# ON THE NOTION OF PRECOHOMOLOGY

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Dedicated to Professor SAUL LUBKIN.

ABSTRACT. For a cochain complex one can have the cohomology functor. In this paper we introduce the notion of precohomology for a cochain that is not a complex, i. e.,  $d^{q+1} \circ d^q$  may not be zero. Such a cochain, with objects and morphisms of an abelian category A, is called a cochain precomplex whose category is denoted by Pco (A). If a cochain precomplex is actually a cochain complex, then the notion of precohomology coincides with that of cohomology, i. e., precohomology is a generalization of cohomology. For a left exact functor F from an abelian category A to an abelian category B, the hyperprecohomology of F is defined, and some properties are given. In the last section, a generalization of an inverse limit, called a preinverse limit, is introduced. We discuss some of the links between precohomology and preinverse limit.

### Introduction

Let  $\mathbf{Z}$  be the ring of integers and let A be an abelian category. Suppose a sequence of objects and morphisms in A is given

which may not satisfy  $d^{q} \circ d^{q-1} = 0$  for certain  $q \in \mathbb{Z}$ . Then one may not be able to take the cohomology at C<sup>q</sup>. We will introduce a functor

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for such a cochain by initially complexifying the cochain to a cochain complex, then taking the cohomology of the complex. For diagram (or element) chasing, we use an exact imbedding of A into the category of abelian groups. It should be noted that precohomology is a self-dual construction and that it is not an exact connected sequence of functors. Furthermore, for each  $n \in \mathbb{Z}$ , Ph<sup>n</sup> is half exact. Hence, they are not derived functors, see §1.

## 1. Precohomology

Let A be an abelian category, and let Co(A) and  $Co^+(A)$  be the categories of cochain complexes and positive cochain complexes of objects in A, respectively.

DEFINITION 1.1. A sequence of objects and morphisms of A,

 $\xrightarrow{dq-1} \overset{dq}{\longrightarrow} \overset{dq}{Cq+1} \overset{dq}{\longrightarrow} \overset{dq}{\longrightarrow} \cdots$ 

is said to be a cochain precomplex, whose category is denoted by Pco (A), and Pco<sup>+</sup> (A) denotes the category of positive cochain precomplexes. A morphism  $(f_q)_{q \in \mathbb{Z}} : (C^q, d^q)_{q \in \mathbb{Z}} \to (D^q, e^q)_{q \in \mathbb{Z}}$  in Pco (A) is a sequence of morphisms  $f_q : C_q \to D_q$  such that the diagram



commutes, i.e.,  $f_{q+1} \circ d^q = e^q \circ f_q$  for  $q \in \mathbb{Z}$ .

NOTE. A cochain precomplex  $(C^q, d^q)_{q \in \mathbb{Z}}$  is a cochain complex if  $d^{q+1} \circ d^q = 0$  for  $q \in \mathbb{Z}$ .

LEMMA 1.2. Let  $(C^q, d^q)_{q \in \mathbb{Z}}$  be an object in Pco (A). Then  $(C^q/Im d^{q-1} \circ d^{q-2}, "d^{q''})_{q \in \mathbb{Z}}$ , abbreviated as  $("C^{q''})_{q \in \mathbb{Z}}$ , is an object in Co (A), where " $d^{q''}$  is the morphism induced by  $d^q$  as will be described below in the proof.

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Proof. Let.

be a cochain precomplex in Pco (A). Then the morphism ''dq'' is defined as the morphism

$$C^{q}/Im d^{q-1} \circ d^{q-2} \xrightarrow{ \ \ \, } C^{q+1}/Im d^{q} \circ d^{q-1}$$

such that  $''d^{q''}([c^q]) = [d^qc^q]$  in  $C^{q+1}/\text{Im} d^q \circ d^{q-1}$  for  $[c^q] \in C^q/\text{Im} d^{q-1} \circ d^{q-2}$ . Note  $''d^{q''}$  is well-defined. It remains to demonstrate that  $''d^{q+1''} \circ ''d^{q''}([c^q]) = 0$ . By the above definition,  $''d^{q+1''} \circ ''d^{q''}([c^q]) = [d^{q+1} \circ d^q (c^q)] = 0$  holds in  $C^{q+2}/\text{Im} d^{q+1} \circ d^q$ .

