the Size Effect and Subramani methods predict the average effect of feed rate quite well, however the Subramani method consistently under-predicts in magnitude due to insufficient weighting of the small chip thicknesses near the tip of the nose,
- the Fu method too strongly weights the small chip thickness region at the tip of the nose hence resulting in over-prediction for the most part, and a strong dependence of prediction accuracy on feed rate, and
- for 1018 steel, the accuracy of the Size Effect method shows a slight dependence on depth of cut.

The final conclusion is that the effect of edge location dependent chip thickness on the specific energies in turning should not be neglected when predicting effective lead angle. Further investigation is recommended to study materials that exhibit noticeably different chip formation, such as cast iron, to ensure that the Size Effect approach works well for these other materials. The depth of cut dependence of the Size Effect method's prediction accuracy should also be considered with special attention given to normal rake angle variation and to methods of calibration that include or in some way account for very small chip thickness levels.
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References


Effective Un-machined Diametral Cutting Edge Equivalent to the Un-machined Surface in Orthogonal Machining

Figure 1(a) Turning Geometry

Effective Cutting Edge

Figure 1(b) Effective or Equivalent Turning Geometry

Figure 2 Geometry of the Colwell Method

Figure 3 Geometry of the Endres Method

Figure 4 Geometry of the Fu Method

Figure 5 Geometry of the Subramani Method
Figure 6(a) The Difference Between Thrust Force Distributions for No Size Effect and Including Size Effect

Figure 6(b) The Difference Between Effective Lead Angle for No Size Effect and Including Size Effect

Figure 7 Predicted vs. Measured Effective Lead Angle for 1018 Steel: (a) Geometry-Based Methods and (b) Distribution-Based Methods.
Figure 8 Prediction Errors vs. Feed Rate (top) and Depth of Cut (bottom) for 1018 Steel: (a) Geometry-Based Methods and (b) Distribution-Based Methods.
Figure 9 Predicted vs. Measured Effective Lead Angle for 305 Stainless Steel at Lead Angles of 0° and 15° and Nose Radii of 1.191 mm (top) and 0.794 mm (bottom): (a) Geometry-Based Methods and (b) Distribution-Based Methods.
Figure 10: Errors vs. Feed Rate (top), Depth of Cut, Nose Radius, and Lead Angle (bottom) for 304 Stainless Steel: (a) Geometry-Based Methods and (b) Distribution-Based Methods.