Formal Models of Authority:

Introduction and Political Economy Applications

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ABSTRACT

Talcot Parsons suggested in 1963 that there are basically three kinds of authority: utilitarian authority, coercive authority, and persuasive authority. In this paper, I show that the models developed by Gibbons and Rutten (1997), Hirshleifer (1991), Skaperdas (1992), Akerlof (1976) and Basu (1986) can be viewed as models where issues such as authority, power, influence and ideology, in the sense of Parsons, can be formally discussed. I also show the existence of an interesting difficulty in providing a contractarian interpretation of the State under the Parsonian view of governmental authority discussed in this paper.

KEY WORDS • Political Economy • microeconomics of government • formal models of authority

1. Introduction

The question of what a government is and how it works is a fascinating one for economists for a variety of reasons. I want to mention three of them. First, there is no hope of understanding the workings of a modern economy without a clear picture of how the design and implementation of policy affects and is affected by the behavior of the non-government side of the social system. Second, one never *sees* a government. What one sees instead is the behavior of a group of individuals that one *interprets* as government behavior. The repeated observation of such behavior defines what a government is. The fact that the government is at the interpreted level of the set of social interactions that we observe in the world is indicative of the fundamental role that beliefs play in defining that which we in ordinary language call institutions.

The third reason why the topic is of great interest to economists is also the main motivation behind this paper: the fact that there is a

peculiar relationship between the demand for government and the extent to which collective action problems are pervasive in a specific social situation. To elaborate on this, recall that for sufficiently low transaction costs we can expect collective action problems to vanish, for the individuals in the collectivity should be able to agree on (1) performing the actions that lead to an efficient allocation of resources, and (2) agree on a transfer scheme that leaves each individual in a position that is no worse off than before, and at least one individual in a better position. This is, of course, the Coase theorem, and it is a perfectly fine argument for solving a variety of collective action problems but not all of them. It is clear from the applications of the principle that the source of authority that makes the contracts enforceable comes from outside the social domain that is the object of study. Once one assumes away an unmodeled source of authority a very different theorem, the so-called 'Hobbes theorem', argues that it is the collective employment of coercive force that will determine the allocation rule.[1]

This situation is not exclusive to the literature on collective action problems. In virtually all models of economics there is an unmodeled source of authority that plays a crucial role at some stage of the analysis. Most of the time the bearer of such authority is a simplified entelechy that we call the government, which we sometimes endow with a set of preferences and actions that act as a simplification of the political system, and which we do not derive as the result of some underlying game among the citizens. Whether we can deal without an unmodeled source of authority in our models is a real challenge, for we do not clearly understand what authority means.

Partly in response to this problem a wide variety of approaches have been proposed to shed partial light on the subject of modeling limited governments. Most of those approaches aim at explaining the mixture of contractual and predatory activities in which governments decide to engage.[2] The first goal of this paper is to review those approaches which explicitly attempt to understand what the source of authority is, and to show that a formalization of the concepts of power and influence, as defined by Parsons (1963), is consequently available because of them. I do this in the subsequent section.

Parsons (1963) argues that there are basically three kinds of authority: utilitarian authority, coercive authority, and persuasive authority. I show that the models developed by Gibbons and Rutten (1997), Hirshleifer (1991), Skaperdas (1992), Akerlof (1976) and Basu (1986) can be interpreted as models where issues such as authority, power, influence and ideology, in the sense of Parsons, can be formally discussed. The

discussion sheds additional light on the relationship between power and beliefs, the role and effectiveness of governments with different sources of authority, and the extent to which different notions of authority can be captured in different solution concepts.

Much insight can be gained by examining those models. The second goal of this paper, however, is to show that two problems still pervade the formal modeling of government in game-theoretic terms: (1) that there is always a version of the Coase theorem that makes the government a redundant entity in the social domain, and (2) that the introduction of agents with authority to solve a collective action problem *creates* a collective action problem itself: that of controlling the government. In the Discussion, I explain the source of the persistence of these problems in the formal modeling of governments. In conclusion, I argue that these results may complicate a contractarian interpretation of the state, which requires us to revise the theory of economic policy that has been developed with such interpretation in mind.

Before entering into the subject matter I want to say that the need for providing adequate foundations for the field of public economics in the way the literature I review strives to attain is more than purely aesthetic. Instead, it relies on the fact that neither private, nor public policy design is invariant to how authority is distributed in society, because this determines the structure of the sets of contracts that can be enforced in courts of law, and the structure of the sets of contracts that can be self-enforced. That is why a good theory of government, whenever available, will be an essential building block of the fields of public economics and contract theory.

2. Modeling Limited Governments

2.1 Power and Beliefs

Why should you do as I say? The literature on power and politics has identified three sources for the authority that I have, as a matter of fact, if you indeed do as I say. First, there is the possibility that you believe I will, in return, do something that is very good for you. Second, there is the possibility that you believe I will do something that is very bad for you if you do *not* do as I say. Third, there is the possibility that you believe that not me but *others* will do things to you such that it is in your best interest to do as I say, and that those beliefs cannot be directly deduced from the examination of the social situation in question. Whichever is the case, I am a powerful man: you do as I say. The

authority that I have is not induced by what I do, but on what you *believe* can happen to you if you do not do as I say. Authority is *instituted* in me through *your* systems of beliefs. Moreover, authority is not a property of mine but, instead, a feature of the relationship between you and I. I can therefore define *authority* as a property of the interaction between you and I in which you do as I say. This simple terminology introduced, we can quickly note that the first source of authority we examined is that which Parsons (1963) calls *utilitarian authority*, the second source of authority is that which Parsons calls *coercive authority*, and the third source of authority that we examined is that which Parsons calls *persuasive authority*.