REMARK. The assignment of an object  $(C^q, d^q)_{q \in \mathbb{Z}}$  in Pco (A) to the object  $(C^q/\text{Im } d^{q-1} \circ d^{q-2}, "d^{q"})_{q \in \mathbb{Z}}$  is a right exact functor.

NOTE. We call this process (functor)  $(C^q, d^q) \xrightarrow[q \in \mathbf{Z}]{} (''C^{q''}, ''d^{q''})_{q \in \mathbf{Z}}$ the complexifying functor of the precomplex  $(C^q, d^q)_{q \in \mathbf{Z}}$ .

DEFINITION 1.3. For an object  $(C^q, d^q)_{q \in \mathbb{Z}}$  in Pco (A), define the q-th precohomology of  $(C^q, d^q)_{q \in \mathbb{Z}}$ , denoted as Ph<sup>q</sup> (C<sup>\*</sup>), by

$$Ph^{q} (C^{*}) = H^{q} (--- \rightarrow C^{q}/Im \ d^{q-1} \circ d^{q-2} \xrightarrow{'' d^{q''}} ---)$$
$$= Ker \ '' d^{q''}/Im \ '' d^{q-1''},$$

i. e., by the q-th cohomology of the cochain complex derived from the cochain precomplex  $(C^q, d^q)_{q \in \mathbb{Z}}$ .

NOTE. We have Ker  $''d^{q''} = \{ [c^q] \in C^q / \text{Im } d^{q-1} \circ d^{q-2} | d^q (c^q - d^{q-1}c^{q-1}) = 0 \text{ for some } c^{q-1} \in C^{q-1} \}$  and Im  $''d^{q-1''} = \{ [c^q] \in C^q / | \text{Im } d^{q-1} \circ d^{q-2} | c^q = d^{q-1} (c^{q-1}) \text{ for some } c^{q-1} \in C^{q-1} \}.$ 

From this note, we plainly have the following proposition.

PROPOSITION 1.4. Precohomology is a generalization of cohomology in the sense that precohomology coincides with cohomology in the case when a cochain precomplex is a cochain complex.

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DEFINITION 1.5. Let  $(C^q, d^q)_{q \in \mathbb{Z}}$  be a cochain precomplex in Pco (A), then the dual-complexifying functor of the precomplex  $(C^q, d^q)_{q \in \mathbb{Z}}$  is defined as  $(\text{Ker } d^{q+1} \circ d^q, 'd^{q'})_{q \in \mathbb{Z}}$ , where 'dq' is the restriction of dq on the subobject Ker  $d^{q+1} \circ d^q$  of Cq. The object which was obtained above is a cochain complex, denoted by  $('C^{q'}, 'd^{q'})_{q \in \mathbb{Z}}$  or simply by  $('C^{q'})_{q \in \mathbb{Z}}$ . Define the q-th dual-precohomology 'Phq (C\*) of a precomplex C\* as

$$Ph^{q}(C^{*}) = \text{Ker} \ 'd^{q'} / \text{Im} \ 'd^{q-1'}.$$

THEOREM 1.6. (Self-Duality of Precohomology). The canonical map from 'Cq' to ''Cq'' induces an isomorphism from 'Phq (C\*) to Phq (C\*) for each  $q \in \mathbb{Z}$ .

