Scale Modeling of Cessna 172

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This report describes how the scale-model aircraft can be built when the Reynolds number is out of range for using dynamic similitude method. Due to lack of time and budget for designing an actual model for testing, Cessna 172 was used to calculate the size of the temporary scale-model aircraft. The method that was used for this project was matching aerodynamic coefficients such as drag coefficient or lift coefficient of the prototype and the model. Based on this method, the takeoff distance, landing distance and the rate of climb of the model came out to be 218 ft, 91 ft and 11.1 fpm respectively. Since the performance data of the actual Cessna 172 is already given, calculating those data from the model was avoided.

I.   Introduction

A scale model is a physical model that represents or copies of an object that is larger or smaller than the actual size of the object which is seeks to maintain the relative proportions of the physical size of the original object. Due to financial and time matters, engineers often create scale models to test the likely performance of a particular design at an early stage of development without incurring the full expense of a full-size performance such as wind tunnel testing.

To do this kind of scale modeling testing, usually for engineers, there has to be a similitude between the prototype and the model. A model is said to have similitude with the prototype if they share geometry similitude, kinematic similitude and dynamic similitude. Geometry similitude refers when the model is the same shape as the application, kinematic similitude refers when the fluid flow of both the model and the prototype should undergo similar time rates of change motions and dynamic similitude refers when the ratios of all forces acting on corresponding fluid particles and boundary surfaces in the two systems are constant.

For this project, Cessna 172 Skyhawk, a four seat, single-engine general aviation aircraft was chosen to calculate the size of the model because there is going to be a new version of Cessna 172 that fully powered by electric power sources which will be helpful for 2010-2011 AIAA competition. Table 1 and Table 2 represent some characteristics and specifications for Cessna 172[1].

Table 1: Cessna 172 Skyhawk characteristic

Length	27 ft 2 in	8.28 m
Wingspan	36 ft 1 in	11.0 m
Height	8 ft 11 in	2.72 m
Wing area	172 ft^2	16.2 m^2
Aspect Ratio		7.32
Airfoil		modified NACA 2412
Empty weight	1,691 lb	767 kg
Gross weight	2450 lb	1,111 kg
Propeller		2-bladed metal

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Table 2: Cessna 172 Skyhawk specification

Cruise velocity	122 knots	140 mph
Stall velocity	47 knots	54 mph
Range	696 nmi	801 mi
Service Ceiling	13,500 ft	4,100 m
Rate of Climb	721 ft/min	3.66 m/s
Takeoff Distance	1,685 ft	514 m
Landing Distance	1,295 ft	395 m

II.   Procedure and Analysis

The major issue for analyzing this project is the relationship between drag coefficient and Reynolds number and picking a Reynolds number where the prototype is going to be operated. Figure 1 shows the general relationship between the drag coefficient and Reynolds number. As it i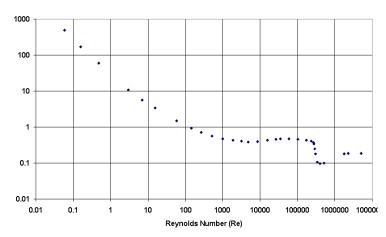s shown in fig 1, the drag coefficient follows general trends before Reynolds number reaches to approximately 5×10^5. Once the Reynolds number reaches to 5×10^5, the drag coefficient loses its trend and it's not a function of Reynolds number anymore. The design point for the prototype is going to be approximate 3.58×10^5 where the drag coefficient is not a function of Reynolds number and see how that Reynolds number would affect the size, operating point and the performance of the model.

Figure 1. The relationship between the drag coefficient and Reynolds number

Once the operating Reynolds number is decided, the other parameters that need to be decided are operating altitude of the prototype and the model, temperature and pressure, testing prototype's velocity and the scale ratio between the prototype and the model. The operating altitudes for both prototype and model are 0 which is sea level at standard day, the prototype's velocity is 150 ft/sec and the model is planned to be $1/10^{th}$ scale of the actual Cessna 172.

