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Effect of chaos and stochastics induced by internal waves on acoustic wave propagation

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ABSTRACT: In the present paper the eikonal equation is considered in the form of a second order, nonlinear ordinary differential equation with harmonic excitation due to internal wave. The harmonic excitation is taken imperfect, i.e. there is a random phase modulation due to Gaussian white noise. The amplitude and wavelength of the acoustic wave are used as the principle signal characteristics in bifurcation analysis. The regions of instability, identified using the bifurcation diagrams, examined through phase diagrams and Poincare maps. The effect of stochastic nature in addition to chaotic one of internal waves is demonstrated by means of comparison of the numerical data obtained for perfectly periodic excitation. Preliminary analysis shows very strong dependence on noise intensity at some values of amplitude and wave length of the internal wave.

1 INTRODUCTION

The sound wave propagation has been used as a tomographic means to study and monitor the ocean for years (Munk and Wunsch, 1979; Munk et al., 1995). The purpose of ocean acoustic tomography is to deduce from precise measurements of travel time, or of other properties of acoustic propagation the state of the ocean traversed by the sound field (Munk and Wunsch, 1979). The tomographic method introduced by Munk and Wunsch (1979) based on the fact that travel time and other measurable acoustic parameters are functions of temperature, water velocity, and other ocean parameters and can be interpreted to provide information about the intervening ocean using inverse methods. Arrival time has been the basic characteristic from which inversions have been performed to reconstruct ocean structure. For long-range propagation it has been observed that ocean fluctuations (internal waves) affect the acoustic wave propagation limiting the resolution of tomographic scheme based on the time of flight. Recent theoretical and experimental studies by Simmen et al. have suggested that the breakdown in identifying signal arrivals at long ranges is due to ray chaos induced by a range-depended ocean structure. The chaotic behavior of rays has been investigated in numerous works (Smith et al., 1991; Duda and Bowlin, 1994; Colosi et al., 1994; Zaslavsky and Abdulaev, 1997; Simmen et al., 1997; Wiercigroch et al.,1999). In their recent work, Wiercigroch et al. (1999) investigated non-linear dynamic behavior of basic ray equations in the presence of a wave-like forcing assuming that a single-mode sound speed perturbation is superimposed onto a genetic range-independent sound speed profile known as the Munk's canonical profile (Munk, 1974). Wiercigroch and his colleagues investigated acoustic wave propagation using the ray equations together with stability analysis conducted by constructing bifurcation diagrams, Poincare maps (Thompson and Stewart, 1987; Wiggins, 1990). Their approach fails to account for nondeterministic contribution from the internal waves.

In the present work it is shown that consideration of only chaotic behavior induced by internal waves somewhat simplifies the real problem, where the stochastic behavior has a substantial input. Our goal is not to precisely quantify acoustic fluctuations due to internal waves but rather to demonstrate that an addition of ideal turbulence model, such as white noise, leads to the different characteristics of the acoustic arrivals. Therefore, in the present paper we investigate the influence of nondeterministic excitation induced by the internal waves on acoustic wave propagation as well as deterministic, chaotic behavior. In the present formulation the eikonal equation is considered in the form of a second order, nonlinear ordinary differential equation with harmonic excitation due to internal wave. The harmonic excitation is taken imperfect, i.e. there is a random phase modulation due to Gaussian white noise. The amplitude and wavelength of the acoustic wave are used as the principle signal characteristics in bifurcation analysis.

2 PROBLEM STATEMENT

In a standard form the eikonal equations can be written as (Wiercigroch, et al., 1999):

$$\frac{dz}{dr} = \frac{\partial H}{\partial p}, \frac{dp}{dr} = -\frac{\partial H}{\partial z} \tag{1}$$

Here H is a Hamiltonian:

$$H(z,p) = \frac{1}{2}p^2 + V(z) + F(z,r) \tag{2}$$

Where z is the ray depth and r is the horizontal range and *F(z,r)* is the external perturbation. The tangent of the ray angle φ with respect to the horizontal axis is $p = \tan\varphi$. Figure 1 shows the diagram of acoustic wave propagation.