Note that these distinctions make finer points about the source of authority than those which the game-theoretic apparatus can formally capture, for every source ought to be persuasive and 'utilitarian' if it is to have any bearing on behavior.[3] Further, the coercive and the utilitarian can be made equivalent, *in a decision-theoretic sense*, without difficulty.[4] We will see below that, despite this fact, it is worth keeping Parson's classification for the purpose of model building.

2.2 Beliefs and Actions

If only one, the message of game theory is that whenever players reach an equilibrium in actions, beliefs about behavior are severely restricted, if not completely determined. Translated into the language of power and politics this means that *there is a limit on the authority that I can exercise over you, regardless of its source*. That is, game-theoretic models of authority are necessarily models of *limited* authority in the sense that they do not allow omnipotence.[5] This is not to say that the effect of my authority on your behavior is independent of the type of authority I use to support it. Indeed, different sources of authority have very different effects on your behavior. Before we turn our attention to those issues it is important to understand the exact role that the authority of a ruler can play in solving collective action problems.

2.3 Authority and Efficiency

The importance of a ruler with authority in enhancing efficiency has long been recognized. The needs to coordinate expectations, facilitate communication, enforce contracts and prevent conflicts have always been associated with the demand for a ruler that will be believed to act concerning that which is considered a social goal. A good example of what has been written about this issue is the work of Douglass North.

According to North (1990: 57), 'the inevitable conclusion that one arrives at in a wealth-maximizing world is that complex contracting that would allow one to capture the gains from trade in a world of impersonal exchange must be accompanied by some kind of third-party enforcement'. This is so, according to North (1990: 57), for two reasons. 'First, it is necessary to form a communications mechanism that provides the information necessary to know when punishment is required . . . Second, because punishment is often a public good in which the community benefits but the costs are borne by a small set of individuals.' These communications, contracting and enforcement mechanisms, it is argued, can be provided by the third-party, or State, in the presumption that the costs of those mechanisms being provided without the State are prohibitively high. There is a danger to this argument, North (1990: 59) adds: if the State has meaningful authority over the rest of the citizens 'then those who run the state will use that force in their own interest at the expense of the rest of society'. The models that I present and discuss below can be viewed as a formalization of North's questions and concerns about the abuse of authority by governments and, more generally, about the need for third-party enforcement.

2.4 A Model of Utilitarian Authority

Gibbons and Rutten (1997) have condensed a great deal of insight about power and politics from both the political science and the political economy literature in a model they use to explain the abuse of authority by governments. I present a simplified version of their work below. Begin by assuming a social situation where cooperation, while beneficial, is not automatic, and where there is an individual empowered by everyone else to punish those who do not cooperate. This situation can be captured by the (stage) game in Table 1. In Table 1, citizen A chooses rows, citizen B chooses columns, and citizen K (the ruler) chooses the tax system $t = (t_c, t_d, t_f)$ representing respectively a tax on mutual cooperation, a tax on mutual defection and a tax on unilateral defection, respectively. Players are assumed to be risk-neutral, and everyone has an outside option O (e.g. leave the country) with utility equal to one.

Table 1. Utilitarian authority

	C	D
C	$3 - t_c, 3 - t_c, 2t_c$	$0, 4 - t_f, t_f$
D	$4 - t_f, 0, t_f$	$1 - t_d, 1 - t_d, 2t_d$

Note that a choice of $t = 0$ leaves citizens A and B essentially playing a Prisoners' Dilemma. The source of authority of K in this example is unmodeled in the sense that, once K has announced the tax system t, the citizens have no option to renege on it. In this context one can see that there is a Nash equilibrium of the static game where the tax system is designed to induce citizens A and B to cooperate. It is also true in this setup that the same authority that can be used to foster the creation of gains from cooperation can be used to extract the gains from the citizens through higher taxes. In particular, it can be shown that whenever the introduction of a rational, self-interested ruler in the static game restores the efficiency, the ruler collects all the gains from cooperation. This leads to the following *irrelevance proposition* about government behavior for the case analyzed above. The proof is reproduced in the Appendix.

> *Proposition 1.* The set of payoffs attainable by the citizens in Nash equilibria of the static game with collective action problems described above is invariant under the addition of a rational, self-interested ruler with unmodeled authority.

Of course, this need not be the case when we examine the situation where the citizens face a positive probability q that the game will be played at least one more time in the future. This possibility allows outcomes where not all the efficiency gains are kept by the ruler, if the probability of continuation is sufficiently high. *There is a limit, however, on the amount that the citizens can keep in equilibrium*. Assume that in a public event citizens A and B accept no tax on unilateral defection and announce independently the following strategy, in an attempt to capture some of the gains from cooperation:

> I will play C if no one has ever played D to date, and if K has never levied a (per-capita) tax on mutual cooperation above τ. Otherwise, I will defect.

