

A SIMPLE TEST OF THE LAW OF DEMAND FOR THE UNITED STATES

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1. INTRODUCTION

ONE OF THE PROPERTIES that one would like the aggregate (mean) demand system $F(p, \mu)$ of any economy to have is that the *Law of Demand* holds, i.e. that

$$(1) \quad (p - p') \cdot (F(p, \mu) - F(p', \mu)) < 0$$

for any two distinct price vectors p and p' and a given distribution μ of household characteristics. This is, after all, a condition under which the classical question of existence, uniqueness, and stability of equilibrium prices in general equilibrium theory can be addressed satisfactorily. It is well known, however, that the theory of Walrasian demand does not require aggregate demand to satisfy the *Law of Demand* as posed above.

Whether the *Law of Demand* holds for a particular economy cannot be determined from direct experience or empirical observations because it refers to hypothetical price changes within the same period. From the standpoint of consumer theory violations of the *Law of Demand* ought to be hard to find because they require the existence of Giffen goods that, while theoretically plausible, lack empirical support. While indirect evidence of this type exists,² whether the Law of Demand holds or not is a question important enough to deserve as direct and parsimonious analysis as possible. Put simply, the question becomes: under what conditions does the actual evolution of prices and demand over time inform us about whether the *Law of Demand* holds or not?

In this paper we explore a related question posed by Hildenbrand (1994), which we call the *Natural Time Law of Demand*: that for any two time periods t and τ ,

$$(2) \quad (p_t - p_\tau) \cdot (Q_t - Q_\tau) < 0,$$

where $Q_t = (Q_t^1, \dots, Q_t^l)$ denotes the vector of demands per period t for a list of l commodities; and $p_t = (p_t^1, \dots, p_t^l)$ denotes the vector of prices in period t . Relation (2) says that the vector $(p_t - p_\tau)$ of price changes and the vector $(Q_t - Q_\tau)$ of changes in quantities demanded point in opposite directions. This does not imply that for every commodity i it is the case that $(p_t^i - p_\tau^i) \cdot (Q_t^i - Q_\tau^i) < 0$, as the example in Figure 1 illustrates.

Clearly, neither does relation (2) imply (1), nor is it implied by the theory of individual or aggregate demand.³ For this reason we formulate a related hypothesis, the *Homoge-*

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² See Nachbar (1998), however, for a difficulty in the interpretation of standard tests for Giffen goods.

³ Yet, Hildenbrand says: “whether the actual evolution of prices and demand over time satisfies relation (2) is an interesting empirical question, which to my knowledge has never been analyzed” (Hildenbrand (1994, p. 5)).

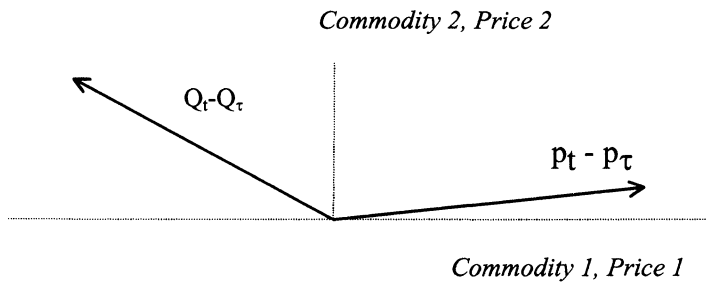


FIGURE 1

neous Law of Demand, which is the *Natural Time Law of Demand* when the prices have been linearly normalized by the current period (disposable) income w . The *Homogeneous Law of Demand* holds when, for any two time periods t and τ ,

$$(3) \quad \left(\frac{P_t}{w_t} - \frac{P_\tau}{w_\tau} \right) \cdot (Q_t - Q_\tau) < 0.$$

We perform nonparametric tests of relations (2) and (3) for the United States with time series data from the period 1959–1998. Relations (2) and (3) impose a negativity constraint on the means of the time series constructed from data on composite commodities. We test this hypothesis using time series techniques.

The empirical results suggest that both the *Natural Time Law of Demand* and the *Homogeneous Law of Demand* are consistent with the data. One may then regard these laws as facts that need to be made consistent with the theory of consumer demand.

