Thinking Outside the Box:
New approaches to very large flexible diaphragms

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Abstract

This paper discusses the past design practices of flexible diaphragms, the current trend to very large diaphragms, and the new tools available to properly evaluate and engineer these large roof systems. Higher shear demands are accommodated by higher plywood/OSB and steel deck shear capacities accepted by the IBC. A new collective chord technique is presented to more accurately reflect large diaphragm behavior. In addition, these very large diaphragms have significant lateral deflection issues that can now be evaluated in the IBC. The new AF&PA deflection equations for wood diaphragms are presented as well as a technique to compute deflections of diaphragms with collective chords or multi-zoned nailing.

Introduction

California's ports account for over 40% of the cargo flowing into the United States. With the ever-increasing demand for Asian imports, larger and larger distribution facilities are being built here in seismically active California to distribute these goods across the Country. Tilt-up buildings in excess of one million square feet are becoming commonplace, and the design of these very large flexible diaphragms is presenting new engineering challenges.

Higher diaphragm shear capacities are now available in both wood and steel deck roof systems. The collective chord approach provides reduced chord forces while more accurately modeling diaphragm behavior. More accurate diaphragm deflection calculations are now possible, as engineers try to mitigate their impact.

1. Special High Diaphragm Shear Capacities:

In California, the hybrid panelized roof is the most common large flat roof system being used today. This consists of wood structural-use panels such as plywood or oriented strand board (OSB) nailed to wood nailers factory installed to the top chord of open-web steel joists. Current trends are for larger buildings with more clear-space and taller clear-heights to facilitate state-of-the-art warehousing and distribution. Wood roof diaphragms are being required to span farther horizontally with higher shear stresses.

Since the 1967 UBC, allowable horizontal diaphragm shears for wood sheathing have remained essentially the same; with a maximum tabular shear capacity topping out at 820 plf (½" Struct I plywood with 10d nailing at 2", 3", 12"). These shear capacities are an outgrowth of testing conducted beginning in 1952 and concluding in 1966 (Tissell, 2000). The traditional UBC table of allowable shears is currently incorporated in the 2006 IBC Table 2306.3.1 with no significant changes.

Over the past couple decades though; engineers of very large wood diaphragms have had another option available for justifying very high allowable shear capacities. Sponsored by the American Plywood Association, the APA funded ICBO Evaluation Service document #1952 permiting allowable shear capacities up to 1800 plf. These very high shear capacities are achieved with ¾" plywood, multiple lines of nails, on 3x and 4x framing, with special inspection. ER-1952 is a direct result of full-scale diaphragm testing by APA, whose primary purpose was to provide higher diaphragm shear capacities in response to the higher demands of the added Seismic Zone 4 into the Building Code. Prior testing was always controlled by nail failure; these tests investigated diaphragm strengths governed by the shear strength of the plywood material. The results and analysis of the testing program are published in APA Research Report 138, (Tissell, 2000). The use of ER-1952 has been widely accepted in large tilt-up and masonry buildings (ICC ER-1952, 2006).

An important recent development has been the incorporation of APA's ER-1952 directly into the IBC. The 2006 IBC contains Table 2306.3.2 that provides direct code approval of these special high shear capacities for all structural-use panels. Originally, only APA stamped ¾" thick plywood products were specifically approved unless a separate justification was used with principles of mechanics. IBC Section 2306.3.1 still allows the calculation of high shears using principles of mechanics, but IBC Table 2306.3.2 has removed much of the need for this separate analysis.

IBC Table 2306.3.2 is applicable to all approved wood structural-use panels conforming to product standards PS-1 (plywood) and PS-2 (OSB, etc.). In addition, high shear values are recognized for 15/32", 19/32", and 23/32" panel thicknesses. This provides broad product approval that better matches the construction materials available today.

Allowable capacities for ½" nominal sheathing are up to nearly 1400 plf, with 5/8" and ¾" nominal up to nearly 1800 plf. In addition, when evaluating wind loads a 40% allowable load increase is permitted. These high values incorporate a factor of safety of 2.8 and are based on a European yield method of analysis, which was confirmed in experimental tests (IBC, 2006, p. 23-55).

One important requirement of using these special high shear capacity diaphragms is the need for special inspection. This special inspection requirement has been necessary since the original ICBO ER-1952 report was first introduced. A lingering problem the industry faces is that many special inspectors do not have the experience or a specific certification with this type of inspection. If some doubt exists as to the competence of the Special Inspector, a preconstruction meeting may be held to clarify the inspection issues with high-load diaphragms.

The requirements for special inspection of high-load diaphragms are discussed in IBC 1704.6.1. The Special Inspector shall verify the wood sheathing's grade and thickness, nail size, nail spacing, and nominal lumber size at the adjoining panel edges. Because most of this is visible after installation, continuous inspection is not required, but rather periodic inspection may be provided (Skaggs, 2006).

A critical element of the high-load diaphragm is the tight nail spacing necessary to achieve high shears. Under the provisions of ICBO ER-1952, the nailing layout is presented graphically showing the necessary staggered spacing and multiple lines offsets. As in all wood diaphragms, closely spaced nails that align with the wood grain could cause wood splitting that compromises the nail's gripping strength. The use of a staggered nailing pattern and wider framing members minimizes the risk of lumber splitting due to tight nail spacings. The 2006 IBC Code Commentary has a useful depiction on this subject (IBC, 2006).

Unfortunately, the incorporation of the high-load diaphragm into the IBC inadvertently omitted the specific mention of staggering nails or minimum spacings between nail lines. APA is currently developing an IBC code errata and/or revision to add the nail spacing diagram from ER-1952 into the IBC (Keith, 2007). Be sure to incorporate into your design drawings requirements to staggered nailing similar to the current ER-1952 (ICC ER-1952, 2006). The minimum spacing between individual lines of nails is 3/8" as measured from center of nail shank to center of nail shank (Skaggs, 2007).

Lastly, IBC 2305.1 recognizes the American Wood Council's publication *Special Design Provisions for Wind and Seismic* (AF&PA, 2005) as an approved alternative publication to the IBC provisions for wood shear walls and diaphragms. This publication provides diaphragm shear values to use under either the LRFD or ASD approaches; however there are no specific tables incorporating the special high shear diaphragm values.

A brief discussion on steel deck: Although less common California's tilt-up and masonry wall buildings, the use of steel decking as the roof diaphragm is gaining some momentum. Steel deck diaphragms are capable of higher shears than wood diaphragms, but concerns of thermal expansion often limit the diaphragm width by introducing thermal expansion joints. Thermal expansion joints can be problematic with the competing needs of good seismic design using continuous crossties. Never the less, steel deck diaphragms are capable of reaching allowable shear values over 3000 plf utilizing heavy steel gauge and special attachments. Recent innovations such as Verco's *PunchLok* (ICC ER-2078P, 2002) and ASC Profiles *DeltaGrip* (ICC ESR-1414, 2006) have increase diaphragm shear capacities while eliminating costly seam welding.