It is not hard to see that for the ruler not to extract all the gains from cooperation and *to preserve incentives* (which in this case requires ensuring that A and B have no reasons to deviate from the strategies announced above), the citizens will want to announce a tax τ that is at most $4q - 2$.

The tax proposed by the citizens cannot be too low, however, because then the ruler would prefer to tax a low output (mutual defection) at a high rate rather than to tax a higher output (the gains from cooperation) at a lower tax rate. It can be shown that, in the example above, the lowest possible value of τ that the ruler will accept is $4 - 3q$. The presence of this lower bound is what keeps the citizens from obtaining a higher share of the gains from cooperation. As a consequence, in an efficient equilibrium at least 50% of the gains from efficiency will

always remain in the hands of the ruler. These results are summarized below (the proofs are reproduced in the Appendix).

> *Proposition 2.* The set of payoffs attainable in subgame perfect equilibria of the repeated game with collective action problems described above as the probability of continuation goes to one is not invariant under the addition of a rational, self-interested ruler with unmodeled authority. In particular, the set of payoffs attainable by the non-rulers is bounded away from the Pareto frontier of the utility possibility set.

To conclude, we see that the ruler levies taxes in both the static and the repeated version of the game by Gibbons and Rutten, but it is naturally the repeated version which teaches us the most. In particular, we learn that repetition allows the citizens to *regain* a great deal of the power that was lost to the ruler since her empowerment. What this means is that the source of authority in the Gibbons and Rutten model is explained as a combination of the unmodeled source of authority that the ruler has to begin with, and the endogenously determined limits to such authority that the citizens can set through the beliefs about their behavior that the ruler will have at a subgame perfect equilibrium of the repeated game. Since the source of authority that is modeled depends mostly on expected utility calculations done by the citizens that can be deduced from the examination of the game, and that whatever role coercion plays is unmodeled, I believe that the source of authority that this model explains is mostly of the *utilitarian* type. One could argue that the beliefs held by the citizen can also be described as coercive in the sense that they can be rewritten in the form of credible threats that prevent the citizens from deviating from the equilibrium strategies. This is true, as I mentioned at the beginning of this section, but I want to reserve the term *coercive* to situations where an explicit model of conflict besides the one implicit in the collective action problem is added to the social situation to be examined.

A complication in the interpretation of this model is that, if in the state of nature citizens A and B decide on a social contract to deal with the collective action problem embedded on the Prisoner's Dilemma, it is not clear that they would have chosen one in which they empower a ruler. To see why, note that for *any* probability of continuation q where a ruler gives them some of the gains from cooperation the citizens can do better by playing the standard trigger strategies against each other and having a disempowered government. This is the case because the *folk theorem* holds in this case for any probability of continuation greater than $\frac{1}{4}$. This means that the analysis by Gibbons and Rutten is useful only when there is a barrier to the application of the folk theorem

that cannot be made dependent on q, or when there is a good reason to assume that the authority was already in the hands of a specific actor. Gibbons and Rutten believe the barrier has to be understood as communication costs: in a large society it will be difficult to keep everyone informed of all the partial histories of play, a necessary datum in the construction of the cooperative equilibrium. The problem with this argument is that, as it is often done, costs that are crucial for their results are left unmodeled. In the next section, I present a model intended to provide an alternative explanation: one based in explicit conflict arising between the citizens, the outcome of which determines the balance of power in society.

2.5 A Model of Coercive Authority

Suppose that at the end of the day, right after you have harvested your bushel of corn, I come and say to you: 'give it to me, or I will beat you up'. Now suppose you do not give it to me. What will happen is that we will engage in a fight, and there is a chance of you winning and a chance of me winning. If you win, you will take all my corn, and if I win, I will be happy to take all your corn. Now this is about real conflict!

An additional feature that one could add to the conflict just described has to do with the realization that those probabilities of winning are not likely to be independent of the size of the output that we are defending. The higher the size of my output relative to yours, the higher my odds of winning, either because a relatively high output can finance a better military technology, or just because I fight harder when there is relatively more at stake.

Models with features that are similar to these exist and have been developed by Hirshleifer (1991) and Skaperdas (1992) to understand the relationship between the productive and predatory sectors of the economy.[6] In this section, I use some of the tools they develop in a very simplified manner. They model the production side of the game assuming the existence of two goods, one that can be consumed directly and one that gives you an edge over your opponent in the conflict but that does not enter your preference function directly. Here I present a simpler version with only one resource which both affects the odds of winning and enters directly in the preference function of the players.[7]

There is a sense in which any game with interesting properties can be given an interpretation that matches the story given above. An even more interesting position is to take a game of interest assuming that the payoffs reflect the existence of unmodeled authority and then replacing

Table 2. Prisoners' Dilemma

	C	D
C	3, 3	0, 4
D	4, 0	1, 1

such unmodeled authority with conflicts of the type described in the previous paragraphs. For instance, take a Prisoner's Dilemma (Table 2) and replace the unmodeled authority with conflict situations everywhere. The game transforms into that shown in Table 3 where $p(x_i, x_{-i})$ is the probability that individual i wins the contest given that the payoff to defend is x_i and that the payoff that citizen -i wants to defend is x_{-i}. The function p is often called a *contest success function*, and it has the following properties: it is increasing in the first argument and decreasing in the other, it is a probability distribution function, and it assigns identical probabilities to individuals of equal resource base.[8]

With this in mind, we can analyze the Prisoner's Dilemma with the conflict depicted above. As can be expected, the coercive structure imposed on the game can lead to an allocation of resources that is different from the one obtained with unmodeled authority. First, note that since $p(x,x) = \frac{1}{2}$ the game turns into that shown in Table 4 where there are no further restrictions on $p := p(0,4)$ besides $p(0,4) < p(0,0) = \frac{1}{2}$. In this context, the interpretation is that $p(4,0) > p(0,4)$ because there is a degree of specificity of the outcome at stake that is of value only to the original producer, and this gets reflected in the value of p.

