The CAFE Formula

David Hafemeister

The Corporate Average Fuel Economy (CAFE) formula mandates that manufacturers comply with a *fleet-average* fuel economy of 27.5 mpg.[1] Since the *gallon* parameter is in the denominator, fleet-average fuel economy is not a simple average of individual fuel economies. Consider the average fuel economy of a 20-mpg car and a 10-mpg car. If both cars traveled 20 miles, the total amount of gasoline consumed would be 1 + 2 = 3 gallons, for an average of 40 mi/3 gal, or 13.3 mpg, not (20 + 10)/2 = 15 mpg. Since the guzzler's mpg dominates the fleet-average fuel economy, manufacturers are encouraged to improve guzzlers more than already efficient cars. The *average fuel economy* for our two cars is obtained by averaging the inverse of fuel economy:

$$\langle(1/\text{fuel economy})\rangle = (1/10 + 1/20)(\text{gal/mi})/2 = 0.075 \text{ gal/m}, \quad (1)$$

with an average fuel economy,

$$\langle\text{fuel economy}\rangle = 1/0.075 \text{ gal/mi} = 13.3 \text{ mpg}. \quad (2)$$

The inverse average fuel economy for a fleet of cars is

$$\langle F_{\text{fleet}}^{-1}\rangle = \sum_i n_i/F_i, \quad (3)$$

where n_i is the fraction of cars in the ith subclass with fuel economy F_i.

A panel of the National Research Council estimated in 2000 that increased fuel economy under CAFE saves the US 2.8 Mbbl/day.[2] This estimate was not obtained by merely doubling fuel economy, because light trucks and SUVs consume at about 20 mpg, a rate midway between 1973's 13.5 mpg and CAFE's 27.5 mpg. Nonetheless, we ignore the SUV effect to examine a larger point. The first doubling of fuel economy cuts gasoline consumption in half. Unfortunately, a point of diminishing returns undercuts further doublings, but certainly does not negate their worthiness. Assume national gasoline consumption is

$$G = C/F, \quad (4)$$

where C is a constant and F is the fleet-average fuel economy. Doubling the fuel economy to CAFE's 28 mpg reduces consumption to $C/2F$, saving $C/2F$. A second doubling to 56 mpg reduces consumption to $C/4F$, saving an additional $C/4F$. A third doubling to 112 mpg reduces consumption to $C/8F$, saving an additional $C/8F$. With each doubling, the effect on fuel economy (for example to 56 mpg) is to make it twice the previous fuel economy (28 mpg), while savings are half as much ($C/4F$) as the previous savings ($C/2F$). If gas consumption $G = C/F$ is 8 Mbbl/day with *today's fleet*, then doubling fuel economy to 56 mpg would save 4 Mbbl/day. A second doubling to 112 mpg would save an additional 2 Mbbl/day, and the third doubling to 256 mpg would save an additional 1 Mbbl/day, a diminished return.

Technology Change Under CAFE. Improvements under CAFE came from the following measures:

- mass downsizing of 25%
- electronic engine–controls for more efficient combustion
- 5-speed manual transmissions
- fuel-injection without a carburetor
- 4 valves per cylinder
- front-wheel drive, reducing drive-train losses
- improved aerodynamics, lowering C_d from 0.4 to 0.3.

Since internal combustion engines can be only marginally improved, there will, at some point, be a departure from complete dependence on IC engines. Such an idea was considered heresy a decade ago. A likely shift is to hybrid cars that get 50 mpg in cities, followed by the plug–in electric car with lithium-ion batteries, which use gasoline only for trips over about 40 miles. The hydrogen-powered fuel cell car is not likely to be deployed for a several reasons.[3] Other options envisioned are very light cars made with carbon fiber, small diesel engines, compressed natural–gas engines, and ethanol/methanol engines. Super-cars could get 80 mpg with vastly reduced emissions.