*Proof.* We will give a proof using [4]. Let us denote the canonical map 'Ph<sup>q</sup>(C\*)  $\rightarrow$  Ph<sup>q</sup>(C\*) by  $\Phi$ , i. e., for the cohomologous class  $\overline{\mathbf{x}}$  of  $\mathbf{x} \in \operatorname{Ker} '\operatorname{dq'} \Phi(\overline{\mathbf{x}}) = \overline{\pi_q}(\operatorname{i_q \mathbf{x}})$ , where i is the monomorphism Ker  $\operatorname{dq^{+1} \circ dq} \rightarrow \operatorname{Cq}$  and  $\pi_q$  denotes the projection  $\operatorname{Cq} \rightarrow \operatorname{Cq} / \operatorname{Im} \operatorname{dq^{-1} \circ dq^{-2}}$ . Notice  $\overline{\pi_q}(\operatorname{i_q \mathbf{x}}) = [\overline{\mathbf{x}}]$ , where  $[\mathbf{x}] \in "\operatorname{Cq''} = \operatorname{Cq} / \operatorname{Im} \operatorname{dq^{-1} \circ dq^{-2}}$ . This map is well-defined since "'dq" ( $[\mathbf{x}]$ ) = 0 holds in "'Cq^{+1''}. This is because  $\mathbf{x} \in \operatorname{Ker} '\operatorname{dq'}$ , i. e., 'dq' ( $\mathbf{x}$ ) = dq ( $\mathbf{x}$ ) = 0 in 'Cq^{+1'}. First we will show that  $\Phi$  is monomorphic. Suppose  $[\overline{\mathbf{x}}] = 0$ , then  $[\mathbf{x}] \in \operatorname{Im} "\operatorname{dq^{-1}''}$ . Hence  $\mathbf{x} = \operatorname{dq^{-1}}(\mathbf{x}^{q-1})$  as in the note after Def 1.3. We need to check  $\mathbf{x}^{q-1} \in \operatorname{Ker} \operatorname{dq^{-1} = 'Cq^{-1'}}$ .  $\operatorname{dqdq^{-1}}(\mathbf{x}^{q-1}) = \operatorname{dqx} = 0$  holds from the above. Secondly, we will prove  $\Phi$  is epimorphic. Let  $[\overline{\mathbf{x}}] \in \operatorname{Phq}(\operatorname{C}^*)$ . Then, since  $[\mathbf{x}] \in \operatorname{Ker} "\operatorname{dq''}, \operatorname{dq}(\mathbf{x} - \operatorname{dq^{-1} \mathbf{x'}}) = 0$  holds for some  $\mathbf{x'} \in \operatorname{Cq^{-1}}$ . Then  $\Phi(\overline{\mathbf{x} - \operatorname{dq^{-1} \mathbf{x'}}) = [\overline{\mathbf{x} - \operatorname{dq^{-1} \mathbf{x'}}] = [\overline{\mathbf{x}}]$  holds since  $-\operatorname{dq^{-1} \mathbf{x'} = \operatorname{dq^{-1}}(-\mathbf{x'})$ . Notice also  $\mathbf{x} - \operatorname{dq^{-1} \mathbf{x'} \in \operatorname{Ker} \operatorname{dq^{+1} \circ dq} = 'Cq'}$ .

PROPOSITION 1.7. (Half-Exacteness). Let  $0 \to C_1^* \xrightarrow{\alpha^*} C_2^* \xrightarrow{\beta^*} C_3^* \to 0$ be a short exact sequence in Pco (A). Then, for each  $q \in \mathbb{Z}$ , the sequence

$$\operatorname{Ph}^{q}(\operatorname{C}_{1}^{*}) \xrightarrow{\overline{\alpha}^{q}} \operatorname{Ph}^{q}(\operatorname{C}_{2}^{*}) \xrightarrow{\overline{\beta}^{q}} \operatorname{Ph}^{q}(\operatorname{C}_{3}^{*})$$

is exact at  $Ph^{q}(C_{2}^{*})$ .

*Proof.* Suppose  $\overline{\beta}^{q}([\overline{\mathbf{x}}]) = [\beta^{q}(\mathbf{x})] = 0$  holds in Ph<sup>q</sup>(C<sub>3</sub><sup>\*</sup>). That is,  $[\beta^{q}(\mathbf{x})] \in \operatorname{Im} "d_{3}^{q-1}"$  holds, which implies  $\beta^{q}(\mathbf{x}) = d_{3}^{q-1}(\mathbf{y})$  for some  $\mathbf{y} \in C_{3}^{q-1}$ . Since  $\beta_{3}^{q-1}$  is an epimorphism, there exists  $\mathbf{x}' \in C_{2}^{q-1}$  such