It is not hard to see that for any $p \in (\frac{1}{4}, \frac{1}{2}]$ the strategy profile (C, C) is a Nash equilibrium of the game. On the other hand, for any p smaller than $\frac{1}{4}$ the only equilibrium of the game is (D,D). Call the game with

Table 3. Prisoners' Dilemma with conflict

	C	D
C	$6p(3, 3), 6p(3, 3)$	$4p(0, 4), 4p(4, 0)$
D	$4p(4,0), 4p(0, 4)$	$2p(1, 1), 2p(1, 1)$

Table 4. Prisoners' Dilemma with conflict, $p(x, x) = \frac{1}{2}$

	C	D
C	3, 3	$4p, 4(1 - p)$
D	$4(1 - p), 4p$	1, 1

collective action problems and conflict over the distribution of resources the game with *coercive* authority and let the measure of symmetry of the distribution of power be given by closeness of p to $\frac{1}{2}$. We have thus proved the following result for the game described above.

> *Proposition 3.* In the static game with collective action problems and coercive authority described above the efficient allocation can be supported as a Nash equilibrium of the coercive game for any sufficiently symmetric distribution of power.

What we have learned from the exercise is that the fact that there will be conflict may decrease the expected benefits from deviating if the likelihood of winning the conflict does not increase too quickly with the amount of resources at stake. Therefore, one could obtain an efficient allocation of resources that is supported by the exercise of mutual coercion, and that works without a ruler. In models like this, we can explain the phenomenon of cooperation in autarchy without appealing to repeated games considerations. Once the game is played repeatedly of course, one is able to solve the collective action problem if the probability that the game will be repeated is sufficiently high, and this will hold regardless of the value of p.

Now assume for the moment that we introduce into the analysis a third citizen, K, who produces nothing but has probability (not coming from a contest success function) arbitrarily close to one of winning a contest against every citizen and therefore collecting the entire output. *We end up with the model by Gibbons and Rutten*. This is, of course, an invitation to add to the model a third citizen with limited probability of winning that still does not produce anything to see if the citizen has any bearing on outcomes. The framework by Hirshleifer and Skaperdas is not, however, appropriate for investigating this question, for a citizen with little resources will have very small odds of winning and therefore of altering the outcome. Let us make sure of this before turning to models where such questions can be posed in a more satisfactory way. Consider the three player game in Table 5, where citizen A chooses rows, citizen B chooses columns, and citizen K *chooses nothing*. What we want to see is the effect that changes in the likelihood of winning have on the equilibria of the game.

Table 5. Coercive authority

	C	D
C	$6p(3, 3, 0)$, $6p(3, 3, 0)$, $6p(0, 3. 3)$	$4p(0, 4, 0)$, $4p(4, 0, 0)$, $4p(0, 4, 0)$
D	$4p(4, 0, 0)$, $4p(0, 4, 0)$, $4p(0, 0, 4)$	$2p(1, 1, 0)$, $2p(1, 1, 0)$, $2p(0, 1, 1)$

It can be shown that for no value of $p(.)$ that ensures cooperation in autarchy and that satisfies the properties of a contest success function does the introduction of a citizen with authority make any difference regarding outcome. However, the gains from cooperation that can be captured by the ruler can get very close to 50% (depending on the structure of $p(.)$). We thus reach the following conclusion, the proof of which is reproduced in the Appendix:

> *Proposition 4.* In the static game with collective action problems and coercive authority presented earlier the efficient allocation can be supported as a Nash equilibrium of the coercive game for any sufficiently symmetric distribution of power, regardless of the introduction of a player without choice set but with coercive authority (a ruler). However, the set of payoffs attainable by the non-rulers is bounded away from the Pareto frontier of the utility possibility set.

Note that both in the framework by Gibbons and Rutten and in the framework by Hirshleifer and Skaperdas, the introduction of an agent with authority is not critical to the solution of the collective action problem: there is always an equilibrium in autarchy that achieves the efficient outcome. Ironically, the introduction of such an agent with authority *creates* a collective action problem: how to prevent the empowered agent from capturing efficiency gains that he did not produce. One would want to see in the models that the ruler 'deserves' the rents he captures in equilibrium by showing that they would have not existed had he not been present. There is a limited sense in which this happens in models with persuasive authority, as I show below.