Since the operating Reynolds number for the model, operating altitudes for prototype and model, velocity of the prototype and the model's dimensions are decided, now the drag force ratio and power ratio between the prototype and the model can be calculated by matching drag coefficient between the prototype and the model as equation (1).

$$C_{D,p} = C_{D,m} \tag{1}$$

where the $C_{D,p}$ is the drag coefficient of the prototype and $C_{D,m}$ is the drag coefficient of the model.
The drag coefficient is defined as,

$$C_D = \frac{D}{\rho v^2 l^2} \tag{2}$$

where D is the drag force, ρ is the density and l is the length of the object. Therefore, the equation (1) can be redefined as

$$\frac{D_p}{\rho_p v_p^2 l_p^2} = \frac{D_m}{\rho_m v_m^2 l_m^2} \tag{3}$$

From equation (3), the drag force can be derived by rearranging the variables and the drag force ratio between the prototype and the model would be

$$\frac{D_m}{D_p} = \frac{\rho_m v_m^2 l_m^2}{\rho_p v_p^2 l_p^2} \tag{4}$$

Since Reynolds number is defined as

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$$Re = \frac{vl}{v} \tag{5}$$

where v is the kinematic viscosity.

By rearranging equation (5), the model velocity can be calculated as

$$v_m = \frac{Re_m v_m}{l_m} \tag{6}$$

The drag force ratio can be found by putting equation (6) into equation (4) and the drag force comes out to be

$$\frac{D_m}{D_p} = \frac{\rho_m}{\rho_p} \left(\frac{v_m Re_m}{v_p l_p} \right)^2 \tag{7}$$

Since the power is defined as

$$P = D \times v \tag{8}$$

where P is the power.

The power ratio between the model and the prototype can be calculated as

$$\frac{P_m}{P_p} = \frac{D_m v_m}{D_p D_p} \tag{9}$$

The takeoff distance can be calculated with the following equation

$$d_{LO} = 1.44 \frac{W^2}{g\rho S C_{L,max}(T - D)} \tag{10}$$

where d_{LO} is takeoff distance, W is weight, g is the gravitational acceleration, S is surface area $C_{L,max}$ is the max lift coefficient, T is thrust and D is drag.

In a similar way, the landing distance can be calculated with

$$d_{land} = 1.69 \frac{W^2}{g\rho S C_{L,max}[D + \mu(W - L)]} \tag{11}$$

where d_{land} landing distance μ is the friction coefficient of the ground and L is the lift.

Lastly, the rate of climb can be calculated from the equation of

$$RoC = \frac{V(T - D)}{W} \tag{12}$$

where RoC is rate of climb.

III. Results and Discussion

A. Data and Results

After organizing required equations for this project, first thing was done is how the Reynolds number of the model would affect the drag force ratio and the weight of the model. Figure 2 shows the relationship between the Reynolds
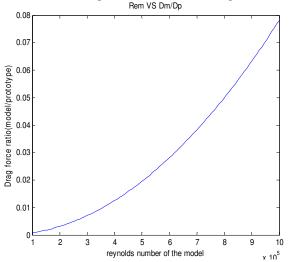

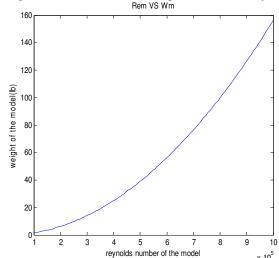**Figure 2. The relationship between the drag force ratio and the model's Reynolds number**

Figure 3. The relationship between the weight of the model and the model's Reynolds number

number of the model and the drag force ratio, and figure 3 shows the relationship between the Reynolds number of the model and the weight of the model. As shown in fig 2 and fig 3, the drag force ratio between the model and the prototype and the weight of the model increase exponentially as the Reynolds number increase. This is because the drag force ratio and the weight of the model are proportional to the square of the model's Reynolds number. Since the power of the model is also proportional to the model's Reynolds number, the power of the model increase with the same trend as fig 2 and fig 3 as shown in fig 4.
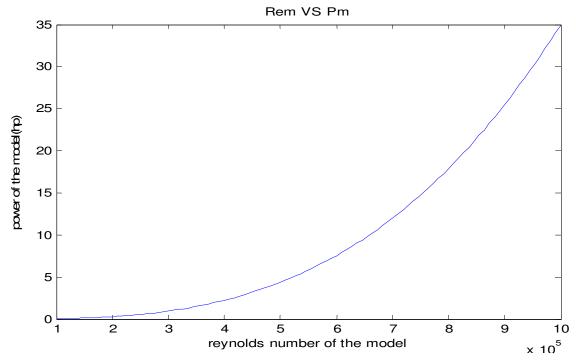
Figure 4. The relationship between the power of the model and the model's Reynolds number