that  $\beta^{q-1}(x') = y$ . Let  $x'' = d_2^{q-1}x'$ . We obtain  $\beta^q(x''-x) = 0$  since  $\beta^q(x''-x) = \beta^q d_2^{q-1}x' - \beta^q(x) = d_3^{q-1}\beta^{q-1}(x') - \beta^q(x) = d_3^{q-1}(y) - \beta^q(x) = 0$ . Therefore one can find  $z \in C_1^q$  such that  $\alpha^q(z) = x'' - x$  by the exactness. We need to prove  $''d_1^{q''}[z] = 0$ , i.e.,

$$d_1^q z - d_1^q d_1^{q-1} z' = 0$$
 holds for some  $z' \in C_1^{q-1}$ .

We have that

$$\begin{split} \alpha^{q+1} d_1^q z &- \alpha^{q+1} d_1^q d_1^{q-1} z' = \alpha^{q+1} d_1^q z - d_2^q d_2^{q-1} \alpha^{q-1} z' = \\ &= d_2^q \left( \alpha^q (z) - d_2^{q-1} \alpha^{q-1} z' \right). \end{split}$$

Therefore, it is sufficient to prove  $[\alpha^q(z)] \in \text{Ker } "d_2^{q''}$ , i. e., to show  $[x'' - x] \in \text{Ker } "d_2^{q''}$ . Choose  $x' - x^0 \in C_2^{q-1}$ , where  $x^0$  is chosen such that  $d_2^q x - d_2^q d_2^{q-1} x^0 = 0$  for  $[x] \in \text{Ker } "d_2^{q''}$  above. Then

$$\begin{aligned} d_2^q \left( \mathbf{x}'' - \mathbf{x} - d_2^{q-1} \left( \mathbf{x}' - \mathbf{x}^0 \right) \right) &= d_2^q \mathbf{x}'' - d_1^q \mathbf{x} - d_2^q d_2^{q-1} \left( \mathbf{x}' - \mathbf{x}^0 \right) \\ &= d_2^q \left( d_2^{q-1} \mathbf{x}' - \mathbf{x} - d_2^{q-1} \left( \mathbf{x}' - \mathbf{x}^0 \right) \right) = 0 \end{aligned}$$

holds. Hence Ph<sup>q</sup> is a half-exact functor.

REMARK 1.8. Consider the following short exact sequence of precomplexes, denoted as  $0 \rightarrow {}^{2}\mathbf{Z} \rightarrow {}^{3}\mathbf{Z} \rightarrow {}^{1}\mathbf{Z} \rightarrow 0$ , of Pco<sup>+</sup>(A):

$$\begin{array}{c} \vdots \\ \vdots \\ 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \\ 0 \rightarrow 2 \rightarrow 2 \rightarrow 2 \rightarrow 0 \rightarrow 0 \\ \uparrow & id \\ 1 \end{pmatrix} \\ 0 \rightarrow Z \rightarrow Z \rightarrow 0 \rightarrow 0 \\ \uparrow & id \\ \uparrow & id \\ 1 \end{pmatrix} \\ 0 \rightarrow 0 \rightarrow Z \rightarrow Z \rightarrow 0 \rightarrow 0 \\ \uparrow & \uparrow & id \\ 0 \rightarrow 0 \rightarrow 0 \rightarrow 2 \rightarrow 2 \rightarrow 0 \\ \uparrow & \uparrow & \uparrow \\ 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \\ \uparrow & \uparrow & \uparrow \\ 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \\ \uparrow & \uparrow & \uparrow \\ 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \\ \uparrow & \uparrow & \uparrow \\ \vdots & \vdots & \vdots \\ \end{array}$$

Then the complexifying functor  $^{\prime\prime}$   $^{\prime\prime}$  applied to the above implies the diagram

	↑ ↑ ↑
	$0 \to 0 \to 0 \to 0 \to 0$
	↑ ↑ ↑
2)	$\mathbf{Z} \rightarrow 0 \rightarrow 0 \rightarrow 0$
	↑ id ↑ ↑
1)	$0 \rightarrow \mathbf{Z} \rightarrow \mathbf{Z} \rightarrow 0 \rightarrow 0$
	t d id id id
0)	$0 \to 0 \to \mathbf{Z} \to \mathbf{Z} \to 0$
	: : :