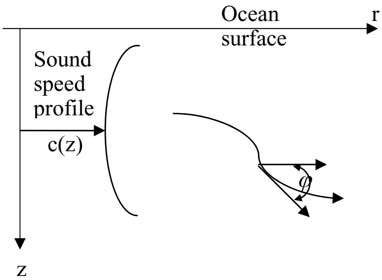

Figure 1. Sketch of underwater acoustic ray propagation.

The potential energy *V(z)* is related to the ray depth through the sound speed profile and may be expressed as (Munk, 1974):

$$V(z) = \frac{1}{2}\left[1 - \left[c_0 / c(z)\right]^2\right],$$
$$c(z) = c_{ch}\left(1 + \varepsilon\left[e^{-2(z-z_a)/B} + 2\frac{z - z_a}{B} - 1\right]\right) \tag{3}$$

where *ε, B* are constants, c_{ch} - the sound speed at the sound channel axis, z_a - the depth at the sound channel axis [Monk]. The ray traveling in the ocean is subject to perturbation due to internal waves. The latter is usually considered as a single-mode wave:

$$F(z,r) = \sqrt{2}Ae^{-3z/2B}\sin\theta r, \theta = \frac{2\pi}{R} \tag{4}$$

Substituting (3) and (4) into (1) one gets two, nonlinear differential equations with external harmonic excitation. Such system has been studied extensively in Wiercigroch et al. (1999). However, it is reasonable to assume that the perturbation due to the internal waves is not perfectly harmonic, but rather imperfectly harmonic. Such an excitation may be modeled by a harmonic function with randomly perturbed phase or random phase modulation, and is written as:

$$\begin{aligned} &F(z,r) = \sqrt{2}Ae^{-3z/2B}\sin q(r), \\ &\frac{dq}{dr} = \frac{2\pi}{R} + \xi(t), \\ &< \xi(t) >= 0, < \xi(t)\xi(t+\tau) >= D\delta(\tau) \end{aligned} \tag{5}$$

where *ξ(t)* is zero mean Gaussian white noise. Thus, the following system will be investigated numerically:

$$\begin{aligned} &\frac{dz}{dr} = p, \\ &\frac{dp}{dr} = -\frac{2}{B}c_0^2 c_{ch}\varepsilon\frac{1 - e^{2(z-z_a)/B}}{c(z)^3} + \\ &+ \frac{3}{\sqrt{2}B}Ae^{-3z/2B}\sin q(r) \\ &\frac{dq}{dr} = \frac{2\pi}{R} + \xi(t) \end{aligned} \tag{6}$$

In the above formula, A and R can be viewed as the mean amplitude and frequency of excitation. Influence of noise intensity D onto the behavior of that system will be considered in the next section.

3 RESULTS OF NUMERICAL SIMULATION

The system of nonlinear, stochastic differential equations (6) will be solved numerically in this section for certain values of parameters *A,R* and *D*. Equations (6) have to be solved for the following values of constants and initial conditions for Monte Carlo simulation:

$B = 1.3, \varepsilon = 0.0074, z_a = 1.0, c_{ch} = 1.53$

$z(0) = 1.0, \varphi(0) = 7.5^0$

It is worth reminding that the purpose of this paper is not to precisely quantify acoustic fluctuations due to random excitation induced by internal waves but rather to show that a ray-based description of sound propagation through internal waves that includes only deterministic excitation, not stochastic, fails to accurately capture the important characteristic. In order to perform the comparison in the following the results for phase planes and bifurcation diagrams will be plotted for two cases: (I) for purely deterministic excitation (D=0), and (II) accounting for deterministic excitation as well as stochastic one. For comparison purposes the physical data such as amplitudes and wavelength of interest as well as initial conditions are taken as mentioned above.
First we investigate the influence of the wavelength, R onto the depth z, for a fixed value of the amplitude, A. The bifurcation diagram shown in Figure 2 constructed based on the ray acoustic equation that does not include the influence of random excitation. It can be noted that the effect of chaotic behavior of internal waves is small up to R=8km. After that, the system responses become irregular.