2. Chord Design:

Chords are required to carry the tension forces developed by the flexural bending moments in the diaphragm under lateral load. In a girder analogy, the chords represent the girder flanges, providing bending strength and stiffness. Chords have been traditionally located at the diaphragm perimeter; however, today's larger buildings result in some very challenging chord forces. A new approach presented here considers multiple chord elements distributed across the diaphragm.

2a. Chord Design: Traditional Approach

Originally, masonry and tilt-up buildings used special reinforcing steel embedded in the walls as the diaphragm chord near the roofline. In the last decade or so, steel ledgers have replace wood ledgers and also have doubled as diaphragm chords. In tilt-up buildings, the chord steel is spliced at the panel joints to provide a continuous chord tie.

In the 1960s and 1970s, two #5 rebar often were all that were necessary to develop calculated chord forces. Since then, code seismic forces have increased significantly, and building sizes have increase dramatically creating new challenges to chord design.

Traditionally, diaphragm chords are thought of as Tension/Compression force couples at the diaphragm extremes. The following model is of a masonry or tilt-up building with a flat roof structure.

Traditional Chord Design Derivation:

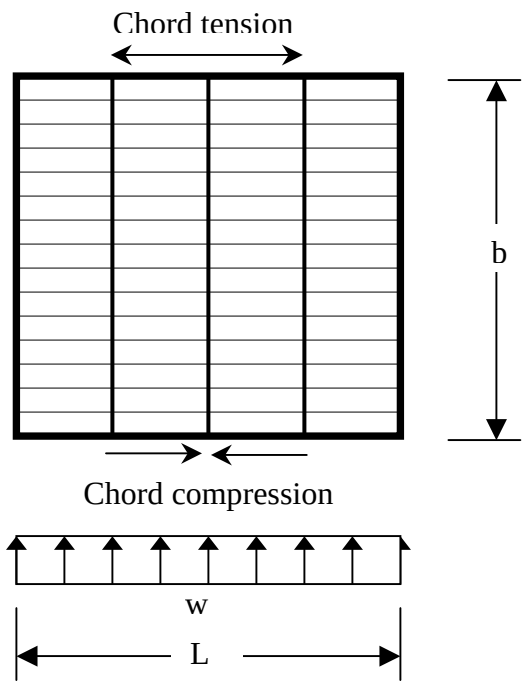

Figure 1: Traditional Chord Forces

w = Diaphragm load (lbs/foot)
L = Diaphragm length (feet)
b = Diaphragm width (feet)
M_{Diaph} = Diaphragm bending moment (ft-kips)
$T = C$ = Chord force couple (kips)

$$M_{Diaph} = \frac{wL^2}{8} \qquad \text{EQ. 2-1}$$

$$T = C = \frac{M_{Diaph}}{b} \qquad \text{EQ. 2-2}$$

2b. New Collective Chord Approach

Today's state-of-the-art distribution/warehouse building is a minimum of 30-foot clear under structure, with 32 and 40-foot clearances becoming routine. Besides the height increase, plan areas of large distribution buildings between 500,000 and one-million square feet are common now. The author's office is currently working on a two-million square foot facility in San Bernardino that pushes the limit of conventional engineering practices. This drive for larger and larger building volumes is challenging traditional chord design methods. In addition, traditional chord design in today's larger buildings is not accurately representing the actual distribution of forces.

In these very large box buildings, the actually behavior is different than would be assumed by an isolated piece of chord steel at the diaphragm extremes. In panelized roof systems, purlins/joists are repetitively spaced at 8-feet on-center and provide perimeter wall anchorage. These purlins/joists are normally well connected across the building's width, serving also as continuity ties as required by ASCE 7-05 Section 12.11.2.2.1. These continuity ties distribute wall anchorage loads into the main diaphragm. Since the 1990's, these purlins/joists have commonly consisted of open-web steel joists with welded top-chord splices across each girder line. For diaphragm chord design, these purlin/joist continuity ties have been traditionally ignored; but in reality, these continuity ties can provide a significant amount of tensile strength to a large diaphragm in bending.

To analyze these forces introduced into these multiple or *collective chord* elements, strain compatibility must be utilized instead of a simplistic tension/compression force couple. Assuming the behavior as linearly elastic, Figure 2 depicts a diaphragm plan view illustrating the force distribution.

Instead of solely relying upon the diaphragm extremes for bending resistance, the continuous elements of the diaphragm's interior are acting as well. In the past, the added complexity necessary to do this analysis was not worth the effort with the small chord loads frequently encountered. However, in today's very large diaphragms the forces are much larger. In addition, the steel continuity ties provide a well-connected diaphragm capable of collectively resisting diaphragm chord forces.

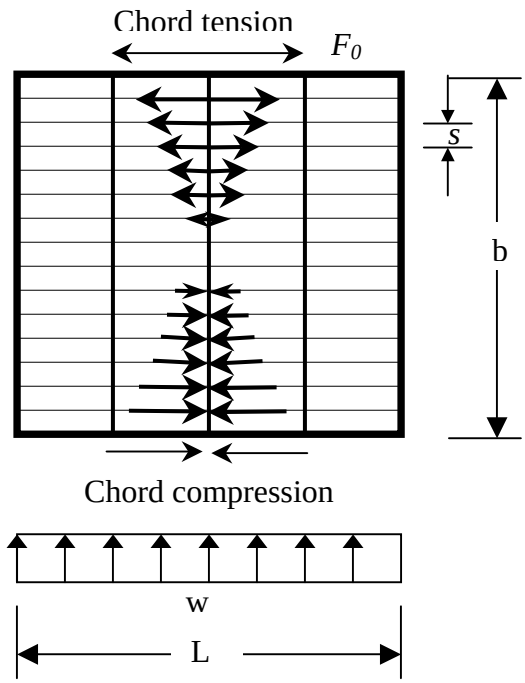

Figure 2: Collective Chord Force Distribution

Collective Chord Design Derivation:

w	= Diaphragm load (lbs/foot)
L	= Diaphragm length (feet)
b	= Diaphragm width (feet)
M_{Diaph}	= Diaphragm bending Moment (ft-kips)
s	= Continuity tie spacing
F_x	= Force in continuity tie "x"
F_0	= Force in extreme continuity tie

Each continuity tie has a different chord force proportional to its distance from the bending neutral axis. In order to simplify the equations later, the various continuity tie forces F_x can be expressed in terms of the extreme continuity tie's force F_0.

