Analysis of Natural Convection in a Low Gravity Environment

E. E. Mattor, W. W. Durgin and P. Bloznalis
Worcester Polytechnic Institute
Worcester, MA

R. Schoenberg
NASA - JSC
Houston, TX
ANALYSIS OF NATURAL CONVECTION IN A LOW GRAVITY ENVIRONMENT

Ethan Mattor†, William Durgin‡ and Peter Bloznalis†
Worcester Polytechnic Institute
Worcester, MA

Richard Schoenberg
Johnson Space Center
Houston, TX

ABSTRACT

Natural convection inside a spherical container was studied experimentally with two apparatuses at low buoyancy levels. The data generated by these experiments, plotted nondimensionally as the Nusselt number versus Rayleigh numbers, gives correlations for Rayleigh numbers between $10^3$ and $10^8$, a range which was previously untested. These results show that natural convection has significant effects at a Rayleigh number of $10^3$ and higher, although the behavior of the Nusselt number as the conduction limit is approached is still unknown for a spherical geometry.

BACKGROUND:

In-space refueling will become essential for the success of many future missions. The Space Station Freedom and the Great Observatories will be strongly dependent upon resupply of consumables and propellants. Refueling systems that will safely, efficiently and reliably deliver these fluids to a wide variety of vehicles will be crucial.¹

When fluid is transferred in a 1-g environment the trapped gas is usually vented to prevent pressure buildup. It is more difficult to vent in space, because of uncertainty of the location of the gas. The vapors are often corrosive, and could damage sensitive, exposed surfaces on the space vehicle when vented. Various schemes for transferring fluids using the no-vent method have been proposed by Griffin¹, including ullage recompression, ullage vent/pressurization, and ullage exchange. Of these three methods ullage recompression is by far the simplest mechanically, posing much less of a possibility of mechanical failure during service. This research focuses on heat transfer in low gravity as the limiting physical mechanism for the method of ullage recompression.

Ullage Recompression

The ullage recompression method has several stages during the transfer. When the liquid first enters the evacuated receiving tank it flashes, that is, turns to vapor. The pressure inside the receiving tank rises until the point of saturation is reached. The process enters the second stage when the transfer fluid starts entering the receiving tank as a liquid. If the fluid is wetting, as is most often the case, a gaseous cavity will form which will tend to be spherical due to surface tension. As the liquid fills the tank the vapor is compressed, causing it to become superheated. Once an equilibrium is reached between the receiving and source tank pressures, the transfer enters the third stage. The superheated vapor slowly cools, transferring heat to the surrounding fluid, and liquid condenses out of the vapor phase.

† Member AIAA
‡ Professor of Mechanical Engineering, Member AIAA

Copyright © 1992 American Institute of Aeronautics and Astronautics, Inc. All rights reserved.
transfer is complete when the gas is completely condensed. There are two surface transfer parameters which affect the fluid transfer rate; the mass transfer coefficient, and the heat transfer coefficient; exactly what these effect will be is unknown. This research effort focuses on the heat transfer coefficient. Chandler used a conduction model because it is the limiting case for this heat transfer, but it is believed that convection, driven by g-jitter and residual filling flows, will probably have a strong additional effect.

In another method of transfer fluid, the fluid is driven with compressed gas, which is separated from the transfer fluid by a flexible membrane. The heat transfer across the membrane, which is spherical during the majority of the transfer, strongly affects the fluid transfer rate. As can be seen, convection heat transfer across spherical interfaces is common to most fluid transfer setups in space.

Natural convection in a spherical cavity has been studied extensively by computer model, but has not been examined as well experimentally. The effect of a low gravity environment adds greater complexity to the problem, because of the chaotic interplay of conduction and convection at these levels of buoyancy. Experimental data on this process would greatly aid designers of ullage recompression systems for use in space.

Previous Research

The first experimental data for heat transfer in a low gravity environment was obtained from liquid oxygen tanks aboard the Apollo missions. Electrical heating elements used to boil the liquid were cylindrically shaped. The amount of energy added to the liquid, and the pressure inside the tank were used to calculate the heat transfer coefficients from the heating elements. Even though the gravitation level was on the order of $10^{-6}g$, there was a significant amount of convective heat transfer from the cylinders.

