TSUNAMIGENIC PROBABILISTIC FAULT DISPLACEMENT HAZARD ANALYSIS FOR SUBDUCTION ZONES

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ABSTRACT

The recent Sumatra earthquake and subsequent tsunami has provoked greater awareness of the hazard posed by coseismic fault displacement associated with sea-floor subduction zones. This catastrophic event has focused renewed efforts on tsunami forecasting, modeling, and detection. Yet the mechanism that causes this type of tsunami, coseismic fault displacement of a sea-floor subduction zone, is still treated deterministically. This paper describes a methodology for probabilistic fault displacement hazard analysis (PFDHA) for a sea-floor subduction zone, and presents example displacement hazard curves for the Cascadia Subduction Zone. The goal of this probabilistic methodology is to quantify the uncertainty and associated hazard of coseismic fault displacement of sea-floor subduction zones. This will provide tsunami modelers with a probabilistic measure of the occurrence of fault displacement, and decision makers with a rational basis for tsunami hazard mitigation measures.

Introduction

Historically, the death toll from tsunamis rivals the death toll from all other earthquake disasters. Mitigating the hazard posed by tsunamis is an urgent task for researchers. Tsunamis are caused by either coseismic sea-floor displacement or seismic induced marine landslides. The forcing function for a tsunami is a seafloor offset that is translated into a gravity wave in the overlying ocean.

The bulk of current tsunami research focuses on estimating run-up heights and generating inundation maps based on deterministic estimates of the forcing function. This limits the decision makers to an all-or-nothing approach when it comes to hazard mitigation. In using a probabilistically based forcing function in conjunction with advanced tsunami modeling, decision makers will be able to assess hazard as a function of recurrence rate, and make cost-benefit based decisions in a more systematic manner. Probabilistic predictions will allow for performance-based engineering and assessment of coastal areas.

The research described herein presents a methodology for estimating tsunamigenic fault

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displacement in a probabilistic manner based on the well-known methodology of probabilistic seismic hazard analysis (PSHA). The results are presented as hazard curves, or curves showing estimated fault displacement as a function of the rate of exceedance.   The ultimate goal of this research is to provide probabilistic estimates of tsunamigenic fault displacement for all subduction zones around the world. The hazard curves can then be used as input into tsunami models for estimating recurrence rate dependent run-up and inundation.

Probabilistic Fault Displacement Hazard Analysis (PFDHA) Methodology

Typical probabilistic seismic hazard analyses assume that earthquake occurrence can be modeled as a Poisson process, or that the probability of exceedance in a specified exposure period (typically corresponding to the useful life of a project) can be estimated as (Yegian, 1979),

$$P\left[A>a,t\right]=1-e^{-\left[v(a)t\right]} \tag{1}$$

where $P[A>a,t]$ is the conditional probability of an earthquake's acceleration (A) exceeding a specified acceleration (a) during a time interval (t) given that an earthquake will occur. The term $v(a)$ is the mean annual rate of exceedance of the specified acceleration level (a).

Computation of the seismic hazard involves accounting for uncertainties in earthquake size, location, frequency, and resulting ground motions. The annual rate at which the ground motion, A, will exceed a particular value, a, can be computed using the theorem of total probability (Cornel 1968, 1971),

$$v(a) = N_{M\,\min} \iint\limits_{M\,R} P\left(A > a|m,r\right)\cdot f_M\left(m\right)\cdot f_R\left(r\right)\cdot dm\cdot dr \tag{2}$$

where N_{Mmin} is the annual number of all earthquakes from a particular source above a minimum magnitude (M_{min}), $P(A>a|m,r)$ is the probability that the ground motion exceeds a certain value given the earthquake magnitude and the distance from the source (as represented by an empirical strong motion "attenuation" relationship), and $f_M(m)$ and $f_R(r)$ are probability density functions describing the relative distributions of magnitude and distance.