For example, the first tie in from the diaphragm's extreme tie has a force F_1:

$$F_1 = F_0 \frac{b/2 - s}{b/2} \qquad \text{EQ. 2-3}$$

For all ties, F_x is as follows:

$$F_x = F_0 \frac{b/2 - sx}{b/2} \qquad \text{EQ. 2-4}$$

Equation (2-4) defines the force in each chord element relative to the extreme element. As can be seen within equation (2-4), when $sx > b/2$, the collective chord forces go negative crossing the neutral axis. It is this point that the chord forces go from tensile to compressive.

Simplifying equation (2-4):

$$F_x = F_0\left(1 - \frac{2sx}{b}\right) \qquad \text{EQ. 2-5}$$

Next, an expression that computes the diaphragm's bending moment in terms of the multiple chord element forces is needed. Figure 3 graphically illustrates the distribution and labeling of collective chord forces.

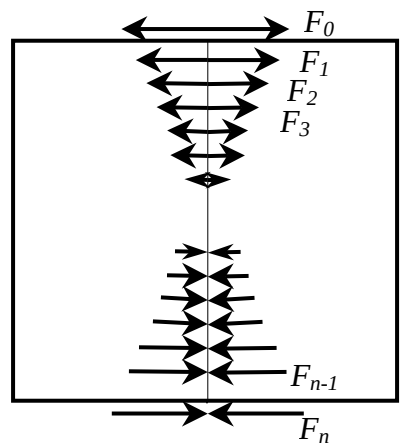

Figure 3: Collective Chord Force Series

Using statics, the following simple series is developed:

$$M_{Diaph} = F_0 b + F_1(b - s) + F_2(b - 2s) + F_3(b - 3s) \ldots + F_n(b - ns) \qquad \text{EQ. 2-6}$$

This may be also expressed simply as a summation:

$$M_{Diaph} = \sum_{x=0}^{n} F_x(b - sx) \quad where \ldots n \approx b/s \qquad \text{EQ. 2-7}$$

Substituting equation (2-5) into equation (2-7) and simplifying:

$$M_{Diaph} = \sum_{x=0}^{n} F_0 \left(1 - \frac{2sx}{b}\right)(b - sx)$$

$$M_{Diaph} = F_0 \sum_{x=0}^{n} \left(1 - \frac{2sx}{b}\right)(b - sx)$$

$$M_{Diaph} = F_0 \sum_{x=0}^{n} \left(b - sx - 2sx + \frac{2s^2x^2}{b}\right)$$

$$M_{Diaph} = \frac{F_0}{b} \sum_{x=0}^{n} \left(b^2 - 3bsx + 2s^2x^2\right)$$

$$M_{Diaph} = \frac{F_0}{b} \sum_{x=0}^{n} \left(2s^2x^2 - 3bsx + b^2\right)$$

Solving for F_0:

$$F_0 = \frac{M_{Diaph} b}{\sum_{x=0}^{n} \left(2s^2x^2 - 3bsx + b^2\right)} \qquad \text{EQ. 2-8}$$

where… $n \approx b/s$

The only variable in equation (2-8) is x, and F_0 can be solved by using a simple summation in the denominator. Using some additional math, the summation in the denominator can be simplified further for a direct solution.

Simplify the denominator:

$$\sum_{x=0}^{n} \left(2s^2x^2 - 3bsx + b^2\right) \qquad \textit{where…}\ n \approx b/s$$

$$= b^2 + \sum_{x=1}^{n} \left(2s^2x^2 - 3bsx + b^2\right)$$

$$= b^2 + \sum_{x=1}^{n} 2s^2x^2 - \sum_{x=1}^{n} 3bsx + \sum_{x=1}^{n} b^2$$

$$= b^2 + 2s^2 \sum_{x=1}^{n} x^2 - 3bs \sum_{x=1}^{n} x + b^2 \sum_{x=1}^{n} 1$$

$$= b^2 + 2s^2\left(\frac{n(n+1)(2n+1)}{6}\right) - 3bs\left(\frac{n(n+1)}{2}\right) + b^2 n$$

$$= 2s^2\left(\frac{(n^2+n)(2n+1)}{6}\right) - 3bs\left(\frac{n^2+n}{2}\right) + b^2(n+1)$$

$$= \frac{s^2}{3}\left(2n^3 + n^2 + 2n^2 + n\right) - \frac{3bs}{2}\left(n^2 + n\right) + b^2(n+1)$$

$$= \frac{2}{3}s^2n^3 + s^2n^2 + \frac{1}{3}s^2n - \frac{3}{2}bsn^2 - \frac{3}{2}bsn + b^2n + b^2$$

The variable *n* represents the number of purlin or joist bays evenly spaced across the diaphragm depth. This may also be expressed as the diaphragm depth divided by the purlin/joist spacing *n = b/s*. Substituting *b/s* for *n*, our denominator equation can be further simplified as follows:

$$= \frac{2s^2b^3}{3s^3} + \frac{s^2b^2}{s^2} + \frac{s^2b}{3s} - \frac{3bsb^2}{2s^2} - \frac{3bsb}{2s} + \frac{b^2b}{s} + b^2$$

$$= \frac{2}{3}\frac{b^3}{s} + b^2 + \frac{1}{3}bs - \frac{3}{2}\frac{b^3}{s} - \frac{3}{2}b^2 + \frac{b^3}{s} + b^2$$

$$= \frac{b^3}{6s} + \frac{b^2}{2} + \frac{bs}{3}$$

$$= \frac{b}{6s}\left(b^2 + 3bs + 2s^2\right)$$

$$= \frac{b}{6s}(b+s)(b+2s)$$

Substituting the simplified summation above into equation (2-8), we can solve for F_0:

$$\boxed{F_0 = \frac{6sM_{Diaph}}{(b+s)(b+2s)}} \qquad \text{EQ. 2-9}$$

This equation is particularly useful in very large diaphragms. Smaller diaphragms can be done by hand. The following example on a small diaphragm will be used to illustrate a solution by hand and checking the answer against equation (2-9):

<u>A Simple Collective Chord Design Example</u>:

This example illustrates the collective chord forces in a simple 48-foot square building. The values presented are in

general agreement with 8-inch thick, 24-foot tall concrete wall panels and a 15 psf roof dead load.