A significant amount of work has been focused on studying transient natural heat transfer from spherical cavities. In 1958, Pustovoit took an analytical approach for studying natural convection inside spherical cavities. In his analysis, the walls of a spherical cavity undergo a step temperature change, and the resulting convective flows were examined. His results match more recent analyses well. In 1972, H. G. Whitley and Vachon studied transient laminar free convection in closed spherical containers using finite difference numerical techniques. Ozoe, Kuriyama and Takami reported results similar to Whitley and Vachon's, albeit more extensive. Although both of these numerical computations demonstrate trends expected from the results of experimentation, neither problem corresponds to the present study of low Rayleigh number convection. Also, the results of the above studies were typically presented as temperature and velocity fields, whereas, experimental studies focus on the overall heat transfer due to the difficulty of measuring the local velocity and temperatures. None of these studies presented overall heat transfer coefficients, and therefore could not be easily compared directly with experimental studies.

Schmidt experimentally studied natural convection with a variety of liquids inside of a sphere. Chow and Atkins experimentally studied pseudosteady-state natural convection inside a sphere by maintaining a constant temperature difference between the bulk temperature of the interior gas and the walls of the container. These experimental studies both used liquids for the contained fluids, and so that the Rayleigh number was significantly high that of interest here.

The experimental studies conducted by Schmidt, and Chow and Atkins give a correlation of the Nusselt number to the Rayleigh number for Rayleigh numbers ranging from $10^6$ to $10^{12}$. Data gathered from the heating of the oxygen tanks aboard the Apollo and Skylab missions show that significant convective heat transfer can be generated at Rayleigh numbers as low as 100, corresponding to gravity levels of $10^{-6}g$. It is the purpose of this study to extend the Nusselt vs. Rayleigh number correlation to Rayleigh numbers which are as low as possible.
Experimental Work

A heat transfer correlation for free convection at spherical surfaces in a low gravity environment is needed for accurate prediction of orbital fluid transfer rates. The first experimental apparatus was a balloon filled with gas surrounded by water, Figure 1. The piston was actuated, reducing the volume of the system and heating the gas inside the balloon. The pressure of the gas inside the balloon was recorded, which could be used to deduce the temperature using a lumped heat capacity analysis.

The second apparatus, presently being prepared for KC-135 Low Gravity Aircraft flight, consists of a pneumatic cylinder to compress the gas, and a spherical test chamber. The spherical material was steel, providing an adequate heat sink such that the wall temperature is essentially constant during testing. A high pressure (100 PSI) air system, consisting of a solenoid and a regulating valve, is used to actuate the piston in a controlled, repeatable manner, Figure 2. A data logging computer is used to record the data from the various instruments, as well as control the experiment.

An experiment consisted of three steps; first, the piston is actuated, compressing the gas; second, the computer records data as the gas came to equilibrium; and last, the piston is brought to the starting position again. The experimentally generated data was then fit to an exponential curve to smooth out the steps caused by the AD conversion, random pressure fluctuations and instrument noise. The computed curve fits the experimental data with a high degree of accuracy in that the sum of the squares of the residuals is about $10^{-2}$. The curve fit generates three constants; an asymptote, a reference temperature, and a time constant. These constants are then used to calculate Nusselt and Rayleigh numbers. By varying the piston stroke length (hence changing the density of the contained gas) and by changing the gas, a large range of test conditions can be tested.

Data Processing

The objective of this work is to determine the heat transfer inside a container based on different levels of buoyancy. In order to correlate data into the most useful and generalized form, it is desirable to formulate dimensionless parameters from the experimental data. Two dimensionless parameters must be chosen; one for the ordinate describing buoyant forces in the fluid, and one for the abscissa which describes the quantity of heat transfer and is solely a function of the ordinate. The traditional choice of these two parameters are the Rayleigh and Nusselt numbers. The Rayleigh number is descriptive of the buoyancy of a fluid, and the Nusselt
number is the ratio of convection to conduction heat transfer. It can be seen that by plotting experimental data as Rayleigh versus Nusselt number, the result will describe the convective heat transfer as a function of the buoyancy.