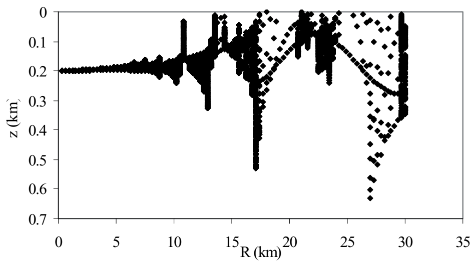

Figure 2. Bifurcation diagram z=f(R); A=0.005; D=0

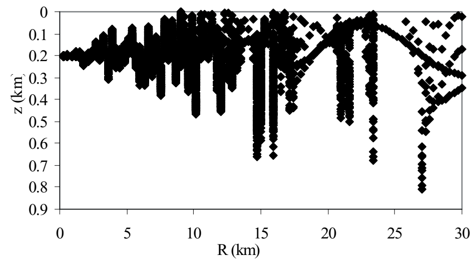

Figure 3. Bifurcation diagram z=f(R) A=0.005; D=0.01 sec^{-2}

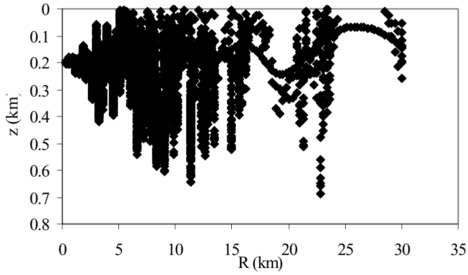

Figure 4. Bifurcation diagram z=f(R) (A=0.005); D=0.1 sec^{-2}

Figures 3 and 4 show bifurcation diagrams for the cases, when Gaussian white noise is included. There are two noise intensities have been examined in the numerical experiment. It is clear that the region of unique solution observed in Figure 2 tends to be replaced with irregular system response with increased noise intensity, D, right from the beginning. As R continues to increase, occasions in which the ray diverges and intersects the ocean surface at z=0 are observed. No white noise system predicts the first occurrence of surface intersection in the region around R=12.5km. In the presence of white noise the first intersection is predicted to take place at R=8.0km, and it becomes much more frequent for $R \geq 10$ km. Finally, for the higher value of the white noise intensity the rays constantly intersect the ocean surface starting at R=5.0km. In order to investigate this behavior in details we plot the phase planes for specific values of the wavelength. We start with phase diagram plotted for the value r=3.0km, which corresponds to the region with the unique solution as it shown in Figure 3.

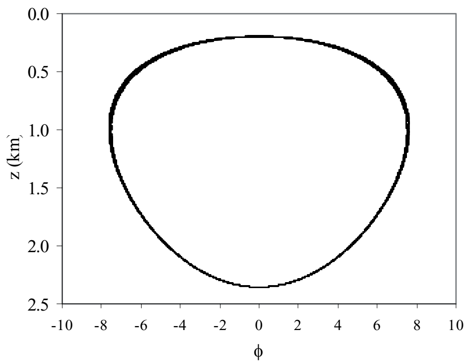

Figure 5. Phase plane; A=0.005, R=3. km; D=0.

Figure 5 presents the wave ray trajectories in the $z - \varphi$ plane, computed for A=0.005. The observation is consistent with the characteristics of a quasi-periodic motion. Quasi-periodicity does not break off or change to a different type of motion.

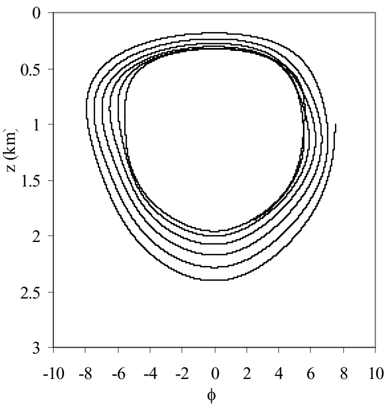

Figure 6. Phase plane: A=0.005, R=12.9 km, D=0.

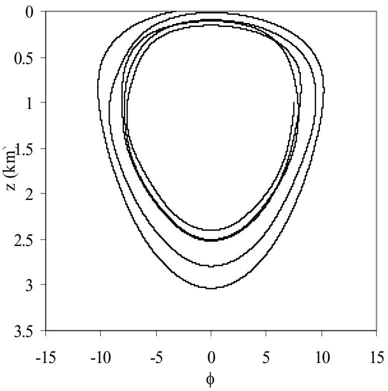

Figure 7. Phase plane: A=0.005, R=12.9 km, D=0.1 sec^{-2}.