w = 700 plf (Diaphragm load)
L = 48 ft. (Diaphragm length)
b = 48 ft. (Diaphragm width)
s = 8 ft. (Continuity tie spacing)
F_0 = Force in extreme continuity tie

$$M_{Diaph} = \frac{wL^2}{8} = \frac{700(48)^2}{8(1000)} = 201.6\,ft - kips$$

Using strain compatibility of a linear elastic model, the diaphragm moment can be related to the continuity tie forces as follows using equation (2-6):

$$M_{Diaph} = 48F_0 + 40F_1 + 32F_2 + 24F_3 + 16F_4 + 8F_5 + 0F_6 \qquad \text{EQ. 2-10}$$

The values for the various F_x terms can be easily determined from Figure 2 using the principles of similar triangles. Equation (2-10) can be simplified as follows:

$$M_{Diaph} = 48F_0 + 40\left(\frac{2}{3}F_0\right) + 32\left(\frac{1}{3}F_0\right) + 0 + 16\left(-\frac{1}{3}F_0\right) + 8\left(-\frac{2}{3}F_0\right) + 0 \qquad \text{EQ. 2-11}$$

$$M_{Diaph} = 74.7F_0 \qquad \text{EQ. 2-12}$$

Solving for F_0:

$$F_0 = \frac{M_{Diaph}}{74.7} = \frac{201.6}{74.7} = 2.7kips$$

As a mathematical check, equation (2-9) will be used to check the answer above.

$$F_0 = \frac{6sM_{Diaph}}{(b+s)(b+2s)}$$

$$F_0 = \frac{6(8\,ft)(201.6\,ft - kips)}{(48+8\,ft)(48+2(8\,ft))}$$

$$F_0 = \frac{9677\,ft^2 - kips}{3584\,ft^2} = 2.7kips$$

<u>Equation confirmed, matches long-hand calculation</u>

<u>A Traditional Chord Design Comparison</u>:

For comparison, the above example illustrating the collective chord approach will be analyzed using a more traditional T-C couple approach. The same values are used for comparison:

w = 700 plf (Diaphragm load)
L = 48 ft. (Diaphragm length)
b = 48 ft. (Diaphragm width)
s = 8 ft. (Continuity tie spacing)
F_0 = Force in extreme continuity tie

$$M_{Diaph} = \frac{wL^2}{8} = \frac{700(48)^2}{8(1000)} = 201.6\,ft - kips$$

Using equation 2-2:

$$T = C = \frac{M_{Diaph}}{b} = \frac{201.6\,ft - kips}{48\,ft} = 4.2kips$$

The 4.2 kip chord force using a traditional T-C force couple method is more realistically a maximum of 2.7 kips when considering all the purlins/joists behaving as a collective chord. This reduction assumes that the repetitive purlins are tied together with minimal slip and are similar in size.

The collective chord example shown assumes every purlin, joist or beam acts as a continuity tie and is thus available for tensile chord force resistance. Some engineers elect to only provide continuity connections at every 2nd, 3rd or 4th purlin, joist or beam, using subdiaphragms to collect and transfer wall forces to the designated continuity ties (ASCE 7-05 Section 12.11.2.2.1). The collective chord approach can still be used as an analytical tool in this situation. The following example will illustrate.

<u>A Collective Chord Design Example with Subdiaphragm Modification</u>:

The same example used previously will now analyze the collective chord force of the diaphragm where every other purlin/joist is interrupted due to the employment of subdiaphragms. This results in half of the purlin/joists being connected for continuity.

w = 700 plf (Diaphragm Load)
L = 48 ft. (Diaphragm length)
b = 48 ft. (Diaphragm width)
s = 16 ft. (Continuity tie spacing) < Revised
F_0 = Force in extreme continuity tie

$$M_{Diaph} = \frac{wL^2}{8} = \frac{700(48)^2}{8(1000)} = 201.6\,ft - kips$$

Using equation (2-9), we may solve for F_0:

$$F_0 = \frac{6sM_{Diaph}}{(b+s)(b+2s)}$$

$$F_0 = \frac{6(16\,ft)(201.6\,ft - kips)}{(48+16\,ft)(48+2(16\,ft))}$$

$$F_0 = \frac{19354\,ft^2 - kips}{5120\,ft^2} = 3.8kips$$

The use of every other purlin/joist still provides a reduction in the maximum chord force, but it is a rather small reduction on this small diaphragm example. Larger diaphragms stand to benefit most from this approach with only every 2nd, 3rd or 4th purlin/joist providing continuity.

Often the resulting chord force developed in the purlin/joist is less than the wall anchorage axial load in the member. In this common occurrence, the wall anchorage axial load governs the purlin/joist design, and the chord force is simply checked against the wall anchorage force. It is important to remember that these chord forces develop tensile and compressive axial loads that the purlins/joists must be designed for.

An inherent benefit of a collective chord system is the redundancy advantage over an isolated chord at the diaphragm's extreme. Researchers pondering the behavior of large flexible diaphragms supported by rigid walls anticipate that the structure's failure mode may occur in the diaphragm instead of the main lateral force resisting system (rigid walls).

Research indicates that the dynamic amplification associated with flexible diaphragms is worst in the longitudinal direction of buildings with large flexible diaphragms (Harris et. al., 1998). Transverse seismic loads begin to go non-linear at maximum seismic loads, thus reducing the amplification. Because tilt-up buildings are often long and narrow, diaphragm designs are more governed by forces in the transverse direction, resulting in conservative overstrength in the longitudinal direction. This results in more elastic diaphragm behavior in the longitudinal direction, creating more amplification, and thus larger diaphragm chord forces at the narrow ends of the building. Today's girder and purlin/joist layouts in these buildings are benefited by a collective chord approach in the more critical longitudinal loaded direction.

3. Diaphragm Deflections Limits:

Diaphragm deflections are limited by the Building Code primarily for two reasons: Maintaining structural integrity and avoiding impact with adjacent buildings. The total horizontal deflection considered is composed of both a diaphragm component and the additional movement of the lateral force resisting system (shear walls, frames, etc.). In typical concrete tilt-up or masonry wall buildings, the in-plane shear wall deflections are insignificant compared to the diaphragm and may be ignored.

Limiting diaphragm deflections to maintain structural integrity has been a part of the UBC back as far as the 1950s; however, the degree of analysis by design engineers has typically been minimal. The primary reason for this has been the code's lack of clarity as to the maximum deflection amount allowed. The language in our building codes is as follows:

> "Permissible deflection shall be that deflection up to which the diaphragm and any attached load distributing or resisting element will maintain its structural integrity under design load conditions, such that the resisting element will continue to support design loads without danger to occupants of the structure." (IBC, 2006, Sec. 2305.2.2; ASCE, 2006, Sec. 12.12.2; AF&PA, 2005, Sec. 4.2.2)

This language is intentionally ambiguous, with the approach left much up to the engineer's own rational judgment. Additionally, the past SEAOC "Bluebook" states, "In lowrise concrete or masonry buildings, deflections that can cause secondary failures in structural and nonstructural walls should be considered." (SEAOC, 1999, Section C108.2.9)

As the diaphragm deflects, perpendicular walls and building columns translate at their tops, thus causing rotation at their bases. Assuming the walls and columns were modeled during design with pinned bases, this base rotation is acceptable even if some unintentional fixity exists.