Since the prototype's velocity is another important variable that affects the drag force ratio and model's characteristics, the relationship between the prototype's velocity and the drag force ratio was found by plotting in MATLAB. Figure 5 shows the relationship between the drag force ratio and the prototype's velocity. As it is shown in figure, the drag for ratio between the model and the prototype decreases as the velocity of the prototype increases because the drag force ratio is proportional to the inverse of square of the prototype velocity.
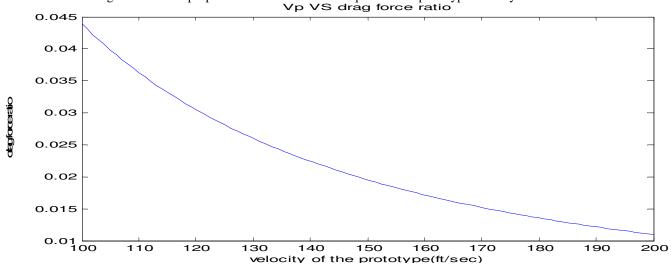

Figure 5. The relationship between the drag force ratio and the velocity of the prototype

In a same way, the weight of the model and the power of the model show the same trend because they are also proportional to the inverse of square of the prototype velocity. Figure 6 and 7 represent the trends of the weight of the model and the power of the model versus the prototype's velocity.
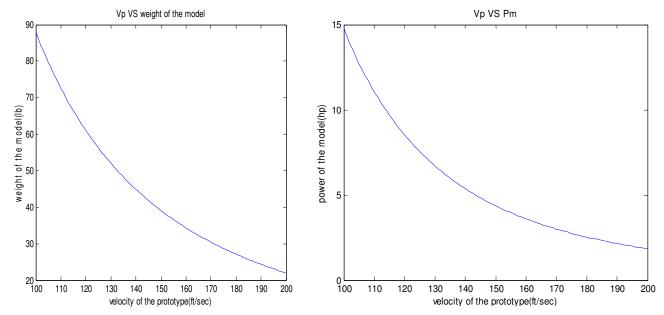

Figure 6. The relationship between the weight of the model and the velocity of the prototype

Figure 7. The relationship between the power of the model and the velocity of the prototype

The next step after finding the relationship between model's characteristics and the Reynolds number is to find a design point for the model by plotting the power of the model and the weight of the model. Since the model is planned to be used for wind tunnel testing or remote controller (RC) airplane, the model aircraft has to have decent amount of weight with the suitable horse power so that it is easy for people to use, fix and change the specification of the model for testing or personal entertainment. As shown in figure 8, the power of the model increases as the weight of the model increases. After researching several RC airplanes, the weight and power for the model aircraft is
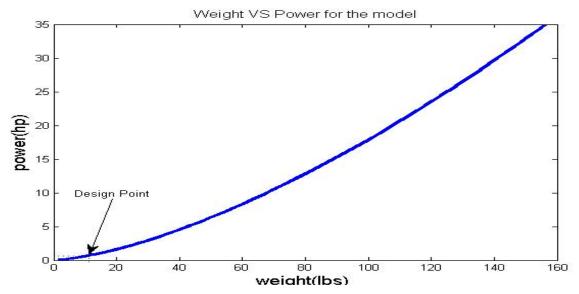

Figure 8. The relationship between the power and the weight of the model

approximately 10.6 lbs, power is 0.7 hp respecively with the velocity of 109.02 ft/sec.

