Empirical Prediction of Shear Modulus and Young's Modulus of Plywood Panels

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Introduction:

An extensive body of experimental data was gathered which calculated the variation of in-plane Shear Modulus $G(\theta)$, and the variation of Young's Modulus $E(\theta)$ of plywood panels, where θ is the angle from the strong axis of the panel . Typically, $G(\theta)$ varies from a minimum value when $\theta=0°$, to a maximum at $\theta=\pm45°$. Previous research has described this variation through θ for solid wood products (1), and for plywood panels (2), yet these formulations require numerous mechanical properties as terms in the variation equation. These properties may be unavailable, or difficult to obtain. We have analyzed a number of different types of plywood panels and propose two simple formulae to predict $G(\theta)$ and $E(\theta)$, These empirical formulae require only the modulus of elasticity along the strong axis, E_0 or $E(0°)$ and along the weak axis E_{90} or $E(90°)$ and two empirically derived constants. The formulae we propose can be of use to design engineers and to researchers who can readily obtain the two moduli of elasticity, by means of a tension or compression test (3), but cannot perform the more cumbersome shear modulus test (3).

Theoretical and Empirical Relationships:

The theoretical relationships describing $G(\theta)$ and $E(\theta)$ shown in Equations 1 and 2, are well known (4), yet are not fully satisfactory for modeling plywood panels, as will be shown by our experimental data. Furthermore, Equations 1 and 2 require Poisson ratio ν_{12} along the 0°-90° axes, and minimum and maximum Shear Moduli (G_0 and G_{45}) which are difficult to obtain.

$$G(\theta) = \frac{G_0 \cdot G_{45}}{G_0 \cdot \sin^2(2\theta) + G_{45} \cdot \cos^2(2\theta)} \quad \ldots\ldots(1)$$

$$\frac{1}{E(\theta)} = \frac{1}{E_0} \cdot \cos^4(\theta) + \left(\frac{1}{G_0} - \frac{2 \cdot \nu_{12}}{E_0}\right) \sin^2(\theta) \cdot \cos^2(\theta) + \frac{1}{E_{90}} \cdot \sin^4(\theta) \quad \ldots\ldots(2)$$

The Poisson ratio plywood panels, such as those in this study, has been shown by Reference 2 (Equation 7 C pg. 11) to be described by:

$$\nu_{12} = \left(\frac{E_0}{E_{90}} + 1\right) \cdot \left(\frac{\sigma_{TL}}{1.036}\right) \quad \text{.......................................(3)}$$

where for yellow-poplar panels, $\sigma_{TL} = .019$

Saliklis and Falk (5) have shown that Equation 2 is not satisfactory for wood-based panels, and they proposed an alternate form that better fits the experimental data and is simpler to use since the Poisson ratio term is eliminated.

$$\frac{1}{E(\theta)} = \frac{\cos^4(\theta)}{E_0} + \frac{\sin^4(\theta)}{E_{90}} + \frac{\cos^2(\theta)\sin^2(\theta)}{\left(A^{2A}\right)G_0} \quad \text{.......................................(4)}$$

where $A = \dfrac{E_{90}}{E_0}$.

In the present study we have further simplified this relationship, as well Equation 1. The proposed simplifications are based on our empirical observation that G_0 and G_{45} can be readily defined in terms of E_0 and E_{90}. There is precedent for such empirical relationships between the Shear Modulus, and Young's Modulus for orthotropic materials, since no general relationship exists between the elastic constants of anisotropic materials. For example, Panc(6) recommended the following:

$$G_0 = \frac{\sqrt{E_0 \cdot E_{90}}}{2\left(1 + \sqrt{\nu_{12} \cdot \nu_{21}}\right)} \quad \text{.......................................(5)}$$

Our study of an extensive body of yellow-poplar plywood data (2) showed that the following relationships were valid for plywood panels.

$$G_0 = 0.2 \cdot \sqrt{E_0 \cdot E_{90}} \quad \text{and} \quad G_{45} = 1.0 \cdot \sqrt{E_0 \cdot E_{90}} \quad \text{.............................(6)}$$

Table 1 describes the goodness of fit of these empirical relationships. In Table 1, the error of using Equation 6 to predict G_0 and G_{45} is quantified. The error is defined as the difference |(Experimental - Predicted)|, and then the Standard Deviation of these differences is reported. For 7-ply, the error is |1030-1320| for G_0, which is 290 MPa, and the error is |7281-6601| for G_{45}, which is 680 MPa. In that calculation, the average of G_{45} and G_{-45} resulted in 7281 MPa.