Examples of unintentional fixity include standard column base plate anchorage, tilt-up wall-to-slab connections, and masonry wall-to-footing connections. The assumption of plastic hinges forming at the base is acceptable, provided that

these hinges do not result in an unstable mechanism within the building (SEAOC, 2006).

In very large flexible diaphragms, the amount of horizontal diaphragm deflection creates a potential *P-delta* concern of the entire system. California's soon-to-be-adopted building code has a new tool that can be used to evaluate diaphragm deflections for stability.

3a. Diaphragm Stability Check

As very large flexible diaphragms translate laterally, the vertical dead load on the roof begins to introduce *P-delta* effects that further introduce a horizontal thrust component from the axially loaded gravity walls and columns.

ASCE Section 12.8.7 (ASCE, 2005) contains a provision to evaluate lateral force resisting systems for instability due to horizontal translations. Although not originally intended to be used for evaluating flexible diaphragm deformations, this provision can be used as a guide to investigate stability of the roof system under diaphragm *P-delta* effects (SEAOC, 2006).

Using this approach, evaluating structural stability is through the use of a stability coefficient θ which is defined using ASCE 7, Equation 12.8-16:

$$\theta = \frac{P_x \Delta}{V_x h_{sx} C_d}$$

where:

P = Vertical load acting on translating system (kips)
V = Seismic shear force acting on the system (kips)
h = Height of the system (in)
C_d = Dynamic amplification factor.
Δ = Average horizontal translation (in).

As mentioned previously, this equation was not set up for flexible diaphragms, but instead was directed towards flexible frame systems with rigid diaphragms. Some modification will be necessary to keep the equation applicable.

In concrete tilt-up and masonry wall buildings, the vertical load P acting on the translating system includes a wall weight portion and a roof weight portion. While the center of mass of the roof undergoes the full translation, the wall's center of mass typically undergoes only half the diaphragm's translation. Thus, the following expression may be used in this situation:

$$P = P_{ROOF} + \frac{P_{WALL}}{2}$$

It should also be noted that P_{ROOF} may only contain roof dead load in accordance with the load combination being considered. The P_{WALL} portion should only consider the walls undergoing the translation, thus the acting shear walls, which are parallel to the force direction, are not included.

Another adjustment to consider when using the stability check equation is the reduction in Δ, the average diaphragm deflection. Because this equation was set up for rigid diaphragms supported by flexible frames, the entire diaphragm often had similar translations. In the rigid wall – flexible diaphragm situation, translations within the diaphragm vary from zero to it's maximum.

The diaphragm's deformed shape may be assumed to be parabolic, and thus the average translation of the columns and walls collectively may be approximated as two-thirds the maximum.

$$\Delta_{averaage} = \frac{2}{3}\Delta_{\max}$$

The dynamic amplification factor C_d is based on the design forces acting on the diaphragm. Often this is simply the same value used in obtaining the building's base shear per ASCE 7 Table 12.2-1. However, there are times when the diaphragm forces are controlled by an upper or lower bound per Section 12.10.1.1, in which case the dynamic amplification factor C_d should be modified to reflect the equivalent design amplification used in the analysis.

When evaluating the stability results for θ, values less than 0.10 are sufficient to ensure stability during *P-delta* translations. In this case *P-delta* effects on story drifts, story shears and moments need not be considered.

This method outlined here provides a quick rational approach to check stability concerns. It is interesting to note that large diaphragm deflections in tall buildings are not as critical as similar deflections in short buildings. The stability check is concerned with the amount of rotation vertical elements undergo while gravity loaded. The combination of vertical load with vertical element rotation results in a horizontal thrust added to the already translated system.

Occasionally, it has been reported that the traditional story drift limits now found in ASCE 7 Section 12.12.1 are applied to flexible diaphragm deflections. This is not correct and results in excessively conservative designs in even small

diaphragm spans. The story drift limits of this section apply to the vertical lateral force resisting system such as frames and shear walls. More information on this justification can be found in the current NEHRP Commentary. This Commentary lists all those items to be included into the story drift calculation, and horizontal diaphragm deflection is not one of them (NEHRP, 2003).

3b. Building Separation Check

Today's large distribution buildings with their resulting large diaphragm deflections create new problems in seismic isolation from adjacent buildings and property lines. Total deflection δ_x is limited by ASCE 7 Section 12.12.3 and computed by Equation 12.8-15:

$$\delta_x = \frac{C_d \delta_{xe}}{I}$$

The C_d and I modifications essentially bring the strength level seismic diaphragm deflection up to maximum considered levels. As discussed previously, the dynamic amplification factor C_d should reflect the effective load reduction used in the diaphragm design also considering any governing upper or lower bound (e.g. ASCE 7 Section 12.10.1.1).

Fortunately, today's very large buildings are seldom immediately adjacent to a property line. This is often to allow sixty-foot side yards to comply with allowable areas and occupancy requirements. In addition, exiting and fire access requirements normally require doors and egress paths on all sides of very large buildings.

A more common occurrence that needs seismic isolation is the location of adjacent buildings on the same property. Often very large distribution buildings have small office structures immediately adjacent. Isolating the two structures is sometimes the best solution for deformation compatibility issues; however, these isolation joints can become very wide.

A similar condition occurs in steel deck buildings with thermal expansion joints that cross the full diaphragm width. There has been some disagreement within the industry as to the amount of seismic isolation necessary where diaphragms meet at the same elevation. It is reasoned that impact damages are a bigger concern where adjacent structures meet at different elevations; for example, a diaphragm impacts the face of an adjacent bearing wall between floors.

This is an area that could benefit from additional research.

4. Wood Diaphragm Deflections

Methods to compute horizontal diaphragm deflections have been around for quite some time, however, engineers have viewed this calculation as necessary only under unusual circumstances. As the code becomes more aggressive in regulating seismic building movement and building separations, the calculation of diaphragm deflection is becoming more commonplace.