Gas Guzzlers. We examine the improvement of two cars, one at 10 mpg and the other at 20 mpg. If the 20-mpg car alone is improved to 21 mpg, the fleet average increases by 0.22 mpg to 13.5 mpg. On the other hand, if only the 10-mpg car is improved to 11 mpg, the fleet average increases by 0.86 mpg to 14.2 mpg, four times the improvement of the former case (0.86/0.22 = 4). The factor of 4 is obtained by taking the differential of the inverse fuel economy, giving the *change in the inverse fuel economy* for *one type of car* proportional to the inverse square of the fuel economy,

$$\Delta(1/F) = -\Delta F/F^2. \quad (5)$$

The ratio of energy savings of ΔF = 1 mpg for *two types of cars* (guzzler and saver) is

$$\text{guzzler/saver} = (1/10^2)/(1/20^2) = 4. \quad (6)$$

To discourage purchase of inefficient autos, the 2000 Gas Guzzler Tax triggers a $1000 guzzler tax on a 22-mpg car (but not on SUVs) and $7,700 on a 12.5-mpg car.

Feebates. An alternative approach to curbing fuel consumption was suggested by Jonathan Koomey and Art Rosenfeld (Lawrence Berkeley National Laboratory), which places penalties on guzzlers and gives rebates for savers. A balance point of 28 mpg was proposed with rebates of $970 for Ford Escorts (35 mpg) and $1250 for Honda Civics (37 mpg), and a $4750 penalty for a Ferrari (15 mpg). On the basis of 1987 sales, $3.4 billion would be paid in fees and $1.7 billion would be disbursed as rebates. This scheme was not revenue-neutral (revenues = benefits) to the government, but the structure could be so modified.

To put these rebates and penalties into perspective, we estimate the fuel cost to run a Civic and a Ferrari over a 150,000-mile lifetime:

$$\text{Civic: } (150{,}000 \text{ mi}/37 \text{ mpg})(\$3/\text{gal}) = \$12{,}000 \quad (7)$$

$$\text{Ferrari: } (150{,}000 \text{ mi}/15 \text{ mpg})(\$3/\text{gal}) = \$30{,}000. \quad (8)$$

Future gasoline payments should be discounted to the present since we can invest money in the present to spend later. *The present net cost energy cost* for the Ferrari is sum of the present value of gasoline (about $20,000) and feebate penalty ($4850), for a total of about $25,000 (at the time of purchase). The Civic's net energy cost is much smaller at $7000 ($8000 for gasoline)– $1250 for the feebate). A political difficulty with feebates is that they penalize large US cars and rebate small Japanese cars.

Electricity vs. Gasoline. The adoption of electric cars would force a shift in energy units from miles/gallon (or liters/100 km) to kWh/mile. If a car loses 15 kW from aerodynamic and rolling drag at 30 m/sec (68 mph, see *PSI* text), a trip of 1 km would consume electrical energy at the rate of

$$E_{\text{elec}} = Pt = (15 \text{ kW})(1000 \text{ m})(1 \text{ sec}/30 \text{ m}) = 0.5 \text{ MJ/km} = 0.15 \text{ kWh/km} = 0.22 \text{ kWh/mi}. \quad (9)$$

If we consider only the cost of fuel, electricity is considerably cheaper than gasoline. At 10¢/kWh, it costs 2.2¢/mile for electrical energy, while gasoline costs 12¢/mile (25 mpg at $3/gal). The 200 million US vehicles, each traveling 12,000 mi/yr, require

$$(200 \text{ million})(12{,}000 \text{ mi/yr})(0.22 \text{ kWh/mi}) = 5.3 \times 10^{11} \text{ kWh/yr.} \quad (10)$$

This amount is increased to allow for energy losses, making total electrical energy needed to 8×10^{11} kWh/yr. Since a 1-GW_e plant produces about 7×10^9 kWh/yr, it would take 115 1-GW_e power plants, or about 20% of the US grid, to sustain an all-electric US vehicle fleet.