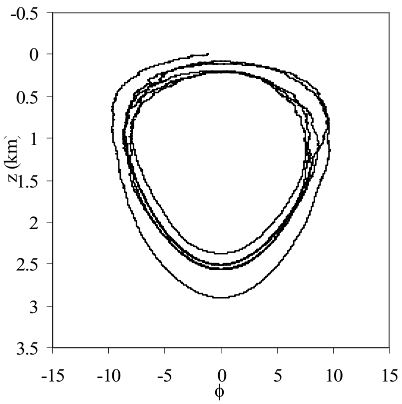

Figure 8. Phase plane: A=0.005, R=12.9 km; D=0.99sec^{-2}

Our observations are in agreement with the conclusion made by Wiercigroch et al., that for an internal wave perturbation of wavelength R=1km and a launch angle 7.5° wave rays remain trapped in the sound channel.

The next we select a value of R=12.9km, i.e., the exact location of a spike in the bifurcation diagram depicted in Figure 2 and plot the phase planes for systems without and with white Gaussian noise respectively. Again, we examine two different white noise intensities. It is seen in Figure 6 that the trajectory loop does not close. The response again reveals the characteristics of quasi-periodic motion. With the increased intensity of added white noise the phase diagrams shown in Figures 7 and 8 exhibit windings and intricate structure, indicating the presence of microcaustics and microfolds; this is not evident from a phase plane obtained as a solution of ray equation that accounts for deterministic excitation only (Simmen et al., 1997).

The second parameter of interest is an amplitude, A. The effect of random excitation on the wave amplitude is examined through the bifurcation diagram $z = f(A)$ for a fixed wave length, R=1km. As it has been done in Wiercigroch et al. (1999), an A range is used out of convenience, A<0.01.

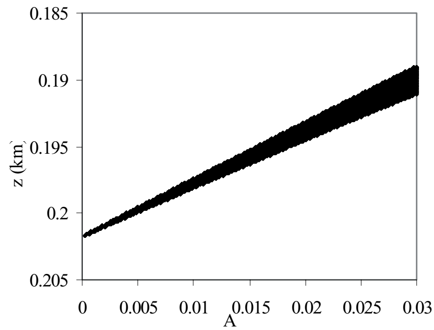

Figure 9. Bifurcation diagram: z=f(A), R=1km, D=0

From the Figure 9 it is seen for the small values of A diagram suggests a stable unique solution, while addition of white nose depicted in Figure 10 for the same wave length results in a sudden jump of amplitude with the following highly unstable behavior.

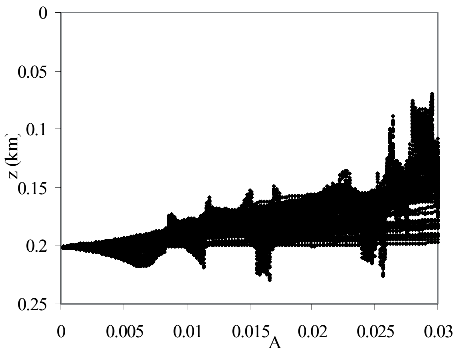

Figure 10. Bifurcation diagram: z=f(A), R=1km, D=0.01sec^{-2}.

4 CONCLUSION

In the present paper the effect of turbulence on chaotic behavior of acoustic rays has been investigated. In order to do that a nonlinear eikonal equation subjected to imperfectly periodic excitation has been considered and analyzed numerically. It has been shown that the results with and without random phase modulation may significantly differ from each other. Namely, imperfect periodicity may significantly reduce propagation range of the ray, forcing it to come to the surface. Moreover, it may develop and/or enhance microcaustics, which significantly complicate identification of signals and its propagation.

An introduced refinement, random phase modulation (white Gaussian noise), into the conventionally viewed nonlinear eikonal equation (perfectly harmonic waves) allowed to formulate more realistic model of underwater sound wave propagation.

Bifurcation diagrams have demonstrated the various regimes of sound ray behavior. The diagrams can be designed for predicting the behavior of ocean sound propagation under various environmental and operational conditions.

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