4a. Wood Diaphragm Deflections: Past Practice

Typically, a horizontal diaphragm is modeled as a flexural girder with equivalent flanges and web. This girder analogy is an acceptable analytical tool and widely used. The sheathing represents the girder web and chords represent the girder flanges. While the web area's resistance to shear deformation is considered, the bending stiffness of the web is neglected, and thus the stiffness calculation is somewhat conservative in that regard (ATC, 1981, p. 20). Additional evidence of this approach's conservatism was also evident in research conducted to determine diaphragm seismic response (Harris et. al., 1998).

The past diaphragm deflection equation in the UBC Standards Volume 3 is now incorporated into IBC 2305.2.2.

IBC EQ. 23-1:

$$\Delta = \underbrace{\frac{5vL^3}{8EAb}}_{\text{Bending}} + \underbrace{\frac{vL}{4Gt}}_{\text{Shear}} + \underbrace{0.188Le_n}_{\text{Nail slip}} + \underbrace{\frac{\sum(\Delta_C X)}{2b}}_{\text{Chord splice slip}}$$

where:

A	= Area of chord cross section (in^2)
b	= Diaphragm width (ft)
E	= Elastic modulus of chords (psi)
e_n	= Nail deformation (in). Reference: IBC Table 2305.2.2(1)
Gt	= Panel rigidity through the thickness (psi). Reference: IBC Table 2305.2.2(2)
L	= Diaphragm length
v	= Maximum shear due to design loads in the direction under consideration (plf)
Δ	= The calculated deflection (in)
$\Sigma(\Delta_C X)$	= Sum of individual chord-splice slip values "Δ_C" on both sides of the diaphragm, each multiplied by its distance to the nearest support "X".

IBC Equation 23-1 estimates diaphragm deflections, and its derivation is found in ATC 7's Appendix A (ATC, 1981). Four separate diaphragm deformation components are combined: bending deformation, shear deformation, nail slip deformation, and chord splice slip deformation. This equation assumes a diaphragm that is fully blocked and uniformly nailed. In the very large diaphragms of today's tilt-up and masonry distribution buildings, the wood roof systems are inherently blocked by their panelized fit-up practices. However, due to their large shears these very large diaphragms never have uniform nailing throughout, creating a problem when using IBC Equation 23-1. Language in IBC Section 2305.2.2 requires the nail-slip component's 0.188 constant to be modified accordingly. More is discussed on multiple nailing zones later in this paper.

An improvement of the 2006 IBC provisions over the 1997 UBC is a more useful table presenting shear deformation stiffnesses of commonly used sheathing materials. Under the 1997 UBC, only plywood sheathing was presented; but 2006 IBC Table 2305.2.2(2) provides values for both plywood and OSB (oriented strand board) sheathing materials. In addition, the IBC has combined the thickness and the shear modulus values together.

Another interesting aspect of the IBC Table is the relative stiffness of plywood sheathing to OSB. In panelized wood roof systems, and OSB is nearly double the stiffness of plywood for commonly used Structural I grade sheathing.

4b. Wood Diaphragm Deflections: New AF&PA

As just discussed, the 2006 IBC specifically provides a deflection equation for horizontal wood diaphragms (IBC Equation 23-1). Alternatively, IBC 2305.1 allows engineers to comply with AF&PA's *Special Design Provisions for Wind and Seismic* (SDPWS), for the design of wood lateral force resisting systems (AF&PA, 2005). In this AF&PA publication, a similar diaphragm deflection equation is provided.

AF&PA SDPWS Equation 4.2-1:

$$\delta_{dia} = \underbrace{\frac{5vL^3}{8EAb}}_{\text{Bending}} + \underbrace{\frac{0.25vL}{1000G_a}}_{\text{Shear \& nail slip}} + \underbrace{\frac{\sum\left(X\Delta_C\right)}{2W}}_{\text{Chord splice slip}}$$

where:

A	= Area of chord cross section (in^2)
W	= Diaphragm width (ft)
E	= Elastic modulus of chords (psi)
G_a	= Apparent diaphragm shear stiffness from nail slip and panel deformation (kips/in)
b	= Diaphragm width (ft)
L	= Diaphragm length (ft)
v	= Maximum shear due to design loads in the direction under consideration (plf)
Δ	= The calculated deflection (in)
$\Sigma(\Delta_C X)$	= Sum of individual chord-splice slip values "Δ_C" on both sides of the diaphragm, each multiplied by its distance to the nearest support "X".

This deflection equation is based on the same source as IBC Equation 23-1, but has been simplified further. The shear deformation contribution has been combined with the nail slip contribution to reduce the 4-term equation to a 3-term equation.

Because nail-slip is non-linear with respect to load per nail, this simplified equation is not numerically identical to the IBC approach. The AF&PA approach has been designed to be identical at the critical strength design level, 1.4 times the allowable shear value for seismic (AF&PA, 2005). This results in deflections that are slightly overestimated.

Excellent commentaries on these deflection equations are available (ATC, 1981; Skaggs & Martin, 2004; AF&PA, 2005).

4c. Diaphragm Deflections: Multiple Nailing Zones

In large wood diaphragms, nail spacing is typically adjusted as the diaphragm shears reduce. Similarly in steel deck diaphragms, welding or other attachments are also adjusted as diaphragm shears reduce. This efficient use of the diaphragm materials, results in non-uniform diaphragm stiffness.

For wood diaphragms, the nail-slip deformation component of the IBC Equation 23-1 or of the AF&PA SDPWS Equation 4.2-1 must be modified to accommodate the non-uniform stiffness across the diaphragm. Two different methods have been published in this regard.

In the IBC Commentary (IBC, 2006), a method that adjusts the nail-slip constant 0.188 is presented graphically. Essentially, the constant 0.188 is adjusted by a ratio of the average non-uniform load per nail to the average uniform load per nail. This method is an outgrowth of a

recommendation from the Applied Technology Council (ATC, 1981).

Another method based on virtual work principles is presented in SEAOC's *2006 IBC Structural/Seismic Design Manual* (SEAOC, 2006). This method creates a table format computing average shears over respective diaphragm lengths. The method is presented in conjunction with the AF&PA equation, and may be applied to the IBC equation as well.

Both of these methods are useful tools to provide greater accuracy in computing wood diaphragm deflections. A similar approach may be developed for steel deck diaphragms with varying connection patterns.

4d. Diaphragm Deflections: Collective Chord

Both the IBC equation and the AF&PA equation for horizontal wood diaphragm deflections contain the same bending deformation component.

$$\Delta_{bending} = \frac{5vL^3}{8EAb}$$

This deformation component is reliant upon the characteristics of the diaphragm's chord properties, based on a girder analogy.

As discussed earlier in this article, today's larger distribution warehouse buildings are more accurately modeled with a collective chord instead of a single isolated chord at the diaphragm's extreme. Instead of solely relying upon the diaphragm extremes for bending stiffness under the girder analogy, the continuous elements of the diaphragm's interior are acting as well.