2.6 A Model of Persuasive Authority

If the source of my authority cannot be directly deduced from the examination of the game, is there really any way in which we can model it in a formal model of authority? Surprisingly, we can. Even if I do not have any actions to choose and will not engage in any contest with anybody to capture part of your output I can nevertheless have authority over you in some circumstances. What kind of circumstances? Well, I need you to believe that there is another 'sucker' in the game who will not choose an action that is very good for you if you do not choose the action I say and give me a fraction of your output. I also need the sucker to believe that you will choose an action that is bad for him if he does not give me a fraction of his output. But, why believe me? I have no good answer for that, but if you two *do* believe, maybe you will do as I say, and then the beliefs will support an authority over you that is of the *persuasive* type.[9]

Models with characteristics that are similar to those just discussed also have been discussed by Akerlof (1976) and Basu (1986). The feature that distinguishes them from the standard models of strategic interaction is their *triadic* nature, that is, models of situations where individuals do not interact pairwise but in triangular or other multiple relations. Once triadic relations are allowed a wealth of phenomena can be explained in game-theoretic language, even if some players have no direct effect on the outcome.

Suppose that citizens A and B are about to play the following game of trade: if both agree to trade (C,C), each citizen gets a payoff of 3. However, if at least one of the citizens does not want to trade, there is no deal and each gets a payoff of zero. The game is shown in Table 6, where citizen A chooses rows and citizen B chooses columns.

Now assume that before the citizens play the game citizen K comes and says privately to each player the following: 'Your fellow citizen will trade with you only if you pay me a tax of 3'. Under the world created by the statement issued by K each player is better off by paying the tax, and therefore trading than otherwise. The outcome does not refute the statement issued by K, for it is indeed a situation where both citizens cooperate and pay the tax and, given that they believe the statement to be true, neither citizen has a unilateral incentive to deviate. The players have reached a state of *self-confirming equilibrium*.[10] It might be assumed that such equilibria are delicate because they rely on threats that are not credible, but this misses an important point about how beliefs are instituted and transmitted in society.

2.6.1 Aside: Cultural Beliefs. Think about all the real life situations where you do not engage in certain behaviors because of the presumption that something really bad could happen to you. In many of those situations you did not acquire this presumption or conjecture by experience (i.e. being actually punished after the 'bad' behavior). Instead, you were told that this was going to happen to you by someone you trusted. *Those conjectures, if shared by the collectivity and not disproved by experience, will support a self-confirming equilibrium of*

Table 6. Game of trade

	C	D
C	3, 3	0, 0
D	0, 0	0, 0

the game. I argue that they are the exact content of that which Greif (1994) calls *cultural beliefs*. For Greif:

> Cultural beliefs are the ideas and thoughts common to several individuals that govern interaction—between these people, and between them, their gods, and other groups—and differ from knowledge in that they are not empirically discovered or analytically proved. (Greif 1994: 915)

Greif insists that there is an analytical benefit from distinguishing between strategies and cultural beliefs:

> Unlike strategies, cultural beliefs are qualities of individuals in the sense that cultural beliefs that were crystallized with respect to a specific game affect decisions in historically subsequent strategic situations. (Greif 1994: 915)

The point is completely made once one realizes that, as with cultural beliefs, the conjectures that support self-confirming equilibria need not be well-defined for situations that have never occurred. Moreover, if one wishes to interpret equilibria as the result of a learning process that takes place while the individuals interact repeatedly, then it is to self-confirming equilibria, rather than to Nash equilibria, to which those processes converge.[11]

2.6.2 Back to 'Models of Persuasive Authority'. In the example above citizen K does nothing but appropriate surplus that does not seem to belong to him; it is somewhat of a bonus associated with being a persuasive authority. This authority, however, can be employed to create value, as the following example shows.

Assume that the collective action problem between A and B has the structure typical of a *game of chicken* (Table 7). Suddenly, citizen K announces to each player separately: 'Your fellow citizen has paid to me a tax of $\frac{1}{2}$ for me to tell him what to do. I will tell you what I will tell him if you pay me a tax of $\frac{1}{2}$. If you do not pay, my recommendation to the other will be *C* with probability $\frac{1}{2}$ and *D* with probability $\frac{1}{2}$'. Now, if each citizen believes the statement made by K, and both know that K will just flip a coin to decide on what recommendation to give between (D,C) and (C,D), then each individual will pay the tax, listen to the recommendation, and indeed play what citizen K has told her to do. In other words, the citizens will be 'frozen'

Table 7. Game of chicken

	C	D
C	3, 3	1, 4
D	4, 1	0, 0

in a *correlated equilibrium* of the game.[12] In it, each citizen gets a payoff of 2, *net of taxes*, which is no smaller than what they were getting in the mixed strategy Nash equilibrium ($\frac{1}{2}C + \frac{1}{2}D$, $\frac{1}{2}C + \frac{1}{2}D$), i.e. 2.

This is no real service, however, for they could have done just as well by flipping the coin themselves and doing without the tax, just as in the models with utilitarian and coercive authority. Indeed, by costless communication, players could achieve any convex combination of the payoffs that can be supported in a mixed strategy Nash equilibrium of the game by correlating their strategies using a randomization device that is publicly observable. Here is, however, a payoff profile that cannot be achieved without help from citizen K: ($2\frac{7}{12}$, $2\frac{7}{12}$), and also a way to achieve it: change the tax to $\frac{1}{12}$ and have everyone know that K will select with probability $\frac{1}{3}$ each of the following profiles of recommendations: (D,C), (C,D) and (C,C). However, this time K will only tell the player her own recommended strategy. Again, the recommendation from K has no binding force, yet both players will pay the tax, play as K recommended, and achieve the payoff profile presented above. The proof is reproduced in the Appendix.