Obtaining such consistency admits a variety of approaches, their common feature being the imposition of restrictions on the distribution of characteristics of the agents in the economy. The traditional approach in consumer theory would be to study the conditions under which a certain property holds for individual demands and to verify that it is a property that is preserved under aggregation. This is the approach pioneered by Gorman (1953), Muellbauer (1976), and Jorgenson, Lau, and Stoker (1982). Alternatively, one may study the conditions under which properties of aggregate demand arise out of aggregation itself, an approach pioneered by Hildenbrand (1983). We provide an argument that combines both approaches, as in Lewbel (1990, 1991, 1992).

The key to our argument is the following: while there is no direct connection between (2) and (1) it turns out that (3) is equivalent to (1) if (i) individuals do not suffer from money illusion, (ii) preferences are stable over time, and (iii) the distribution of income changes slowly relative to the speed at which aggregate income changes over time.⁴ This is so because under (i) and (ii) one can control for an increase (resp. decrease) in individual income by ‘deflating’ (resp. ‘inflating’) the general price level—hence the formulation p/w in (3)—and under (iii) one can exactly match the changes in individual income with changes in aggregate income. Under such conditions we provide an extremely parsimonious test of the *Law of Demand* that relies on very mild assumptions about preferences in the economy. Our results indicate that the *Law of Demand* is strongly consistent with the US data.

⁴ For empirical evidence that supports this assumption, see Lewbel (1992).

The structure of the remainder of the paper is as follows. In Section 2 we describe the data employed. In Section 3 we describe the econometric methodology and report empirical results. In particular, we apply the general trend function testing procedures of Vogelsang (1998a) to the case of testing a hypothesis about the mean of a time series. We derive asymptotic distributions and tabulate critical values for three new tests. In Section 4 we give a condition on the distribution of income under which the *Homogeneous Law of Demand* is equivalent to *Law of Demand*. Section 5 concludes.

2. THE DATA

The data employed were obtained from the National Income and Product Accounts for the United States, from the third quarter of 1959 to the second quarter of 1998. The data are quarterly. We selected the fourteen composite commodities shown in Table I, and this choice was based on the maximum disaggregation available for the data.

Each series of real personal consumption expenditure was paired with an implicit price deflator that we employ to compute price changes for such composite commodities. From the raw data on prices and quantities we constructed six series to test relations (2) and (3). Each of the series is a collection of data points of the form $\sum_{i=1}^{14} (y_t^i - y_{t-d}^i)(p_t^i - p_{t-d}^i)$, where y_t^i is a measure of consumption of composite commodity i at date t , p_t^i is a measure of price for such composite commodity i at date t , and d represents the frequency at which the data points are computed. To control for population growth we use *per-capita* consumption level. Measures of consumption are in real terms, and prices are normalized by the implicit price deflator of the index of total real consumption expenditures, to control for inflation. As mentioned in the Introduction, the formulation p/w controls for the effect of changes in real income under conditions discussed in Section 4. We grouped the series in the following way:

- *The NL Series* are three series, computed at a quarterly, semi-annual, and annual frequency ($d = 1, 2,$ and 4 respectively), where y_t^i and p_t^i are indices of real *per-capita* consumption and inflation-adjusted prices as reported in the National Income and Product Accounts. The *NL Series* are used to test the *Natural Time Law of Demand*.

- *The HL Series* are three series, computed at a quarterly, semi-annual, and annual frequency, where y_t^i is the index of real *per-capita* consumption of commodity i and p_t^i is the inflation-adjusted price index of commodity i divided by the index of total real consumption expenditures (the proxy for income that is used here). The *HL Series* are used to test the *Homogeneous Law of Demand*.