Substituting these empirical relationships of Equation 6 into Equations 1 and 3 resulted in a useful set of formulae for $G(\theta)$ and $E(\theta)$ that are based solely on the principal moduli of elasticity, E_0 and E_{90}.

$$G(\theta) = \frac{0.2 \cdot E_0 \cdot E_{90}}{0.2 \cdot \sqrt{E_0 \cdot E_{90}} \cdot \sin^2(2\theta) + \sqrt{E_0 \cdot E_{90}} \cdot \cos^2(2\theta)} \quad \text{......................(7)}$$

$$\frac{1}{E(\theta)} = \frac{\cos^4(\theta)}{E_0} + \frac{\sin^4(\theta)}{E_{90}} + \frac{\cos^2(\theta)\sin^2(\theta)}{\left(A^{2A}\right)0.2 \cdot \sqrt{E_0 \cdot E_{90}}} \quad \text{.............................(8)}$$

Equations 7 and 8 were compared to our experimental data. Table 2 quantifies the goodness of fit of these relationships, as well as the weakness of the theoretical relationship. In Table 2, typical calculations are summarized in detail. Here is shown the calculations for 7-ply. Error is estimated as the square root of the sum of the squares, divided by n through the entire range of θ. Figure 1 shows the variation of $G(\theta)$ and $E(\theta)$ for a typical set of data.

Discussion and Conclusions:

The proposed empirical formulae for $G(\theta)$ and $E(\theta)$, shown in Equations 7 and 8, are able to capture the variations in material modulus for four different sets of experimental data. The newly proposed formulae are simpler to use than theoretical formulae because only the two orthogonal moduli of elasticities E_0 and E_{90} are required inputs. Two empirical constants were needed, but these were valid for all four sets of data investigated.

Table 1. Goodness of Equation 6

	3 – ply	5 – ply	7 – ply	9- ply	Standard Deviation of difference
G(0)	63 MPa	-164 MPa	290 MPa	239 MPa	178 MPa
G(45)	3422 MPa	4959 MPa	680 MPa	937 MPa	1779 MPa

Table 2. Estimation of Error

Angle (degrees)	$E(\theta)_{exper.}$ (MPa)	$E(\theta)_{empir.}$ (MPa)	$E(\theta)_{theor.}$ (MPa)	Angle (degrees)	$G(\theta)_{exper.}$ (MPa)	$G(\theta)_{empir.}$ (MPa)	$G(\theta)_{theor.}$ (MPa)
		3 PLY				3 PLY	
0	9510	9510	9510	-45	13782	6716	10139
15	5325	5388	7107	-30	4192	3358	3713
30	2785	2843	4608	-15	1607	1679	1638
45	2132	2215	3678	0	1280	1343	1280
60	2315	2472	3706	15	1600	1679	1638
75	3936	3608	4306	30	3385	3358	3713
90	4743	4743	4743	45	6495	6716	10139
SSRS/ n		54 MPa	473 MPa			1017 MPa	741 MPa
		5 PLY				5 PLY	
0	7601	7601	7601	-45	14610	6274	11234
15	4640	5015	6302	-30	4347	3137	4116
30	2684	2958	4631	-15	1795	1569	1816
45	2265	2397	3932	0	1419	1255	1419
60	2532	2711	4054	15	1743	1569	1816
75	3880	3957	4718	30	3878	3137	4116
90	5179	5179	5179	45	7857	6274	11234
SSRS/ n		74 MPa	502 MPa			1230 MPa	684 MPa
		7 PLY				7 PLY	
0	7617	7617	7617	-45	8336	6601	7281
15	4612	5324	5714	-30	3258	3301	2893
30	2598	3299	3780	-15	1434	1650	1312
45	2112	2720	3162	0	1030	1320	1030
60	2339	3078	3493	15	1358	1650	1312
75	3756	4434	4702	30	3075	3301	2893
90	5721	5721	5721	45	6226	6601	7281
SSRS/ n		220 MPa	348 MPa			264 MPa	222 MPa
		9 PLY				9 PLY	
0	8801	8801	8801	-45	4902	6344	5640
15	5459	5072	6922	-30	3999	3172	3186
30	3112	2707	4733	-15	1808	1586	1704
45	2744	2118	3826	0	1382	1269	1382
60	3169	2370	3790	15	1555	1586	1704
75	3958	3471	4247	30	3082	3172	3186
90	4573	4573	4573	45	6378	6344	5640
SSRS/ n		248 MPa	604 MPa			290 MPa	403 MPa