From this sequence of complexes, if  $Ph^*$  were an exact connected sequence of functors, one would obtain

$$\begin{array}{cccc} 0 \longrightarrow \operatorname{Ph}^{0}(^{2}\mathbf{Z}) \longrightarrow \operatorname{Ph}^{0}(^{3}\mathbf{Z}) \longrightarrow \operatorname{Ph}^{0}(^{1}\mathbf{Z}) \longrightarrow \operatorname{Ph}^{1}(^{2}\mathbf{Z}) \longrightarrow \dots \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & &$$

Hence,  $Ph^n$ ,  $n \in \mathbb{Z}$ , is not an exact connected sequence of functors.

Remark 1.9. The right derived functors of  $\mathrm{Ph^{0}}$  on  $\mathrm{Pco^{+}}\left( \mathrm{A}\right)$  are given by

$Ph^{0} = Ker (d^{0}),$	n = 0
Coker (d <sup>0</sup> ),	n = 1
0,	$n \ge 2$ .

## 2. Hyperprecohomology of a left exact functor

Let A and B be abelian categories and let  $F : A \longrightarrow B$  be a left exact additive functor.

DEFINITION 2.1. Let  $(C^q, d^q)_{q \in \mathbb{Z}} \in Pco^+(A)$ . By the complexifying functor, denoted by "" in the previous section, one has  $(C^q / \text{Im } d^{q-1} \circ d^{q-2}, "d^{q''})_{q \in \mathbb{Z}}$  as an object  $Co^+(A)$ . We will abbreviate the above associated cochain complex as "C\*". Then F"C\*" is an object of  $Co^+(B)$ . The q-th hyperprecohomology of F evaluated at C\*, denoted as Ph<sup>q</sup>F (C\*), is defined as the q-th hyperderived functor of F evaluated at "C\*".

NOTE 1. We have the following diagram of categories and functors

$$\begin{array}{ccc} & & & & & \\ Pco^+(A) & & & & \\ & & &$$

where functors  $(C^q, d^q)_{q \in \mathbb{Z}} \longrightarrow "C^{*''} \in Co^+(A), "C^{*''} \longrightarrow F''C^{*''} \in H^0$  $\in Co^+(B)$  and  $F''C^{*''} \longrightarrow Ker F'' d^{0''}$  are defined as in Definition 2.1, and  $H^0$ :  $Co^+(A) \longrightarrow A$  is defined by  $H^0$  ("C\*") = Ker "d^{0''} = Ker "d^{0''}

 $= H^0(C^*) = \text{Ker } d^0 \text{ and } F : A \longrightarrow B \text{ by Ker } d^0 \longrightarrow F (\text{Ker } d^0).$ Notice F (Ker  $d^0$ )  $\xrightarrow{\approx}$  Ker F  $d^0$  holds since F is left exact. Then there are induced spectral sequences

$$(2.1.1) \quad E_2^{p. q} = H^p \left( R^q F \left( {''C}^{\cdot \prime \prime} \right) \right) =$$
$$= H^p \left( \dots \rightarrow R^q F \left( {''C}^{p\prime \prime} \right) \rightarrow R^q F \left( {''C}^{p+1\prime \prime} \right) \rightarrow \dots \right)$$
$$(2.1.2) \quad {'E_2^{p. q}} = (R^p F) \left( Ph^q \left( C^* \right) \right)$$

with their abutement the hyperprecohomology  $Ph^{n}F(C^{*})$ , where  $R^{p}F$  denotes the p-th derived functor of F.

Furthermore, (2.1.1) can be extended to

 $(2.1.1'') \quad E_{i}^{p-q} = (R^{q}F) \; (''C^{p''}),$ 

see [2, pp. 118].

REMARK. We have the commutative diagram of categories and functors:



See Definition 2.1 and the above Note 1 for the description of each functor. The composition of functors leaving  $Pco^+(A)$  to B, counterclockwise, defines the zero-th hyperprecohomology  $Ph^{0}F(C^{*})$  of F at C<sup>\*</sup> in  $Pco^+(A)$ . The composition of functors leaving  $Pco^+(B)$  to B, clockwise, defines the zero-th precohomology of FC<sup>\*</sup>.