TABLE I
THE FOURTEEN COMPOSITE COMMODITIES

• Motor Vehicles and Parts	• Other Nondurable Goods
• Furniture and Household Equipment	• Housing
• Other Durable Goods	• Electricity and Gas
• Food	• Other Household Operation
• Clothing and Shoes	• Transportation
• Gasoline and Oil	• Medical Care
• Fuel Oil and Coal	• Other Services

3. ECONOMETRIC METHODOLOGY AND EMPIRICAL RESULTS

Each of the *NL* and *HL* time series are modeled as simple univariate processes given by

$$(4) \quad y_t = \beta + u_t$$

where $\{u_t\}$ is a mean zero random process so that $E(y_t) = \beta$. If the laws of demand hold, then strictly speaking each data point in the *NL* and *HL* series should be negative. This stringent requirement is unlikely to hold at all times. However, if on average the series are negative ($\beta < 0$), then we take that as evidence in support of the laws of demand. We take as the null hypothesis failure of the laws to hold and the alternative hypotheses that the laws hold, i.e. $H_0: \beta \geq 0$, $H_1: \beta < 0$. This simple hypothesis test is complicated by the fact that $\{u_t\}$ may be serially correlated and would be further complicated if a unit root in $\{u_t\}$ could not be ruled out. Therefore, we test H_0 using statistics that are robust to serial correlation in $\{u_t\}$. Several of the statistics are also robust to $\{u_t\}$ having a unit root.

3.1. The Statistics

Let $\hat{\beta} = T^{-1} \sum_{t=1}^T y_t$ denote the OLS estimate of β where T is the sample size. If $\{u_t\}$ is stationary, under fairly general regularity conditions $T^{1/2}(\hat{\beta} - \beta) \xrightarrow{d} N(0, \sigma^2)$, where $\sigma^2 = \sum_{j=-\infty}^{\infty} \gamma_j$ and $\gamma_j = E(u_t u_{t-j})$. Asymptotically valid inference regarding β can be obtained using a t statistic defined as $t = (\hat{\beta} - \beta) / (\hat{\sigma}^2 T^{-1})^{1/2}$ where $\hat{\sigma}^2$, a consistent estimate of σ^2 , replaces the usual OLS estimate of the error variance. We construct this t statistic using $\hat{\sigma}^2 = \sum_{j=-(T-1)}^{T-1} \kappa(j/s_T) \hat{\gamma}_j$, where $\hat{\gamma}_j = T^{-1} \sum_{t=j+1}^T \hat{u}_t \hat{u}_{t-j}$, s_T is the truncation lag,

$$\kappa(x) = 25 \frac{\sin(6\pi x/5)/(6\pi x/5) - \cos(6\pi x/5)/(6\pi x/5)}{(12\pi^2 x^2)}$$

(the quadratic spectral kernel),

and $\{\hat{u}_t\}$ are the OLS residuals. Following Andrews (1991) we choose s_T using a data dependent method based on an AR(1) plug-in method. See Andrews (1991) for details. We denote the t statistic using this estimate of $\hat{\sigma}^2$ by t_{QS} . We also constructed t statistics using an estimate of σ^2 similar to $\hat{\sigma}^2$ which employs AR(1) prewhitening as suggested by Andrews and Monahan (1992). We denote this t statistic by t_{QS-PW} .

Kiefer, Vogelsang, and Bunzel (2000) provide an alternative approach to constructing t statistics for model (4) that does not require explicit estimates of σ^2 . Let $\hat{s}_t = \sum_{j=1}^t \hat{u}_j$ and define $\hat{c} = T^{-2} \sum_{t=1}^T \hat{s}_t^2$. Consider the statistic $t^* = (\hat{\beta} - \beta) / (\hat{c} T^{-1})^{1/2}$. Kiefer, Vogelsang, and Bunzel (2000) derive the asymptotic distribution of t^* under the assumption that $\{u_t\}$ is stationary and they tabulate critical values.

If a unit root in $\{u_t\}$ cannot be ruled out, then the previous statistics are invalid. Statistics that are valid whether $\{u_t\}$ is stationary or has a unit root can be constructed using the trend function testing framework in Vogelsang (1998a) because model (4) is a special case of a univariate trend function model. While model (4) is the simplest trend function model one can write down, explicit asymptotic distribution of tests proposed by Vogelsang (1998a) have not been derived for this case, nor have critical values been tabulated. We perform these calculations in subsection 3.2.