First, using fundamental thermodynamic arguments, a general curve is found to which the experimentally found temperature history can be fit. It is then shown how the average Nusselt number is calculated from the experimental data generated from both experimental setups, based on the temperature difference of the gas and the sphere wall. Finally, the formulation of the Rayleigh number, based on the slope of the temperature history, is shown.

The first law of thermodynamics states that for a closed system,

\[ \frac{dU}{dt} = \frac{dQ}{dt} - \frac{dW}{dt} \tag{1} \]

The work term is zero after the piston has completed the compression of the gas. The rate of change of the internal energy is equal to the heat loss, given by,

\[ \frac{dQ}{dt} = m c_p \frac{dT_{\text{bulk}}}{dt} \tag{2} \]

The convective heat loss is given by Newton's law of cooling,

\[ \frac{dQ}{dt} = h A_w (T_{\text{bulk}} - T_w) \tag{3} \]

where
- \( h \) = average heat transfer coefficient
- \( A_w \) = Surface area of the wall
- \( T_{\text{bulk}} \) = Gas characteristic temperature
- \( T_w \) = Wall temperature

Combining equations (1), (2), and (3) and solving for the temperature of the gas yields the following exponential function,

where
- \( T_i \) = Initial Temperature of the gas

\[ T_{\text{bulk}} = T_a + (T_i - T_a) e^{-\frac{t}{\tau}}, \quad \tau = \frac{c_p m}{(UA)} \tag{4} \]

\[ t = \text{Time} \]
\[ \tau = \text{Time constant of heat transfer} \]

Equation (4) shows that the temperature history is expected to be an exponential decay, assuming that the heat transfer constant, \( h \), is constant for the portion of data being analyzed. Equation (4) is the mathematical model to which the experimentally generated data is fit. The Nusselt number is given by,

\[ Nu = \frac{h d}{k} \tag{5} \]

where
- \( d \) = diameter of the sphere
- \( k \) = Conductivity of the gas.

The balloon experimental setup has convective heat transfer in both the contained gas, and in the water. The temperature of the gas - water interface varied as the gas cooled, changing the reference wall temperature. Also, complicating matters, was the fact that the overall heat transfer coefficient given in equation (4) is comprised of both the water and gas coefficients;

\[ U = \frac{h_i h_s}{h_i + h_s} \tag{6} \]

It can be shown\(^{11}\) using an energy balance of the gas - water interface, that the balloon wall temperature and interior heat transfer coefficient is given by;

\[ T_{\text{wall}} = \frac{T_{\text{gas}} + K T_{\text{liq}}}{K + 1} \tag{7} \]

\[ h_i = \frac{U K}{K + 1} \tag{8} \]
Where
\[ U = \text{the overall heat transfer coefficient} \]
\[ K = \text{the ratio of the liquid to gas conductivities} \]

When eq. (7) is substituted into the definition for the Nusselt number, eq. (5), the following relationship for the Nusselt number is obtained;
\[ Nu = \frac{U d (k_{\text{liq}} + k_{\text{gas}})}{k_{\text{liq}} k_{\text{gas}}} \] (9)

which can be used to calculate the Nusselt number from the experimentally determined time constant of the cooling. Note that, because the heat transfer coefficient is assumed to be roughly constant during an experimental run, the Nusselt number is also constant.

![Nusselt Number vs. Temperature](image)

Figure 3 - Free convection limit

In the second experimental setup, the steel sphere is assumed to conduct the heat away from the surface quickly enough so that the wall temperature could be modeled as constant. The analysis of these data is complicated by residual velocities induced by the piston stroke, however.

The heat transfer coefficient is a function of both the forced convection and the free convection. Because the residual velocity from the piston stroke dissipates, the Nusselt number will change as the gas cools. In order to find the natural heat transfer coefficient the forced convection was separated from the natural convection. Although the velocity induced from natural convection is unknown, the natural heat transfer coefficient, as found by the first apparatus, can be used to show that these velocities are negligible. A Nusselt number of 58.7 is predicted for a test run in which the steel sphere apparatus runs with a full stroke length, a compression ratio of 1.5.