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After the weight, power and the operating velocity are found the takeoff distance, landing distance and the rate of climb for the model is going to be analyzed. To analyze the takeoff distance and the landing distance, the drag coefficient needs to be found first from the protype. To find drag coefficient, Cessna 172 performance data at sea level with 100 mph velocity was used to find drag coefficient and the drag coefficient came out to be 0.029[2]. Since the drag coefficient is same for both prototype and model, the model drag coefficient is also 0.029. From this drag coefficient, the model's drag force is 2.45 lbf with the wetted area 6.19 ft^2 and the thrust is 3.53 lbf which is converted from 0.7 hp. With these thrust, drag and the assumed C_{Lmax} value of 1.5, the measured takeoff distance with using equation (10) is approximately 218 ft. With the same drag, μ of 0.4, and lift force of 9 lbf based on Cessna 172 L/D during landing, the landing distance comes out to be approximately 91 ft. Lastly, with the same thrust, drag, velocity and weight, the rate of climb of the model is calculated to be 11.1 feet per minute.

IV.   Conclusion

When the Reynolds number is too high so the drag coefficient is not a function of Reynolds number anymore, other methods can be used to do scale modeling such as matching aerodynamic coefficient between the prototype and the model. As shown in figures above, the drag force ratio between the prototype and model, the weight of the model and the power of the model tend to increase as the Reynolds number increase. On the other hand, the drag force ratio, the weight of the model and the power of the model tend to decrease as the prototype's velocity increase. Also, the calculated takeoff distance for the 1/10th scale-model is 218 ft, calculated landing distance is 91 ft and the calculated rate of climb is 11.1 feet per minute. Since this scale modeling study is fully based on Cessna 172 whose performance data is already calculated and given, transferring model performance characteristics to the prototype's performance characteristics has not been done. However, if other unknown-aircraft is to be used for this project, the way to transfer a model's characteristic to the prototype's characteristic is to use the force ratio or using the kinematic similitude between the prototype and the model which is based on the fluid characteristic.

References

[1]Cessna Aircraft Company , "Performance," [online data base]. URL: http://www.cessna.com/single-engine/skyhawk/skyhawk-specifications.html [cited 18 March 2011].

[2]John McIver B.Eng., "Cessna 172 Skyhawk II/100 Performance Assessment," [online database]. URL: http://www.temporal.com.au/c172.pdf [cited 18 March 2011].

Appendix

MATLAB code

```
function [PmPp,Pm,Wm] = scale_power(Pp,hm,hp,mum,mup,vp,lp,Wp,n)
% use all SI units, except power is in hp
[Tm,Pm,rhom] = stdatm(hm*3.281);    %input should be in feet
[Tp,Pp,rhop] = stdatm(hp*3.281);
%density will come out as slug/ft^3
%temperature will come out as R
%gotta convert them back into SI

rhom = rhom*1000/1.940; %converting into kg/m^3
rhop = rhop*1000/1.940; %converting into kg/m^3
Tm = Tm/1.8;     %converting into Kelvin
Tp = Tp/1.8;     %converting into Kelvin

Rep = rhop*vp*lp/mup;
if Rep > 1e5
    vm = 1e5*mum/(rhom*(lp*n));
else
    vm = (vp*(rhom/mum))/((rhop/mup)*n);
end
Fdm_Fdp = rhom*vm^2/(rhop*vp^2*(1/n)^2);
Wm = Wp*(Fdm_Fdp);
```

```matlab
PmPp = Fdm_Fdp*(vp/vm);
Pm = PmPp*Pp;
end

%% all velocity
clc
clear all
close all

n = linspace(0.05,0.9,10);
v = [0,10,20,30,40,50]; %velocity of the model, m/sec

for i = 1:length(n)
    for j = 1:length(v)
    [PmPp(i,j),Pm(i,j),Wm(i,j)] = halfv_scale_power(160,0,2000,1.789*1e-
5,1.726*1e-5,v(j),1.1303,2450,n(i));
    end
end

figure(1)
plot(n,Pm)
xlabel('scale(model/prototype)')
ylabel('power of model(hp)')
title('scale VS power')
legend('0m/s','10m/s','20m/s','30m/s','40m/s','50m/s','Location','Best');
figure(2)
plot(n,Wm)
xlabel('scale(model/prototype)')
ylabel('weight(lb)')
title('scale VS weight')
legend('0m/s','10m/s','20m/s','30m/s','40m/s','50m/s','Location','Best');
figure(3)
plot(n,PmPp)
xlabel('scale(model/prototype)')
ylabel('power ratio(model/prototype)')
title('scale VS power ratio')
legend('0m/s','10m/s','20m/s','30m/s','40m/s','50m/s','Location','Best');
%% for cruise speed

clc
clear all
close all

n = linspace(0.05,0.9,10);


for i = 1:length(n)

    [PmPp(i),Pm(i),Wm(i)] = halfv_scale_power(160,0,2000,1.789*1e-5,1.726*1e-
5,62.78,1.1303,2450,n(i))

end

figure(1)
plot(n,Pm)
```