In this study we are interested in the probability of exceedance of surface displacement, particularly tsunamigenic reverse fault displacement along a subduction zone. Following the earthquake approach presented in Youngs et al. (2003), we substitute the density function of displacement for acceleration in Equation 2. For this hazard calculation we are interested in the displacement at a particular location, therefore we neglect the probability density function of distance. The annual rate at which surface displacement, D, will exceed some value, d, can then be computed as,

$$v(d) = N_{M\,\min} \int\limits_{M} P\left(D > d|m\right)\cdot f_M\left(m\right)\cdot dm \tag{3}$$

The term $P(D>d|m)$ is analogous to the ground motion attenuation relationship but here it is an

empirical relationship that estimates fault displacement. This term is composed of two parts; the first is the conditional probability of surface rupture given the occurrence of an earthquake, and the second is the conditional probability distribution of the amount of displacement given that surface rupture has occurred. The total probability of these two terms can be written as,

$$P(D > d|m) = P(SR|m) \cdot P(D > d|m, SR)$$

(4)

Magnitude Recurrence

In this study, the relative distribution of magnitudes for a fault is modeled using the Youngs and Coppersmith (1985) characteristic magnitude model. This is a composite model formed of two parts; (1) an exponential part in low to moderate magnitudes, and (2) a characteristic part with constant probability density in a small range of magnitudes around the characteristic event. The model was developed initially to account for observations of the magnitude and frequency characteristics of paleo-earthquakes observed in the Quaternary geologic record (i.e., paleoseismology) that could not be determined from the instrumental and historical records of seismicity. Because the data are fault-specific, the characteristic earthquake model is appropriate to model fault sources having assigned magnitude and slip rate information. In the characteristic model, the frequency of events near the maximum earthquake magnitude is larger than the expected frequency predicted from an extrapolation of smaller magnitude events. The characteristic model was chosen over the truncated exponential model because most tsunamigenic subduction events are documented using paleoseismic data.

The magnitude density function for this model is given by:

$$f(m) = \begin{cases} \dfrac{1}{1+c_2} \cdot \dfrac{\beta \cdot \exp\left(-\beta(\overline{M}_{char} - M_{min} - 1.25)\right)}{1 - \exp\left(-\beta(\overline{M}_{char} - M_{min} - 0.25)\right)} & \text{for} \quad \overline{M}_{char} - 0.25 < M < \overline{M}_{char} + 0.25 \\ \dfrac{1}{1+c_2} \cdot \dfrac{\beta \cdot \exp\left(-\beta(M - M_{min})\right)}{1 - \exp\left(-\beta(\overline{M}_{char} - M_{min} - 0.25)\right)} & \text{for} \quad M_{min} \le M \le \overline{M}_{char} - 0.25 \end{cases}$$

(5)

$$\text{where} \quad c_2 = \frac{0.5 \cdot \beta \cdot \exp\left(-\beta(\overline{M}_{char} - M_{min} - 1.25)\right)}{1 - \exp\left(-\beta(\overline{M}_{char} - M_{min} - 0.25)\right)}$$

$$\text{and} \quad \beta = \ln(10) \cdot (b - value)$$

The *b-value* is the slope of a semi-log plot of historical seismicity (i.e., the truncated exponential recurrence model; Gutenberg and Richter, 1954), M_{min} is the minimum considered magnitude, $\overline{M}_{char}$ is the mean characteristic magnitude which is the maximum magnitude minus 0.25. The annual number of earthquakes larger than the minimum magnitude, N_{Mmin}, is computed by equating the accumulated seismic moment to the moment released from earthquakes.