It would seem that a person of influence can do a lot of good, or a lot of harm to the rest of the members of society solely by virtue of her persuasive authority, but there are two reasons why this is not always true. First, pure influence has no effect in many games of interest, and that is the reason why I switched from the Prisoner's Dilemma to other games. Importantly, if the source of citizen K's authority is purely persuasive there is *nothing* he could say: no massaging of beliefs, no propaganda, or ideology, or promise or threat, that he could employ to influence citizens A and B away from the choice of (D,D) in the static Prisoner's Dilemma. Furthermore, this follows from the *revelation principle* for games with complete information. To see why, recall that according to the revelation principle, if there is a communication system with or without a mediator such that a certain outcome can be achieved as an equilibrium of the game with communication, then it can also be achieved by a correlated equilibrium of the game. But C is strictly dominated in the Prisoner's Dilemma, and therefore not rationalizable. This means that no strategy in a correlated equilibrium will place positive probability on action C, and so the collective action problem will remain invariant under the addition of a player with persuasive authority. This *may* suggest that the Prisoner's Dilemma is not the right collective action problem to employ when formulating models of limited government.[13] The second reason why the govern-

ment again may be a redundant entity is that, in games with more than three citizens, communication mechanisms have been identified that would implement any correlating device that citizen K could possibly employ at no cost to the citizen except, perhaps, unmodeled communication costs.

What we have seen here is a discussion that stresses the role of beliefs held by certain players in the game that can have effects on the outcomes, yet those beliefs cannot be directly deduced from the examination of the game. The literature refers to those as supporting self-confirming equilibria and correlated equilibria.[14] More importantly, they seem to be the natural vehicle to discuss the content of the terms *influence*, *cultural beliefs* and *ideology*. The role of ideology has traditionally been neglected in formal models of institutions, not because anybody believed it was a useless concept,[15] but because it is very elusive. This situation has not changed, of course, but at least now there is a chance for formal insight where there was none.

3. Discussion

The relationship between the presence of a limited government and the type and size of collective action problems in the equilibria of social systems is far less than transparent: It depends a great deal on the type of actions and beliefs that support the authority held by the government. In all the models studied above, the outcomes achievable thanks to government intervention were also achievable without it, and this conclusion needed no additional assumptions to those already in the pertinent theory. But are these conclusions general enough, or are they an artifact of the specific models of utilitarian, coercive and persuasive authority studied above? In this section, I provide an answer to this question.

3.1 Preliminaries

An examination of the common features of all the models studied above reveals that the effects that utilitarian and coercive authority have on the outcomes of the situations analyzed are qualitatively different from the effects that persuasive authority may have. Specifically, the introduction of a ruler with utilitarian or coercive authority affects the outcomes of the situation through explicit changes on the *structure of the game* that is played by the citizens. The introduction of a ruler with persuasive authority alters the outcomes through a different

channel: that of affecting the *beliefs* that the citizens have about each other's actions. Despite this difference, the argument that explains the redundancy of governmental authority is very similar in both cases, but the difference makes it necessary to separate the analyses. I proceed first with the easiest case.

3.2 Communication Costs and the Demand for Government

I previously gave examples of models of persuasive authority that show how the revelation principle can be used to understand why persuasive authority has no bite in many games of interest. The examples imply no loss of generality in the sense that the same arguments can be used to show that persuasive authority will have no bite in general. The key to the argument is to realize that, when the original game among the citizens is enhanced to include a ruler with persuasive authority, the ruler is playing the role of a *communication system* in the sense of Myerson (1991). But the revelation principle for games with complete information states that the equilibrium achieved by the game with the ruler can be simulated by a correlated equilibrium of the original game. Therefore, *if communication costs are negligible relative to the rents captured by the ruler*, then the ruler's presence is eminently redundant.

3.3 Contracting Costs and the Demand for Government

The actions chosen by a ruler with utilitarian authority alter the structure of the game that ends up being played by the citizens, and therefore the outcomes that one obtains in equilibrium. One may believe that these changes in the structure of the game are necessary for cooperation to be established. This is, however, not the case: the conditions under which the actions of the ruler induce the cooperative outcome also allow the cooperative outcome to be an equilibrium of the game in anarchy. This is so because, technically, the ruler can only change the structure of the game in the same ways in which the introduction of costless contracting changes the structure of the game.

The assumption that costless contracting is available is strong, but it is not required in the present context because the type of costless contracting that is available to the players in the model of utilitarian authority is that of costless *implicit* contracting, which is in turn possible due to a sufficiently high probability of the game being repeated. As a consequence, if an outcome cannot be attained as an equilibrium of the contract-signing game, then it cannot be attained as

an equilibrium of the game with a utilitarian or coercive ruler. Therefore, *if contracting costs are negligible relative to the rents captured by the ruler*, then the ruler's presence is eminently redundant.