Consider the following regression model based on partial sums of $\{y_t\}$,

$$(5) \quad z_t = \beta t + S_t,$$

where $z_t = \sum_{j=1}^t y_j$ and $S_t = \sum_{j=1}^t u_j$. Let $\tilde{\beta} = \sum_{t=1}^T t z_t / \sum_{t=1}^T t^2$ denote the OLS estimate of β from regression (5). Let RSS_y and RSS_z denote respectively the sum of squared OLS residuals from regressions (4) and (5). Define $s_y^2 = T^{-1}RSS_y$ and $s_z^2 = T^{-1}RSS_z$. Standard t statistics constructed from these OLS estimates of β are defined as $t_y = (\hat{\beta} - \beta) / (s_y^2 T^{-1})^{1/2}$ and $t_z = (\tilde{\beta} - \beta) / [s_z^2 (\sum_{t=1}^T t^2)^{-1}]^{1/2}$.

Vogelsang (1998a) proposed the statistics⁵ $T^{-1/2}t_y$, $t-PS \equiv T^{-1/2}t_y \exp(-bJ_T)$, and $t-PSW \equiv t_y \exp(-bJ_T) [s_y^2 / (100T^{-1}s_z^2)]^{1/2}$ where $J_T = (RSS_y - RSS_J) / RSS_J$, RSS_J is the sum of squared residuals from the OLS regression $y_t = \hat{\beta} + \hat{c}_1 t + \hat{c}_2 t^2 + \hat{c}_3 t^3 + \dots + \hat{c}_9 t^9 + \hat{u}_t$. Note that T times J_T is the Wald statistic for testing the hypothesis $c_1 = c_2 = \dots = c_9 = 0$.

In the next subsection we derive asymptotic distributions of these three statistics. The asymptotic results illustrate how the statistics are used in practice and the role played by the $\exp(-bJ)$ scaling factor. Readers only interested in the empirical results can skip to subsection 3.3. Critical values required for hypothesis testing are given in Table II.

3.2. Asymptotic Distributions

The asymptotic distributions of the $T^{-1/2}t_y$, $t-PS$, and $t-PSW$ statistics depend on which of the following two conditions hold for $\{u_t\}$:

$$(6) \quad T^{-1/2} \sum_{t=1}^{[rT]} u_t \Rightarrow \sigma W(r),$$

$$(7) \quad T^{-1/2} u_{[rT]} \Rightarrow \lambda W(r),$$

where $r \in [0, 1]$, $[rT]$ denotes the integer part of rT , $W(r)$ is the standard Wiener process, \Rightarrow denotes weak convergence, and $\lambda^2 = \lim_{T \rightarrow \infty} E[T^{-1}(u_T)^2]$. Condition (6) holds when $\{u_t\}$ is a stationary process and satisfies certain regularity conditions, e.g. mixing conditions popularized by Phillips (1987). Condition (7) holds when $\{u_t\}$ has unit root. Define $V(r) = \int_0^r W(s) ds$. Let $\hat{W}(r)$ be the residuals from the projection of $W(r)$ onto the space spanned by the constant function on the space $[0, 1]$. Let $\tilde{W}(r)$ and $\tilde{V}(r)$ denote the residuals from the projections of $W(r)$ and $V(r)$ respectively onto the space spanned by the function r on the space $[0, 1]$. Let $W^*(r)$ denote the residuals from the projection of $W(r)$ onto the space spanned by the functions $(1, r, r^2, \dots, r^9)$ on the space $[0, 1]$. Define $J = [\int_0^1 \hat{W}(r)^2 dr - \int_0^1 W^*(r)^2 dr] / \int_0^1 W^*(r)^2 dr$. The asymptotic distributions are as follows:

THEOREM 1: A. Suppose that $\{u_t\}$ satisfies condition (6). Then as $T \rightarrow \infty$, $T^{-1/2}t_y \Rightarrow 0$, $t-PS \Rightarrow [(1/3) \int_0^1 \tilde{W}(r)^2 dr]^{-1/2} \int_0^1 r W(r) dr$ and $t-PSW \Rightarrow [100 \int_0^1 \tilde{W}(r)^2 dr]^{-1/2} W(1)$.