## 3. Preinverse Limit

Let  $(C^q, d^q)_{q \in \mathbb{Z}}$  be a cochain precomplex and be regarded as an inverse system:

$$\xrightarrow{dq-1} dq$$

$$\xrightarrow{dq} Cq \xrightarrow{q+1} \xrightarrow{} Cq$$

DEFINITION 3.1. Let A be an abelian category such that denumerable direct products of objects exist and such that the denumerable direct product functor is exact. Let  $C^0 = C^1 = \prod_{q \in \mathbb{Z}} C^q$  and define a morphism

$$\delta^0$$
 :  $\mathbf{C}^0 \to \mathbf{C}^1$ 

by  $\pi_{q+1} \circ \delta^0 = d^q \circ \pi_q - d^q d^{q-1} \circ \pi_{q-1}$ , where  $\pi_q : \prod_{q \in \mathbb{Z}} C^q \to C^q$  is the projection. Let  $\mathbb{C}^n = 0$  for  $n \neq 0,1$  and  $\delta^n = 0$  for  $n \neq 0$ . Then

$$0 \longrightarrow \mathbf{C}^{0} \xrightarrow{\delta^{0}} \mathbf{C}^{1} \xrightarrow{\delta^{1}} 0 \longrightarrow \cdots$$

is a cochain complex, denoted by  $C^{\ast}.$  Define the preinverse limit, denoted as Pim,

$$\underset{q \in \mathbf{Z}}{\operatorname{Pim}} \ C^{q} = \operatorname{H}^{0}(\mathbf{C}^{*}) = \operatorname{Ker} \ \delta^{0}$$

and define the 1-st preinverse limit, denoted as Pim<sup>1</sup>,

**.** 

$$\operatorname{Pim}^{1} \operatorname{Cq} = \operatorname{H}^{1} (\mathbf{C}^{*}) = \mathbf{C}^{1} / \operatorname{Im} \delta^{0}.$$

NOTE.  $\lim_{\leftarrow} C^q \subset Pim C^q$  and  $\lim_{\leftarrow} C^q \longrightarrow Pim^1 C^q$  hold, where  $\lim_{\leftarrow} and \lim_{\leftarrow} are$  the usual inverse limits.

THEOREM 3.2. Let  $(C^q, d^q)_{q \in \mathbb{Z}}$  be a cochain precomplex, regarded as an inverse system. There exists an isomorphism

$$\Pi \ "Cq" / \operatorname{Pim} \ "Cq" \xrightarrow{\sim} \operatorname{Pim} \ "Cq" / \Pi \ \operatorname{Phq} (C^*)$$

$$q \in \mathbb{Z} \qquad \longleftarrow \qquad q \in \mathbb{Z}$$

where "" is the canonical epimorphism  $\Pi C^{q} \rightarrow \Pi "C^{q}$ ".

Proof. Consider the following diagram.



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From the definition of "dq", one has  $\Pi$  Ker "dq" = Ker " $\delta^{0"}$  and  $\Pi$  Im "dq" = Im " $\delta^{0"}$ . Hence, the commutative diagram



implies, by a well-known lemma applied to the second and third short exact sequences, the following exact sequence,

$$\begin{array}{ccc} 0 \to \operatorname{Ker} l' \to \operatorname{Ker} l \to \operatorname{Ker} l'' \to \operatorname{Coker} l' \to \operatorname{Coker} l \to \operatorname{Coker} l'' \to 0. \\ & \| & \| & \| \\ 0 & 0 & 0 \end{array}$$

Hence, one obtains the isomorphism

$$\operatorname{Coker} l = \prod_{q \in \mathbf{Z}} \operatorname{''Cq''} / \operatorname{Pim} \operatorname{''Cq''} \xrightarrow{\simeq} \operatorname{Coker} l'' = \operatorname{Pim^1} \operatorname{''Cq''} / \prod_{q \in \mathbf{Z}} \operatorname{Phq} (C^*).$$

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