In order to verify that the residual velocity decays to a nominal value, data were collected from the experiments and divided into short segments for analysis. The Nusselt number was found for each segment by fitting the data to equation (4). Figure 3 is a plot of the Nusselt number versus the temperature as the sphere cools for a variety of stroke times. As can be seen, the Nusselt number approaches the natural heat transfer limit predicted by the first experiment as the temperature approaches the ambient value. This would suggest that the natural heat transfer coefficient can be found by extrapolating to a zero temperature difference. Thus, the extrapolated value is, indeed, that due to natural convection only.

The definition of the Rayleigh number is given by;
\[ Ra = \frac{\rho^2 c_p g \beta (T_{\text{gas}} - T_w) d^3}{\mu k} \] (10)

where;
\[ \rho = \text{density of gas} \]
\[ g = \text{gravitational acceleration} \]
\[ c_p = \text{heat capacity} \]
\[ \beta = \text{Thermal coefficient of expansion} \]
\[ T_w = \text{wall temperature} \]
\[ \mu = \text{viscosity of gas} \]
\[ k = \text{conductivity of the gas} \]

It can be seen that the Rayleigh number is dependent on the temperature, which changes during an experimental run. For each sampling of data, an average Rayleigh number for the gas is calculated by taking the mean of the maximum, at the end of the piston stroke, and the minimum, at the end of the run. Noting that the final temperature difference is zero;
**Experimental Results:**

The experiments were run with both air and helium as test media. A range of compression ratios were used, thus changing the Rayleigh number via the density. A variety of different Rayleigh numbers were tested for each gas. The Nusselt number was then plotted as a function of the Rayleigh number. First the results from the balloon experiment is shown, then from the steel sphere apparatus.

The data from the apparatuses was plotted on logarithmic axes, and a best straight line fit was plotted using linear regression analysis. Using data obtained from the first apparatus yielded the following relation for convection inside a sphere;

\[
Nu_d = 1.158 \left( Ra_d \right)^{0.210}
\]

with a standard deviation of 0.0482. A plot of the data is shown in figure 4.

\[
Nu = 1.3127 \left( Ra_d \right)^{0.206} \quad \text{for } 10^6 < Ra_d < 10^8
\]  

A plot of both the correlations found by this work, along with results found by previous researchers, can be found in figure 6.

The data obtained from the second apparatus corresponds to a higher range of Rayleigh numbers, due to the larger sphere size. These data shows an only
Conclusions and Recommendations:

The correlation found by Schmidt for heat transfer inside a sphere, which was valid for Rayleigh numbers from $10^{12}$ to $10^8$ has been extended for Rayleigh numbers ranging from $10^8$ to $10^3$. The correlation found by this work is compared to that of previous experimental work, namely the low gravity, Schmidt, and Chow and Atkins, Figure 6. The published analytical and numerical results are not shown because the data presented in these works show the temperature gradients, but are not correlated into overall heat transfer coefficients.

These results compare favorably with the work performed by Chow and Atkins in 1975. The difference in the above expressions and the one found by Chow and Atkins can be attributed to the transition from laminar to turbulent flow and differences in the experimental setup.

Although the Nusselt to Rayleigh number correlation has been found for considerable lower Rayleigh numbers, the lower limit of the Rayleigh number for free convection is still unknown. Low gravity data for convection on the exterior of cylinders suggest that the lower limit is around 100, but the geometry of containers plays an important role in low Rayleigh number convective flows. More extensive studies must be conducted in order to investigate natural convection inside spheres as the buoyancy approaches zero due to a reduction in gravitational acceleration. These tests can be performed with the steel sphere apparatus onboard a low gravity test facility, such as NASA's KC-135 low gravity test aircraft.

References


8. Schmidt, Ernest, "Versuche zum Wärmeübergang bei natürlicher Konvektion," Chemie-Ing.-Techn., 28 Jahrg. 1956, Nr. 3.1