B. Suppose that $\{u_t\}$ satisfies condition (7). Then as $T \rightarrow \infty$, $T^{-1/2}t_y \Rightarrow [\int_0^1 \hat{W}(r)^2 dr]^{-1/2} \int_0^1 W(r) dr$, $t-PS \Rightarrow [(1/3) \int_0^1 \tilde{V}(r)^2 dr]^{-1/2} (\int_0^1 r V(r) dr) \exp(-bJ)$, and $t-PSW \Rightarrow [100 \int_0^1 \tilde{V}(r)^2 dr]^{-1/2} V(1) \exp(-bJ)$.

The Theorem follows directly as a corollary to Theorems 1 and 2 of Vogelsang (1998a).

The asymptotic results illustrate how the tests are used in practice. When $\{u_t\}$ is stationary *or* has a unit root, $t-PS$ and $t-PSW$ have well defined limiting distributions.

⁵ The 100 in $t-PSW$ is for normalization purposes and has no effect on the performance of the statistic.

TABLE II
ASYMPTOTIC DISTRIBUTIONS: t - PS , t - PSW , $T^{-1/2}t_y$, AND J_T STATISTICS

	1.0%	2.5%	5.0%	10.0%
t - PS	-5.918	-4.782	-3.891	-2.969
(b)	(0.337)	(0.255)	(0.190)	(0.134)
t - PSW	-0.980	-0.782	-0.635	-0.469
(b)	(0.368)	(0.291)	(0.214)	(0.154)
$T^{-1/2}t_y$	-2.899	-2.548	-2.236	-1.861
J_T	0.975	1.402	1.911	2.645

NOTES: The critical values were simulated as follows. In the asymptotic representations, partial sums of iid $N(0,1)$ random deviates were used to approximate the standard Wiener process, and the integrals were approximated by normalized sums using 1,000 steps. 10,000 replications were used. The b 's were also computed using simulations. Right tail critical values of t - PSW , t - PSW , and $T^{-1/2}t_y$ follow from symmetry of the distributions. The critical values for t - PS and t - PSW are valid for both stationary and unit root errors using the values of b , in parentheses, for each percentage point. The critical values for $T^{-1/2}t_y$ are valid for unit root errors. A unit root in $\{u_t\}$ is rejected for small values of J_T .

However, the distributions are different in the two cases. It is here that b plays a crucial role. The limiting distributions do not depend on b when $\{u_t\}$ is stationary because $J_T \Rightarrow 0$ and $\exp(-bJ_T) \Rightarrow 1$; they do depend on b when $\{u_t\}$ has a unit root. Given a percentage point for either t - PS or t - PSW , there exists a value of b (specific to each statistic) such that the critical value for that percentage point is the same when $\{u_t\}$ is stationary and when $\{u_t\}$ has a unit root. On the other hand, $T^{-1/2}t_y$ has a well defined asymptotic distribution when $\{u_t\}$ has a unit root but remains robust (conservative) to the case where $\{u_t\}$ is stationary in which case $T^{-1/2}t_y \Rightarrow 0$. Using these tests, inference can be carried out without a prior knowledge about whether or not $\{u_t\}$ has a unit root.

Limiting distributions given by Theorem 1 are nonstandard. Critical values were simulated using Monte Carlo methods and are tabulated in Table II. See the notes to the table for simulation details. In Table II we also tabulate critical values for J_T , which is a unit root test in the class of tests proposed by Park and Choi (1988) and Park (1990).

3.3. Empirical Results

We now turn to the empirical results. Consider the results for the full series (1959–1998). The t - PS and t - PSW statistics were computed for the significance levels of 1% and 5%. The results are reported in the first six rows of Table III. With the exception of the $T^{-1/2}t_y$ statistic, $\beta \geq 0$ can be rejected in all cases, often at the 1% significance level. The rejections obtained using t_{QS} , t_{QS-PSW} , and t^* could be spuriously caused by a unit root (or near unit root) in $\{u_t\}$. This is unlikely for two reasons. First, $\beta \geq 0$ can be rejected using t - PS and t - PSW which are robust to a unit root in $\{u_t\}$. Second, using the J_T statistic (see Table III) unit roots can be rejected in all cases at the 1% significance level.

