A Sheaf Theoretic Approach to Consciousness

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Abstract

A new fundamental mathematical model of consciousness based on category theory is presented. The model is based on two philosophical - theological assumptions: a) the universe is a sea of consciousness, and b) time is multi-dimensional and non-linear.

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1.0 Introduction

The purpose of this paper is to propose a fundamentally new approach to consciousness and the problem of deep reality. Our approach will provide a mathematical model based on category theory (we will specifically use the category of presheaves as model for a conscious universe), which appears to be very promising. On one hand, category theory (originally developed in the fifties by Eilenberg and Mac Lane for purely algebraic reasons) is abstract enough to allow treatment of a fundamentally abstract phenomenon such as consciousness. On the other , category theory is now so advanced and fully developed, that it provides a theory rich enough for the complexity of the problem at hand.

Given the preliminary nature of this first announcement, we have decided against providing a full background on the mathematical notions we will use; rather, we would like to refer the reader to the fundamental treatise of Mac Lane [5] as far as categories are discussed, and to the introduction of Bredon [1], for the necessary notions from sheaf theory.

As we said, this article proposes a model which is based on a very specific world-view; the model and its implications will need much refinement, verification, and discussion before its full strength can be established. Nevertheless, we have decided to present it here in the hope that it will be sufficiently controversial to generate insightful discussion.

The second author wishes to express his gratitude to his colleague Professor Kafatos for the many illuminating discussions, and for sharing an early copy of [2], which further stimulated his thinking. The proposal presented here should not be seen in opposition to [2], but rather as a complement and it is our hope that our formalism may in the future be used to support the ideas put forward in [2].

2.0 The Model

Our model is based on two fundamental philosophical-theological assumptions:

a) the universe must be interpreted as a sea of consciousness,

b) time is multi-dimensional and non-linear

Both assumptions are consistent with Buddhist world views [6], [7], and with many recent interpretations of quantum

physics [3], but we still wish to briefly discuss what we have in mind. Assumption a) is essentially a conscious universe view, and rejects the existence of individual conscious entities as we usually perceive them (though our model will provide an explanation for their perception); rather than agreeing to the existence of such individual entities, we postulate the essential unity of all consciousness out of which individual conscious entities appear and disappear (consistently with a fundamental principle of impermanence of all things). Assumption b), on the other hand, is refuting the existence of time as we perceive it (though, once again, our model will justify such perception) and postulates a time in which there is no past, present, future, but only a collection of presents, all equivalent to each other.

In order to mathematically describe such a model, we consider a topological space T of arbitrarily large dimension (or, possibly, of infinite dimension), which we call "time" and we consider the category U, for universe, of presheaves on T. The most appropriate way to regard a presheaf is to consider it as a contravariant functor from the category of opens sets in T to a target category C to be specified at a later stage. In our interpretation, the category U represents the "sea of consciousness", its objects represent the individual conscious entities, while its arrows, or maps, represent communications between such conscious entities. It is appropriate to think of inanimate objects as particularly "poor" presheaves. For example, a single particle in a stable state could be represented by a constant sheaf whose stalk is the trivial object in C (either defined on all of T, or only on a subset of T, if we want to consider its existence as confined to a subset of time). Similarly, a particle with two possible states (say spin $\pm$ 1) could be represented by a constant sheaf whose stalk is Z_2 if C is, for example, the category of abelian groups. On the other hand of the spectrum, complex conscious entities (such as a "brain") could be represented by very "rich" and complex presheaves. The interpretation of brains like presheaves or sheaves is particularly interesting and lends itself to an abstract discussion of the notion of "thoughts" and "understanding" in terms of sections and restriction maps, [4].

In order to understand the apparent paradox of the perception of the everyday reality (which we can call "shallow" reality, in contrast with the "deep" reality discussed in [2]), we can postulate an injective map

$$i : IR \to T.$$

If we now consider the categories U_T and U_{IR} of presheaves on T and IR respectively, we can define a functor

$$S : U_T \to U_{IR},$$

which we shall call the "shallow" functor, by taking the pull-back of every object in U_T So, if $P \in Ob(U_T)$, we define

$$S(P) := i^*(P) \in Ob(U_{IR}),$$

and we interpret $S(P)$ as the perception of P in the "shallow" reality; note that since $S(P)$ is a presheaf on IR, it is not surprising that all time is perceived linearly in the shallow reality.

Other apparent paradoxes of modern physics can be easily understood within this framework. The famous non-locality Aspect experiment described, for example, in [3], can be interpreted by considering the two photons as two constant sheaves P_1 and P_2 with stalk Z_2, and regarding the experiment as simply the map $f : P_1 \to P_2$ defined as the identity on each stalk. The existence of such map can be interpreted as the possibility of performing the experiment, which shows why one particle "learns" instantaneously from the other.

Let us conclude this short note by pointing out a few of the issues which the authors are exploring. We have mentioned the interpretations which sections and restrictions have within this model. Even more intriguing (for its philosophical and theological interpretation) is the discussion of initial and terminal objects in U. When information is transmitted between two or more objects, it may be worthwhile to eliminate insignificant or redundant information; such a process can be accomplished by introducing a cohomology theory for sequences (not necessarily complexes) in U. Such a theory has been developed independently by the first author and finds here an interesting application. Pushing even further, we will develop a theory for the derived categories which arise in this context. Finally, the authors will propose other interesting functors which, as the shallow functor, are susceptible of relevant interpretations. Of particular interest is the problem of analyzing the meaning of autofunctors (i.e. functors from U to itself).

References

[1] S. Bredon, Sheaf Theory, Springer Verlag.

[2] M. Draganescu and M. Kafatos, manuscript (in this volume), 1999.
[3] M. Kafatos and R. Nadeau, The Conscious Universe, Springer Verlag, 1990.
[4] G. Kato and D.C. Struppa, The brain as a presheaf, in preparation.
[5] S. Mac Lane, Categories for the Working Mathematician, Springer Verlag, 1971.
[6] Oka, The Heart of Japan (Nihon no Kokoro), translation to appear in Harusame Publishing Company.
[7] D.T. Suzuki, The Zen Doctrine of No Mind, S. Weiser, 1991.