A 30-mpg gasoline car consumes energy at a rate

$$E_{\text{distance}} = (130 \text{ MJ/gal})(1 \text{ gal/30 mi}) = 4.3 \text{ MJ/mi} = 2.7 \text{ MJ/km.} \quad (11)$$

This gasoline-powered car consumes 5.4 times the energy of the electric car at 0.5 MJ/km. The electric car did better than the gasoline car because electrical motors are 90% efficient as compared to 15% for cars. However, this compares *gasoline energy to electrical energy*. If the efficiency of a power plant is 33%, the electric car advantage drops from 5.4 to 1.8. If a combined cycle gas makes electricity at 60% efficiency, the electric car's advantage rises upward to 3.2. The favorable efficiency of electric cars would be decisive except for the lifespan and cost of batteries. The ability to generate electricity on board greatly reduces battery requirements, making the hybrid viable. A satisfactory lithium-ion battery for autos is the tipping point for the plug–in electric car.

It is hoped the advent of the lithium-ion battery will broaden the mission of hybrid cars to plug-in electric cars. Demand for electricity in the summer in California peaks at about 50 GW between 2–4 PM and bottoms out at about 26 GW between 2–4 AM. The Electric Power Research Institute estimated that 13% of the unused power (3.2 GW of the 24 GW reduction) could be used to charge auto batteries, saving 5 million California cars (20 miles/day) from the need for gasoline. This is particularly promising since considerable night-time power is wasted since it comes from base-loaded power plants. The 3.2 GW acting over 6 hours produces

$$(3.2 \times 10^6 \text{ kW})(6 \text{ hr/day}) = 2 \times 10^7 \text{ kwh/day,} \quad (12)$$

which could be consumed by 5 million cars (20 miles/day)

$$2 \times 10^7 \text{ kwh/day}/(20 \text{ miles/day})(1 \text{ kwh/5 miles}) = 5 \times 10^6 \quad \text{cars.} \quad (13)$$

Cost of Conserved Energy. Would we be willing to spend \$4000 extra to obtain a 50-mpg hybrid, when compared to a CAFE traditional car that gets 20 mpg in the city? The 20-mpg car that goes 40 miles/day over 250 days travels 10,000 mi/yr and consumes 10,000/20 = 500 gal/yr. The 50-mpg car consumes 10,000/50 = 200 gal/yr, saving 300 gal/yr. If the extra investment for a Prius is \$4000, the cost of the extra loan and repayment *in constant dollars* (without inflation) is about \$500/yr for 10 years. This puts *the cost of conserved energy* for the 50-mpg car at

$$\text{annual cost/annual energy saving} = (\$500/\text{yr})/(300 \text{ gal/yr}) = \$1.67/\text{gal.} \quad (14)$$

Since the 50-mpg car has a CCE 50% of the market price (\$33.50 in 2007), this is a good choice. If the car lasts two decades and is driven more than 10,000 mi/yr, it is a very good purchase. For those living in Europe and Japan , paying \$5 per gallon, the investment is very, very good.

David Hafemeister
California Polytechnic State University
San Luis Obispo, CA 93407
dhafemei@calpoly.edu

[1] This paper was adapted from Ch. 15 of *Physics of Societal Issues: Calculations on National Security, Environment and Energy* by D. Hafemeister (Springer, New York, 2007). It was my pleasure to watch Allan Hoffman majestically steer CAFÉ through the Senate and the Senate–House Conference.

[2] National Research Council, *Effectiveness and Impact of Corporate Average Fuel Economy Standards*, (National Academy Press, Washington, 2002). Also see, *Automotive Fuel Economy: How Far Should we go?,* (National Academy Press, 1992).

[3] G. Crabtree, M. Dresselhaus, and M. Buchanan, Physics Today 57, p; 39–44, December 2004. APS Panel on Public Affairs Report, *The Hydrogen Initiative*, www.aps.org/