During the energy crisis of the early 1970's and the inflationary period of the late 1970's and early 1980's, commodity prices were highly variable, and the NL and HL series have large outliers around 1973 and 1981. These outliers take on negative values and may be biasing the tests toward rejection of $\beta \geq 0$. To show that the outliers are not driving the results we also report in Table III test statistics for the subperiods 1959–1972 and 1982–1998. As before, $\beta \geq 0$ is rejected in all cases with the exception of $T^{-1/2}t_y$, and many of the rejections occur at the 1% significance level. Therefore, we conclude that the results for the full series are not simply an artifact of outliers.

TABLE III
EMPIRICAL RESULTS

Series	T	t_{QS}	t_{QS-PW}	t^*	t -PS (1% b)	t -PS (5% b)	t -PSW (1% b)	t -PSW (5% b)	$T^{-1/2}t_y$	J_T
<i>Full Series:</i>										
NL: Quarterly	155	-3.362 ^a	-3.210 ^a	-7.554 ^b	-4.183	-4.221 ^b	-0.765	-0.772 ^b	-0.298	0.060*
Semi-Annual	154	-2.787 ^a	-2.258 ^a	-7.606 ^b	-4.345	-4.402 ^b	-0.738	-0.748 ^b	-0.333	0.087*
Annual	152	-2.309 ^a	-1.366 ^c	-6.567 ^b	-3.886	-3.983 ^b	-0.631	-0.648 ^b	-0.371	0.168*
HL: Quarterly	155	-4.687 ^a	-4.461 ^a	-12.115 ^a	-7.922 ^a	-7.995 ^b	-1.290 ^a	-1.302 ^b	-0.403	0.063*
Semi-Annual	154	-4.103 ^a	-3.339 ^a	-9.777 ^a	-7.375 ^a	-7.497 ^b	-1.150 ^a	-1.170 ^b	-0.471	0.111*
Annual	152	-3.900 ^a	-2.355 ^a	-8.771 ^a	-7.058 ^a	-7.303 ^b	-1.069 ^a	-1.108 ^b	-0.620	0.232*
<i>Pre-1973 Series:</i>										
NL: Quarterly	53	-3.779 ^a	-3.932 ^a	-5.916 ^b	-4.567	-4.750 ^b	-1.026 ^a	-1.069 ^b	-0.399	0.267*
Semi-Annual	52	-3.833 ^a	-3.756 ^a	-6.421 ^b	-4.027	-4.271 ^b	-0.851	-0.905 ^b	-0.535	0.401*
Annual	50	-2.402 ^a	-1.362 ^c	-5.008 ^c	-1.864	-2.563	-0.417	-0.582	-0.557	2.168***
HL: Quarterly	53	-4.752 ^a	-4.193 ^a	-10.000 ^a	-5.969 ^a	-6.709 ^b	-1.142 ^a	-1.291 ^b	-0.751	0.795*
Semi-Annual	52	-3.783 ^a	-2.605 ^a	-9.897 ^a	-4.471	-5.177 ^b	-0.817	-0.952 ^b	-0.773	0.997**
Annual	50	-3.205 ^a	-1.341 ^c	-10.128 ^a	-2.323	-3.491	-0.383	-0.587	-0.888	2.770***
<i>Post-1981 Series:</i>										
NL: Quarterly	62	-2.874 ^a	-2.845 ^a	-7.086 ^b	-4.377	-4.513 ^b	-0.692	-0.714 ^b	-0.353	0.208*
Semi-Annual	62	-3.299 ^a	-3.040 ^a	-10.521 ^a	-6.318 ^a	-6.559 ^b	-1.018 ^a	-1.058 ^b	-0.514	0.255*
Annual	62	-3.547 ^a	-3.373 ^a	-10.806 ^a	-5.782	-6.001 ^b	-1.031 ^a	-1.072 ^b	-0.498	0.253*
HL: Quarterly	62	-4.031 ^a	-4.122 ^a	-10.967 ^a	-7.814 ^a	-7.998 ^b	-1.228 ^a	-1.259 ^b	-0.492	0.158*
Semi-Annual	62	-4.469 ^a	-3.727 ^a	-13.043 ^a	-9.240 ^a	-9.559 ^b	-1.461 ^a	-1.514 ^b	-0.711	0.231*
Annual	62	-4.153 ^a	-3.668 ^a	-11.313 ^a	-5.424	-6.093 ^b	-0.873	-0.987 ^b	-0.807	0.791*

NOTES: The full series span 1959:3–1998:2. A rejection at the 1% level using t -PS and t -PSW trivially implies a rejection at the 5% level. We applied the mean shift tests of Vogelsang (1998b) and found no evidence suggesting the means of the series were unstable over time.