3.4 The Costs of Conflict and the Demand for Government

In the model of coercive authority what allows cooperation to occur is that the presence of explicit conflicts lowers the relative benefits of defection. This is so because conflict is modeled in such a way that the producer of most of the output gets the highest probability of success in the conflict, and therefore the defector gets a lower expected payoff from defection than in the case without conflict. In addition to this, whenever the value of defecting has been sufficiently lowered to induce cooperation in the absence of a ruler, the introduction of a ruler makes no difference regarding outcomes. The intuition is that the fact that the ruler does not engage in any productive activities places a limit on the amount of rent that it can capture in any situation, and this leaves unaltered the relative merits of the different strategies that the citizens can follow. Therefore, *if the costs of engaging in conflict activities are negligible relative to the rents captured by the ruler*, then the ruler's presence is eminently redundant.

4. Conclusions

Parsons (1963) suggested that there are basically three kinds of authority: utilitarian authority, coercive authority, and persuasive authority. The first goal of this paper was to show that the models developed by Gibbons and Rutten (1997), Hirshleifer (1991), Skaperdas (1992), Akerlof (1976) and Basu (1986) can be viewed as models where issues such as authority, power, influence and ideology, in the sense of Parsons, can be formally discussed.

These game-theoretic representations of the forms of authority studied by Parsons can be used to analyze the extent to which the presence of a government with Parsonian authority can solve the collective action problems embedded in a situation of anarchy. In all the models studied in the paper, the outcomes achievable thanks to government intervention were also achievable without it, regardless of the type of authority suporting governmental actions, and this conclusion needed no additional assumptions to those already in the pertinent

theory. I also argued that these conclusions are not an artifact of the specific models of utilitarian, coercive and persuasive authority studied, but that they ultimately depend on how unmodeled communication, contracting and coercion costs would compare to the rents that the ruler obtains from the interaction.

Aside from the important issue of how it is that a ruler may economize on communication, contracting and coercion costs, this leads to an interpretation of the state that cannot be contractarian in nature: citizens would not empower a ruler to solve collective action problems in any of the models discussed, for the ruler would always be redundant *and* costly. The results support a view of the state that is eminently predatory, case in which whether the collective action problems are solved by the state or not depends upon whether this is consistent with the objectives and opportunities of those with the (natural) monopoly of violence in society. This conclusion is also reached in a model of a predatory state by Moselle and Polak (1997). How the theory of economic policy changes in light of this interpretation is an important question left for further work.

I would like to conclude by stressing that it does *not* follow from the results presented in the present paper that a contractarian interpretation of the state cannot be achieved in the language of game theory and supported by a reasonable notion of authority. It does follow, however, that it is not appropriate for the arguments that support such interpretations to crucially depend on unmodeled and arbitrarily high transaction costs. The results from this paper can therefore be viewed as pointing to the kinds of environments and costs that need to be explicitly studied to fully support the activities and the authority of the State, whether contractarian or predatory, in non-cooperative models of rationality, self-interest and equilibrium.

NOTES

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1. See Cooter (1982).
2. Some of those efforts include the work of the Washington School (Barzel 1989; Levi 1988; North 1981; Olson 1993), the Virginia School (Brennan and Buchanan 1985; Tollison 1982; Tullock 1980), the Chicago School (Stigler 1971; Becker 1983; Peltzman 1976; Baron 1989), the California School (Calvert 1994; Greif et al. 1994),

the neo-Hobbesian school (Hampton 1986; Hardin 1991; Heckathorn and Maser 1987), and those works reviewed in the present paper.

3. This is so because the act of obeying is represented, from a decision-theoretic standpoint, as one that tautologically yields a *payoff* with *sufficient certainty*. Persuasion and utilitarian rewards are, in a decision-theoretic sense, always present.
4. This is so because standard decision theory makes no distinction between pleasure-seeking and pain-avoiding motivations for behavior.
5. In contrast, a government modeled as a traditional social planner is omnipotent. See Dixit (1996: 8).
6. Other social situations that can be explained using similar tools include rent-seeking (e.g. Tullock 1988; Pérez-Castrillo and Verdier 1992), tournaments (Rosen 1986), and electoral campaigns (Skaperdas and Grofman 1995).
7. Conflict is not a choice variable in this context, not because I believe this is unimportant but because the main point of this sub-section does not hinge on it.
8. The properties of contest success functions have been thoroughly studied by Hirshleifer (1989) and Skaperdas (1996).
9. The pioneering work of Hermalin (1998) on the study of leadership can be interpreted as being precisely about what the sources of persuasive authority are in an environment of incomplete information. In contrast, I show that persuasive authority can have a role even in complete information environments.
10. For a formal definition, and its relationship with standard solution concepts, see Fudenberg and Levine (1993a). Basu (1986) and Akerlof (1976) in their papers used a solution concept that is in a sense a precursor of the concept of self-confirming equilibrium, see also Hahn (1987).
11. See, for example, Fudenberg and Levine (1993b) and Kalai and Lehrer (1993).
12. For a formal definition and an excellent discussion of the subject, see Myerson (1991).
13. For an excellent discussion that uses different arguments than those I give here about the dangers of selecting the Prisoners' Dilemma when other models of collective action seem more appropriate, see Baird et al. (1994).
14. Importantly, these concepts are deeply related, see Fudenberg and Levine (1993a).
15. See, for example, the discussion in Skaperdas and Syropoulos (1995).