To model the diaphragm's bending stiffness considering collective chord elements, the parallel axis theorem for the moment of inertia is utilized. Assuming the behavior as linearly elastic, Figure 4 illustrates the chord stiffness distribution.

Beginning with the traditional deflection equation of a uniformly loaded beam, a suitable equation can be developed that incorporates a collective chord:

$$\Delta_{bending} = \frac{5wL^4}{384EI}$$

All variables utilize pounds and inches. In order to accommodate more customary units of feet for length ***L*** and pounds per linear foot for uniform load ***w***, the bending deflection equation is modified as follows for unit consistency:

EQ. 4-1:

$$\Delta_{bending} = \frac{5\left(\frac{w}{12}\right)(L \times 12)^4}{384EI} = \frac{5wL^4(1728)}{384EI} = \frac{45wL^4}{2EI}$$

where:

w	= Applied uniform loading (plf)
L	= Diaphragm length (ft)
E	= Elastic modulus of chords (psi)
I	= Moment of inertia (in^4)

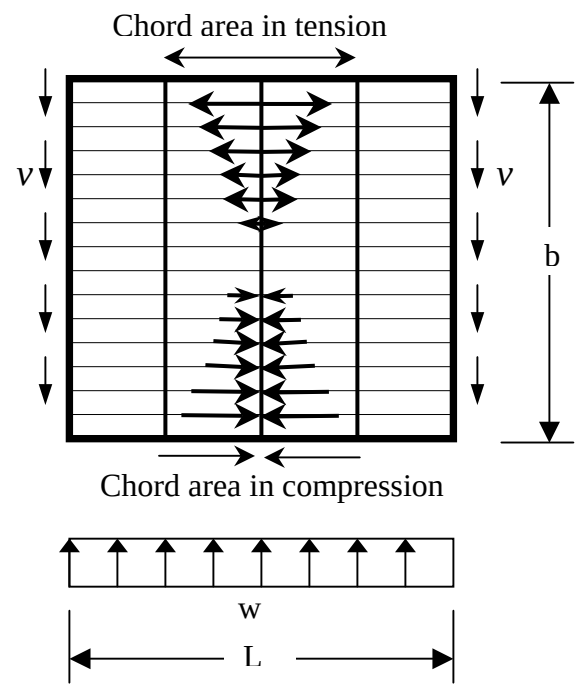

Figure 4: Collective Chord Area Distribution

It is desirable to have an equation in terms of the maximum diaphragm shear ***v*** (plf) instead of the applied uniform load ***w*** in order that the diaphragm deflection of other non-uniform loading conditions can be approximated.

$$v = \frac{V}{b} = \frac{wL}{2}\frac{1}{b}$$

where:

V	= Diaphragm shear reaction (lbs)
b	= Diaphragm width (ft)

This may be rewritten as follows:

$$w = \frac{2vb}{L}$$

Substituting into Equation 4-1:

$$\Delta_{bending} = \frac{45(2vb)L^3}{2EI} = \frac{45vbL^3}{EI} \qquad \text{EQ. 4-2}$$

In horizontal wood diaphragms, the chords traditionally are considered to dominate the bending stiffness while the "web" area of the girder model analogy is ignored. This is especially appropriate in wood diaphragms where much of the web's stiffness would rely upon cross-grain tension at the sheathing edges for continuity.

For the classic single chord located at each of the diaphragm's extremes, the moment of inertia based on the parallel axis theorem may be computed as follows:

$$I = \sum\left(I_x + A_x d_x{}^2\right)$$

$$I = A\left(\frac{b}{2}\right)^2 + A\left(\frac{b}{2}\right)^2$$

$$I = \frac{Ab^2}{2}$$

To convert the diaphragm width ***b*** into customary units of feet, we have the following:

$$I = \frac{Ab^2 144}{2} = 72Ab^2$$

Substituting into Equation 4-2:

$$\Delta_{bending} = \frac{45vbL^3}{E\left(72Ab^2\right)} = \frac{5vL^3}{8EAb} \qquad \text{EQ 4-3}$$

Notice that this expression matches the bending deformation component of both the IBC equation and AF&PA equation for horizontal wood diaphragm deflections.

As discussed earlier in this paper, larger buildings may be more accurately modeled with a collective chord approach. Instead of solely relying upon the diaphragm extremes for bending stiffness, the continuous elements of the diaphragm's interior are acting as well. Using the parallel axis theorem for moment of inertia, a new expression for the collective chord's moment of inertia can be derived. The moment of inertia of each individual chord element $\boldsymbol{I_x}$ is insignificant and assumed as zero.

$$I = \sum\left(I_x + A_x d_x{}^2\right)$$

$$I = \sum\left(A_x d_x{}^2\right)$$

From Figure 5, the following series represents the collective chord moment of inertia, where ***s*** represents the uniform spacing of the continuous chord elements.

$$I = A_0\left(\frac{b}{2}\right)^2 + A_1\left(\frac{b}{2} - s\right)^2 + A_2\left(\frac{b}{2} - 2s\right)^2 + \ldots + A_n\left(\frac{b}{2} - ns\right)^2$$

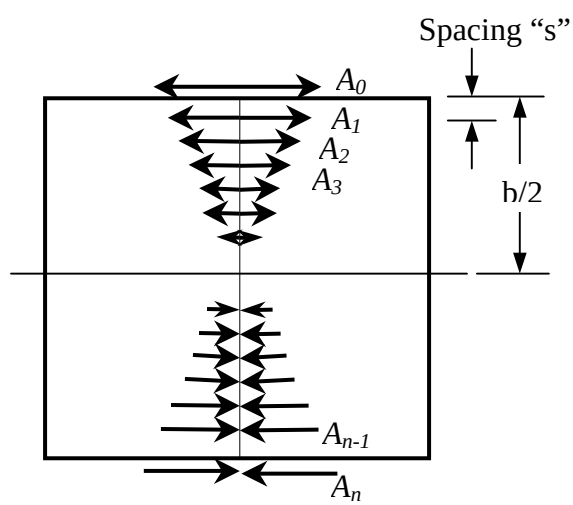

Figure 5: Collective Chord Area Series

Expressed as a summation:

$$I = \sum_{x=0}^{n} A_x\left(\frac{b}{2} - sx\right)^2$$

This can be simplified further to remove the summation operator. In addition, the chord element areas can be

assumed to be all equal, or conservatively consider the smallest occurring chord element area.