^a, ^b, and ^c denote rejection of $\beta \geq 0$ at the 1%, 5%, and 10% levels respectively.

*, **, and *** denote rejection of the unit root hypothesis at the 1%, 5%, and 10% levels respectively.

The asymptotic 1%, 5%, and 10% critical values for t^* are -8.544, -5.374, and -3.890 respectively.

Overall, the empirical results suggest that both the *Homogeneous Law of Demand* and the *Natural Time Law of Demand* are strongly consistent with the data.

4. ON THE “HOMOGENEOUS LAW OF DEMAND”⁶

In this section we give conditions under which the *Law of Demand* is equivalent to the *Homogeneous Law of Demand*.

We begin by defining a microeconomic model of a large and heterogeneous population of households as in Hildenbrand (1994, Ch. 2). Every household h at time t is described by a level of disposable income w_t^h and a demand function $f(p_t, w_t^h, \alpha_t^h) \in \mathfrak{N}_{++}^l$, where $p_t \in \mathfrak{N}_{++}^l$ denotes the vector of prices of the l commodities at time t and α_t^h is an element of the set Θ of demand functions that are homogeneous of degree zero in prices and income and satisfy the budget identity. As in much of the literature on empirical demand analysis, current income is assumed to be independent of the current price system.

⁶ We want to thank a referee for a suggestion that led to a substantially improved version of this section.

A population of households at time t is described by a joint distribution μ_t of income and individual demand functions, that is, μ_t is a probability measure on the σ -field of Borelian subsets of $\mathfrak{R}_+ \times \Theta$. We can now define aggregate (mean) demand at time t by $F(p_t, \mu_t) := \int_{\mathfrak{R}_+ \times \Theta} f(p_t, w, \alpha) d\mu_t$. Assume that the marginal distribution ρ_t of such distribution μ_t on current disposable income has a finite mean, equal to w_t , and that the integral defining mean demand is well-defined and finite.

In this framework, the *Law of Demand* holds whenever $(p - p') \cdot (F(p, \mu) - F(p', \mu)) < 0$ for any two distinct price vectors p and p' , given μ . Similarly, the *Natural Time Law of Demand* holds when for any two different periods t and τ it is the case that $(p_t - p_\tau) \cdot (F(p_t, \mu_t) - F(p_\tau, \mu_\tau)) < 0$.

The *Law of Demand* neither implies nor is implied by the *Natural Time Law of Demand* because the distributions μ_t and μ_τ might be very different. This is not so for the *Homogeneous Law of Demand*. Assume that the change from μ_t to μ_τ is such that all households keep their preferences and that the change in the ratio of the income of every household to mean income is very small. This can be modeled by making the following assumptions.

(A1) $\mu_t|_{w_t} = \mu_\tau|_{w_\tau} = \mu_\Theta$, and

(A2) for every household h and every pair of dates t and τ it is the case that $w_t^h/w_t = w_\tau^h/w_\tau$ so that we can write $\rho_t = \rho(w_t)$, where $\rho(w_t)$ is a density with mean equal to its location parameter w_t .

These assumptions are reasonable whenever preferences are stable and the distribution of income varies slowly relative to the speed at which aggregate income changes over time.

Then, given μ_t and the new level of aggregate income w_τ we have that the distribution μ_τ is given by $\mu_\tau(A \times B) = \int_A \rho(w_\tau)(x) dx \cdot \mu_\Theta(B)$ for every measurable subset $A \times B$ of $\mathfrak{R}_+ \times \Theta$. We are now ready to show the equivalence between (1) and (3).