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```matlab
xlabel('scale(model/prototype)')
ylabel('power of model(hp)')
title('scale VS power for cruise speed')
figure(2)
plot(n,Wm)
xlabel('scale(model/prototype)')
ylabel('weight(lb)')
title('scale VS weight for cruise speed')
figure(3)
plot(n,PmPp)
xlabel('scale(model/prototype)')
ylabel('power ratio(model/prototype)')
title('scale VS power ratio for cruise speed')
%% contour plot
clc
close all
clear all

n = linspace(0.05,0.9,100);
v = linspace(0,80,100);

for i = 1:length(n)
    for j = 1:length(v)
    [PmPp(i,j),Pm(i,j),Wm(i,j)] = halfv_scale_power(160,0,2000,1.789*1e-
5,1.726*1e-5,v(j),1.1303,2450,n(i));
    end
end
figure(4)
[v,n] = contour(v,n,PmPp);
xlabel('velocity(m/sec)')
ylabel('scale(model/prototype)')
title('contour of velocity, scale and power ratio')
axis([0 15 0 1])
clabel(v,n)

function [Dm_Dp,Pm_Pp,Wm,Pm] = reynold_drag(Rem,hm,hp,vp,lp,Wp,P_pro,n)
[Tm,Pm,rhom] = stdatm(hm);
[Tp,Pp,rhop] = stdatm(hp);

muo = 0.01827;  %reference dynamic viscosity, unit in centipoise
muo = muo*(3.81843e-7/0.01827); %convert into lbf*sec/ft^2;
To = 524.07;    %reference temperature, unit in Rankine

mum = muo*((0.555*To+120)/(0.555*Tm+120))*(Tm/To)^1.5;
mup = muo*((0.555*To+120)/(0.555*Tp+120))*(Tp/To)^1.5;

nup = mup/rhop;
num = mum/rhom;

Rep = vp*lp/nup

vm = num*Rem/(lp*n)

% vm = vp*sqrt(n) %using Froude number scaling
```

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```matlab
    Dm_Dp = (rhom/rhop)*(vm/vp*n)^2;

    Pm_Pp = Dm_Dp*(vm/vp);

    Pm = Pm_Pp*P_pro;

    Wm = Wp*Dm_Dp;
end

clc
clear all
close all

Rem = linspace(1e5,10e5,100);
for i = 1:length(Rem)
    [Dm_Dp(i),Pm_Pp(i),Wm(i),Pm(i)] =
reynold_drag(Rem(i),0,7000,80,3.8,2000,160,1/72);
end
figure(1)
plot(Wm,Pm)
title('Weight VS Power for the model')
xlabel('weight(lbs)')
ylabel('power(hp)')
% figure(2)
% plotyy(Rem,Wm,Rem,Pm)
% xlabel('Rem')
% ylabel('model weight(lbs)')
%% Different scale
clear all


Rem = 9e5;
scale = linspace(1/50,1/10,50);

for i = 1:length(scale)
    [Dm_Dp(i),Pm_Pp(i),Wm(i),Pm(i)] =
reynold_drag(Rem,0,0,150,3.8,2000,160,scale(i));
end
figure(3)
plotyy(scale,Pm,scale,Wm)
```