$$I = A\sum_{x=0}^{n}\left(\frac{b}{2} - sx\right)^2$$

$$I = A\sum_{x=0}^{n}\left(s^2x^2 - bsx + \frac{b^2}{4}\right)$$

$$I = A\sum_{x=1}^{n}\left(s^2x^2 - bsx + \frac{b^2}{4}\right) + A\frac{b^2}{4}$$

$$I = A\left(s^2\sum_{x=1}^{n}x^2 - bs\sum_{x=1}^{n}x + \frac{b^2}{4}\sum_{x=1}^{n}1\right) + A\frac{b^2}{4}$$

$$I = A\left(s^2\frac{n(n+1)(2n+1)}{6} - bs\frac{n(n+1)}{2} + \frac{b^2}{4}n\right) + A\frac{b^2}{4}$$

$$I = A\left(s^2\frac{2n^3 + 3n^2 + n}{6} - bs\frac{n^2 + n}{2} + \frac{b^2}{4}n + \frac{b^2}{4}\right)$$

$$I = A\left(\frac{s^2n^3}{3} + \frac{s^2n^2}{2} + \frac{s^2n}{6} - \frac{bsn^2}{2} - \frac{bsn}{2} + \frac{b^2n}{4} + \frac{b^2}{4}\right)$$

The number of chord element spaces n can be approximated as follows, thus simplifying the equation further:

$n = \frac{b}{s}$, and substituting…

$$I = A\left(\frac{s^2b^3}{3s^3} + \frac{s^2b^2}{2s^2} + \frac{s^2b}{6s} - \frac{bsb^2}{2s^2} - \frac{bsb}{2s} + \frac{b^2b}{4s} + \frac{b^2}{4}\right)$$

$$I = A\left(\frac{b^3}{3s} + \frac{b^2}{2} + \frac{bs}{6} - \frac{b^3}{2s} - \frac{b^2}{2} + \frac{b^3}{4s} + \frac{b^2}{4}\right)$$

$$I = A\left(\frac{4b^3}{12s} - \frac{6b^3}{12s} + \frac{3b^3}{12s} + \frac{b^2}{4} + \frac{bs}{6}\right)$$

$$I = A\left(\frac{b^3}{12s} + \frac{b^2}{4} + \frac{bs}{6}\right)$$

$$I = \frac{Ab}{12s}\left(b^2 + 3bs + 2s^2\right)$$

$$I = \frac{Ab}{12s}(b + 2s)(b + s) \qquad \text{EQ. 4-4}$$

This expression represents the collective chord moment of inertia. Customarily the diaphragm width ***b*** and collective chord spacings ***s*** are in feet, but the area ***A*** and inertia ***I*** are in inches^2 and inches^4 respectfully. To maintain consistency of units Equation 4-4 is modified as follows:

$$I = \frac{Ab(12)}{12s(12)}(b + 2s)(12)(b + s)(12)$$

$$I = \frac{12Ab}{s}(b + 2s)(b + s) \qquad \text{EQ. 4-5}$$

Substituting into Equation 4-2, a new bending deformation component for horizontal diaphragms is achieved.

$$\Delta_{bending} = \frac{45vbL^3}{EI} = \frac{45vbL^3}{E\left(\frac{12Ab}{s}\right)(b + 2s)(b + s)}$$

$$\boxed{\Delta_{bending} = \frac{15vL^3s}{4EA(b + 2s)(b + s)}} \qquad \text{EQ. 4-6}$$

This equation finally represents the bending deformation component in a horizontal diaphragm. This is useful in both wood and steel deck diaphragms.

As a check, both ***s = b*** and ***s = b/2*** should result in a bending deformation component similar to a traditional diaphragm with only a chord at each extreme.

Verification Check #1

$$s = b$$

$$\Delta_{bending} = \frac{15vL^3 b}{4EA(b+2b)(b+b)}$$

$$\Delta_{bending} = \frac{15vL^3 b}{4EA(3b)(2b)}$$

$$\Delta_{bending} = \frac{5vL^3}{8EAb}$$

This matches Equation 4-3, thus there is agreement.

Verification Check #2

$$s = b/2$$

$$\Delta_{bending} = \frac{15vL^3 \frac{b}{2}}{4EA\left(b+2\frac{b}{2}\right)\left(b+\frac{b}{2}\right)}$$

$$\Delta_{bending} = \frac{15vL^3 \frac{b}{2}}{4EA(2b)\left(\frac{3b}{2}\right)}$$

$$\Delta_{bending} = \frac{5vL^3}{8EAb}$$

This matches Equation 4-3, thus there is agreement.

Equation 4-6 can now be substituted into IBC Equation 23-1 to obtain the complete horizontal wood diaphragm deflection equation. Because chord deformations are assumed to be linear elastic, the chord slip deformation component is a problem. Today's hybrid panelized roof systems in seismically active California are typically composed of steel joist spliced with welds. In this situation, chord slip is approximated as zero. A collective chord model of a wood roof system with bolted splices is not accurately modeled with this approach due to collective problems with chord slip.

For horizontal wood diaphragm with no-slip collective chords, the deflection equation is as follows:

$$\boxed{\Delta = \frac{15vL^3 s}{4EA(b+2s)(b+s)} + \frac{vL}{4Gt} + 0.188Le_n} \qquad \text{EQ. 4-7}$$

4e. Diaphragm Deflections: Steel Deck

Unlike wood sheathing, horizontal steel deck diaphragms have no direct code section providing equations for their computation. A good source of information to estimate steel deck diaphragm deflections are the deck manufacturer's ICC approvals. Within these approvals, the manufacturer often has a table providing appropriate equations and coefficients applicable to their product.

Two of the more popular steel deck manufacturer's in California are Verco Manufacturing Company and ASC Profiles. Both of these deck manufacturer's have ICC ES Reports with the same deflection limitations, however the deflection equations and stiffness coefficients vary between manufacturer (ICC ES-2078P, 2002; ICC ESR-1414, 2006),

Typically, steel deck diaphragms also employ a girder analogy for computing deflections. A flange component and a web component are both defined in the manufacturer's literature. Similar to wood diaphragms, the flange component consists of the diaphragm chord deformation, while the web component consists of the steel decking deformation.

Because the bending deformation component is primarily reliant upon the chords, the use of a collective chord and the equations derived in the previous section are still applicable.

5. Conclusion

As tilt-up and masonry wall buildings grow larger and larger, more and more demands are placed on their large roof diaphragms. Larger shear stresses, chord forces, and deflections are challenging today's engineers. Fortunately, new tools are constantly being developed to assist designers to meet these challenges by providing greater modeling accuracy and larger strength capacities. The collective chord approach presented here can provide a more realistic model for diaphragm stiffness and bending stresses.

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