THEOREM 2: *Under (A1) and (A2) The Law of Demand is equivalent to the Homogeneous Law of Demand.*

PROOF: First notice that from (A1), (A2) and the homogeneity of the individual demand functions it follows that $F(p_\tau, \mu_\tau) = F\left(\frac{w_t}{w_\tau} p_\tau, \mu_t\right)$. This is so because

$$\begin{aligned} F(p_\tau, \mu_\tau) &= \int_{\mathfrak{R}_+ \times \Theta} f(p_\tau, w, \alpha) d\mu_\tau = \int_{\mathfrak{R}_+} \int_{\Theta} f(p_\tau, w, \alpha) d\mu_\tau|_{w p_\tau}(w) dw \\ &= \int_{\mathfrak{R}_+} \int_{\Theta} f(p_\tau, w, \alpha) d\mu_\Theta \rho(w_\tau)(w) dw \\ &= \int_{\mathfrak{R}_+} \int_{\Theta} f\left(\frac{w_t}{w_\tau} p_\tau, \frac{w_t}{w_\tau} w, \alpha\right) d\mu_\Theta \rho(w_\tau)(w) dw \\ &= \int_{\mathfrak{R}_+} \int_{\Theta} f\left(\frac{w_t}{w_\tau} p_\tau, w, \alpha\right) d\mu_\Theta \rho(w_t)(w) dw = F\left(\frac{w_t}{w_\tau} p_\tau, \mu_t\right). \end{aligned}$$

As a consequence, the *Homogeneous Law of Demand*

$$\left(\frac{p_t}{w_t} - \frac{p_\tau}{w_\tau}\right) \cdot (F(p_t, \mu_t) - F(p_\tau, \mu_\tau)) < 0$$

can be written as

$$\frac{1}{w_t} \left(p_t - \frac{w_t}{w_\tau} p_\tau \right) \cdot \left(F(p_t, \mu_t) - F\left(\frac{w_t}{w_\tau} p_\tau, \mu_t \right) \right) < 0,$$

which is precisely the *Law of Demand*.

Q.E.D.

REMARK: Assumption (A2) in this paper can be replaced by the assumption of *mean scaling* (c.f. Lewbel (1990)) to obtain exactly the same result. Mean scaling differs from (A2) in that it assumes that changes in the distribution of income scaled by income are statistically independent of changes in mean income, instead of assuming that they are negligible in magnitude. Because both assumptions permit analyses involving changes in mean income to treat the distribution as it were fixed, assumption (A2) in this paper could be used instead of mean scaling to generate the results in Lewbel (1990, 1992).⁷

5. CONCLUSIONS

In this paper we explored empirically a question posed by Hildenbrand (1994) that we have called the *Natural Time Law of Demand*: that for any two time periods the vector of price changes and the vector of changes in quantities demanded point in opposite directions. We also formulated and tested a related hypothesis, the *Homogeneous Law of Demand*, which is the *Natural Time Law of Demand* for income-normalized prices.

We performed tests of the *Natural Time Law of Demand* and the *Homogeneous Law of Demand* for the United States with time series data for fourteen composite commodities for the period 1959–1998 and tested the hypotheses at a quarterly, semi-annual, and annual frequency using time series techniques.

According to our empirical results both the *Natural Time Law of Demand* and the *Homogeneous Law of Demand* are strongly consistent with the data. What do these empirical results tell us about whether or not the *Law of Demand* holds? We argue that theoretical and empirical conditions are met under which a test of the *Homogeneous Law of Demand* is a simple test of the *Law of Demand* for the United States. These conditions are extremely general in the sense that they do not include any assumptions about the distribution of preferences in the economy besides stationarity of preferences and the absence of money illusion.

Because the empirical results suggest that the *Homogeneous Law of Demand* is consistent with the US data, we conclude that the *Law of Demand* appears consistent with the US data as well, an empirical finding consistent with the conventional wisdom in the literature (c.f. Lewbel (1994)). More precisely, if the distribution of income varies slowly relative to the speed at which aggregate income changes over time, we strongly reject the hypothesis that the *Law of Demand* fails to hold for the United States.

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