Conditional Probability of Surface Rupture

An empirical method for estimating the conditional probability that the rupture will reach the surface was developed by Wells and Coppersmith (1993) using a logistic regression model. Given the occurrence of an earthquake, the probability of rupture reaching the surface can be calculated using the following cumulative probability distribution,

$$P(surface\_rupture) = \frac{e^{f(x)}}{1 + e^{f(x)}}$$ (6)

where $f(x) = a + b \cdot M$

In Equation 6, M is the earthquake magnitude, and a and b are parameters estimated from the data. From the Wells and Coppersmith (1993) database that included 276 worldwide earthquakes, the values reported for a and b are -12.51 and 2.053 respectively. These are the values currently used in this study. In the near future we plan to reevaluate the fit of this distribution to reverse and/or subduction events as part of ongoing research.

Probability of Displacement Exceedance

In this study we are only concerned with the probabilistic displacement along each segment of the subduction zone. Therefore the effect of spatial variability of rupture displacement with respect to location along the fault segment is not considered.

The conditional probability of exceedance is based on empirical distributions from Wells and Coppersmith (1994). We calculate the probability of displacement for each fault segment by treating least squares regression results as lognormally distributed (Youngs et al., 2003). Equations reported in Wells and Coppersmith (1994) for the median displacement are of the form,

$$\log(D) = a + b \cdot M$$ (7)

where D is the displacement of interest, a and b are the regression coefficients, and M is the magnitude. The conditional probability of exceedance is then the cumulative lognormal distribution of D which can be written as (Ang and Tang , 1975),

$$P(D > d | M, Slip) = 1 - \Phi\left(\frac{\log(d) - \mu}{\sigma}\right)$$ (8)

where μ is the log(D) calculated using Equation 7, σ is the standard deviation of the random error, and Φ is the standard normal distribution function.

For this study we reevaluated the regression of moment magnitude vs. displacement with respect to subduction events in two ways; 1) the existing data was revised, and 2) new data was added.

To provide the largest sample size from the existing data set we used only the Maximum Displacement (MD) data from Wells and Coppersmith, which had a third more data points than

the Average Displacement (AD). For some of the MD data points there was no reported moment magnitude (M_w), which meant that these data points were not used in the regression. To remedy this we converted the reported M_s to M_w using a conversion compatible with Heaton et al. (1986). This provided us with maximum available sample size from the Wells and Coppersmith data set.

The existing data set contained little or no displacement measurements from subduction events, and we needed to extend the regression equation to a magnitude range more representative of subduction events. We collected a small data set of coseismic displacements associated with subduction events, these are measured maximum displacements or maximum displacements inferred from paleo-seismic information. This set of subduction data is by no means complete, but gives an indication of the relative appropriateness of the regression line with respect to subduction displacements. In future investigations we will collect more subduction displacement data to provide a better estimate of M_w vs. MD for subduction events.

Shown in Figure 2 are the results from the reevaluation of the Wells and Coppersmith regression of moment magnitude vs. maximum displacement. The coefficients used in this analysis are compared to the previous values in Table 1. These results are only preliminary, and are presented here to demonstrate future research needs. The plot shows that the reevaluated regression is not too dissimilar from the previous results, and the regression equation has been extended to an upper magnitude of 9.2 (previously 8.1), now within the range for predicting displacements for subduction events.
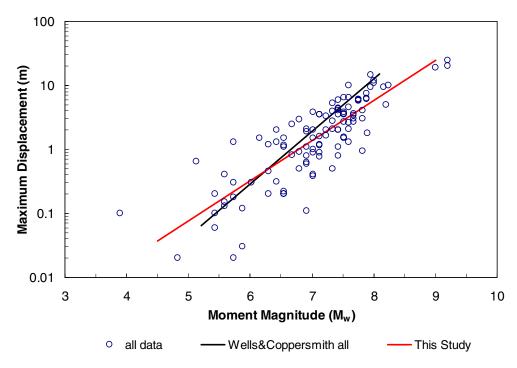

Figure 3. Plot of moment magnitude vs. maximum displacement. All data includes the revised data set from Wells and Coppersmith and the sub-set of data for subduction zones. The regression line from Wells and Coppersmith (1994) is for all slip types.