APPENDIX

Proof of Proposition 1

The game played by the citizens in the absence of a ruler is a Prisoners' Dilemma with an outside option. In this case the set of payoff profiles attainable by the citizens is $\{(1,1)\}$, corresponding to the equilibria $\{(D,D),(D,O),(O,D),(O,O)\}$. I want to show now that in any Nash equilibrium of the game with a ruler, the set of payoff profiles attainable by the citizens is also $\{(1,1)\}$. To see this we have to consider two cases. First there is a case where one of the players select their outside options, and their payoff profile is (1,1), no matter what the ruler does. The other case involves no citizen selecting their outside option. Unilateral cooperation cannot be made an equilibrium for any tax system t because

against defection the remaining citizen can always take the outside option and not cooperate. Mutual defection can be an equilibrium, but only if $t_d = 0$. Similarly, mutual cooperation is sustainable as an equilibrium only if $t_c = 2$ and $t_f = 3$. In all these cases the payoff profile for the citizens is (1,1).

Proof of Proposition 2

First, I want to show that the citizens will not want to support a tax τ that is greater than $4q - 2$. To see this, one notices that any citizen gets a payoff from following the strategy outlined above equal to:

$$V = (3 - \tau) + qV, \text{ or}$$

$$V = \frac{3 - \tau}{1-q}$$

The payoff that a citizen obtains from deviating from such strategy is at most:

$$4 + \frac{1}{1 - q}$$

The first term corresponds to the expected payoff the citizen gets from defecting, obtaining a payoff of 4. The second term corresponds to the expected payoff the citizen obtains if the game continues, citizens A and B stick to their strategy, and the ruler places no tax on mutual defection. Therefore, a citizen will not want to deviate from the announced strategies if $\frac{3-r}{1-q} \geq 4 + \frac{1}{1-q}$, that is, if $\tau \leq 4q - 2$.

I now want to show that the lower bound on τ imposed by the ruler's ability to accept higher taxes on smaller outcomes is $4 - 3q$. The expected payoff for the ruler that deviates from setting a tax on cooperation no greater than τ is at most $6 + \frac{2}{1-q}$.

The first term corresponds to the expected payoff the ruler gets if completely abusing the citizens capturing the entire output after they cooperate. The second term corresponds to the expected payoff the ruler gets if the game continues, citizens A and B stick to their strategy, and the ruler, again, completely abuses the citizens capturing the entire output. Now note that the payoff for the ruler that accepts a tax on cooperation of τ is:

$$V = 2\tau + qV, \text{ or}$$

$$V = \frac{2\tau}{1-q}$$

so that it is best for the ruler not to deviate from a tax on mutual cooperation of τ if:

$$\frac{2\tau}{1-q} \geq 6 + \frac{2}{1-q}, \text{ that is, if } \tau \geq 4 - 3q.$$

As a consequence of the calculations presented earlier, the set of admissible tax rates on mutual cooperation that can be supported in any efficient subgame perfect equilibrium is $[4-3q, 4q-2]$ for $q \geq \frac{6}{7}$, which is a set that goes to [1,2] as q approaches 1. The gains from cooperation per capita in this setup are 2 per period, so any tax on cooperation captures at least 50% of those gains for any probability of continuation q that sustains the equilibrium, and therefore the payoffs attainable by the citizens are bounded away from the Pareto frontier of their utility possibility set.

Proof of Proposition 4

Because of the structure of $p(.)$ we can rewrite the game as:

	C	D
C	$6p_1, 6p_1, 6p_1(1-2p_1)$	$4p, 4(1-2p), 4p$
D	$4(1-2p), 4p, 4p$	$2p_2, 2p_2, 2(1-2p_2)$

Where $p_1 := p(3, 3, 0)$, $p_2 := p(1, 1, 0)$ and $p := p(0, 4, 0)$, and the following relationships follow:

$$(1-2p) > p_1 > \frac{1}{3} > p \text{ and } p_1 > \frac{1}{2}$$

Now note that for no p that ensures cooperation in autarchy and that satisfies the properties of the function $p(.)$ the introduction of a citizen with authority makes any difference regarding outcomes. To see why note that for any p in $(\frac{1}{4}, \frac{1}{3})$ we know that $\frac{2}{9} < \frac{2}{3}(1-2p) < \frac{1}{3} < p_1$, so that $4(1-2p) < 6p_1$ and hence C is a best response against C. Moreover, since $\frac{1}{2} > p_2$ we know that $4p > 1 > 2p_2$ and hence C is also a best response against D. Therefore, the game is dominance solvable with solution (C,C). However, there are no further restrictions on p_1 besides $p_1 < 1 - 2p$ and $p_1 < \frac{1}{2}$, so the value of p_1 can get close to $\frac{1}{3}$ as p is closest to $\frac{1}{3}$. This means that the expected payoff for the ruler can get close to 2, which is exactly 50% of the gains from cooperation.

Citizens Obey The Ruler in a Correlated Equilibrium of the Game in Models of Persuasive Authority

The argument is taken from Myerson (1991: 251), although he does not have the mediator charge anything for his services.

When citizen A is told 'C' she knows that citizen B was told to do either D or C, with equal probability, and does not win from deviating to D because from either choice she gets a payoff of $1\frac{11}{12}$. On the other hand, if citizen A is told 'D', then she knows that citizen B was told to do C, case in which his best response is D. Therefore, citizen A obeys the command by citizen K if she expects B to obey, too. A similar argument applies to citizen B.

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