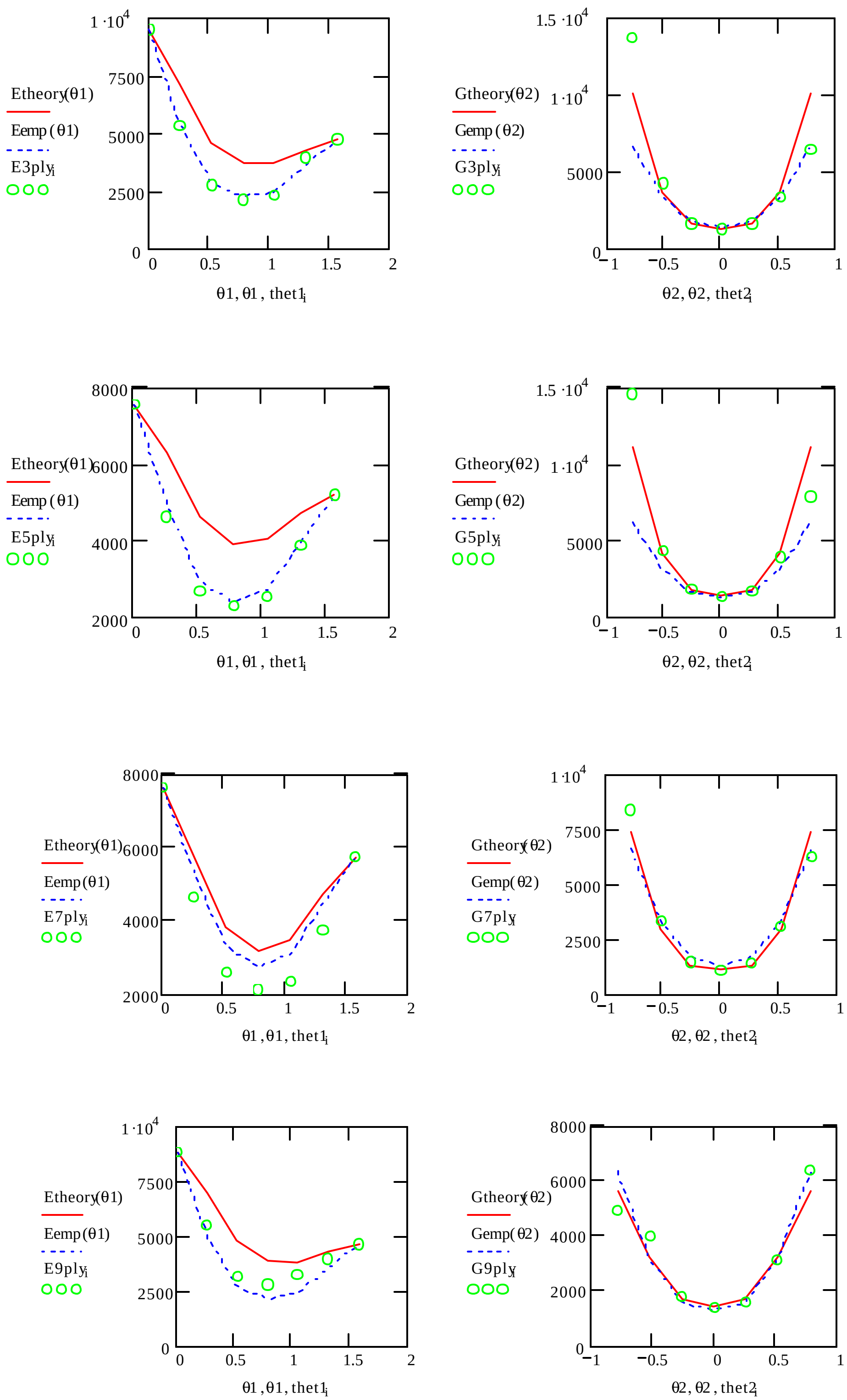

Figure 1. Experimental Data, with Empirical and Theoretical Predictions.

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