Table 1. Values for the *a* and *b* coefficients in Equation 7.

	a	*b*	standard deviation
This Study	-4.27	0.63	0.50
Wells and Coppersmith	-5.46	0.82	0.42

With these regression results we now have all the components for calculating the maximum coseismic surface fault displacement for subduction events.

Hazard Curves

To demonstrate the results of this methodology, we used the Cascadia subduction zone as a test case. In this example we are treating the Cascadia rupture as a single event occurring along the entire length of the subduction zone. The source parameters used for the Cascadia subduction zone are; fault rupture area of roughly 1000 km by 50 km, shear modulus of $3.5^{\cdot}(10)^{11}$ dynes/cm^2, minimum and maximum magnitude of 5.0 and 9.0 ($\overline{M}_{char}$) respectively, b-value of 0.8, and slip rate of 33 mm/yr. These parameters are not necessarily the most accurate but are used here for demonstration purposes. The slip rate is an average between the southern section of the subduction zone in Oregon and the northern section of the subduction zone off the Olympic peninsula as discussed in Miller et al. (2001). The b-value is consistent with events within the upper 35 km of the crust in the Western U.S. as used by Frankel et al. (2002).

The results of the hazard calculations are shown in Figure 2. Hazard curves are given for the annual probability of exceedance, and the exposure periods of 10, 100, and 500 years. The risk level of 10% in 50 years is commonly used in seismic design of critical structures such as hospitals and schools. Here the 10% in 50 years (i.e. ~475 year return period) risk level corresponds to roughly 20 meters of maximum surface fault rupture displacement. This agrees with the estimate of maximum displacement from the last major Cascadia Subduction event circa 1700 (Satake et al., 2003).

These hazard curves are a demonstration of probabilistic displacement calculations that can be used in conjunction with tsunami modeling to provide a probabilistic estimate of run-up. This methodology accounts for:
- The probability of earthquake occurrence,
- The probability of surface fault rupture, and
- The probability of exceeding a specific displacement value.

The authors hope that this methodology will be incorporated into tsunami forecasts to provide a realistic estimate of the uncertain nature of the forcing function of surface fault rupture along subduction zones.
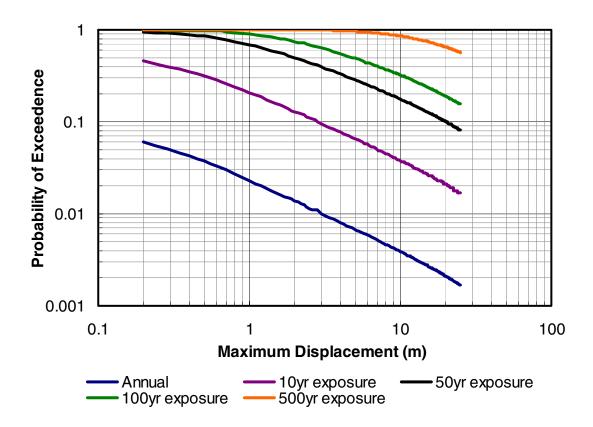
Figure 2.   Probability of exceedance of maximum surface fault rupture displacement for
different exposure periods.

Summary and Future Research

We have presented a methodology for calculating coseismic fault displacement of a sea-floor
subduction zone in the form of probabilistic fault displacement hazard analysis (PFDHA).
These calculations result in hazard curves for different return periods. To demonstrate the
methodology we used the Cascadia Subduction Zone. The probabilistic displacement results for
the Cascadia Subduction Zone agree with estimates for the last major event along this
subduction zone. This research is an ongoing project and as this project continues we hope to:
1) Collect more data to augment the regression of moment magnitude vs. maximum
   displacement.
2) Reevaluate the fit and distribution of the conditional probability of surface fault rupture
   model with respect to subduction events.
3) Evaluate the translation of surface fault displacement into a tsunami wave, assessing
   sensitivity of the analysis to rupture obliquity.
4) Compile source parameters for subduction zones around the world and calculate hazard
   curves for fault segment capable of tsunamigenic displacement.

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