The Usage of Smartphones in the Calculation of Relativistic Time Dilation effects at Meager Velocities

A Senior Project

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by

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Abstract:

The theories of special and general relativity postulate that events are not simultaneous for different observers, and that different observer's times tick at different rates depending on relative velocity, and the magnitude of the gravitational field they are in. This project seeks to design a program for use with smartphones that will calculate this change in time, in real time, and keep track of the discrepancies in time experienced between the observer and a stationary point on the surface of the earth.

Introduction:

More than a hundred years ago, Albert Einstein revolutionized physics and how people thought about the natural world with his theories of relativity. The two postulates of special relativity can be paraphrased as: 1) The laws of physics are the same for all inertial reference frames, and 2) The speed of light in empty space is the same for all inertial reference frames, regardless of the speed of the observer. The second postulate was quite revolutionary from Newtonian physics; space and time were no longer the absolutes that they were thought to be; the new absolute was the speed of light. From this postulate, the phenomenon of time dilation was theorized. Time dilation is when an observer sees an object moving in relation to him experience less time than he himself experiences. Because the speed of light is so great compared to our usual velocities, we never observe this phenomenon in everyday life, and it is still common and practical to think of time as invariant. Einstein proposed *another* variance of time with general relativity, which states that time in a stronger gravitational field will tick more slowly than time in a weaker gravitational field. It is the goal of this project to elucidate these principles at work by using smartphones to calculate the effects of time dilation in real time.

There have been numerous experiments that have verified time dilation, including the Hafele-Keating experiment which flew a cesium atomic clock (which 'tick' over 9 billion times per second) around the world both westward and eastward. The increased altitude provided a

slightly weaker gravitational field, which sped up the clock in the plane, and the speed of the plane relative to the speed of the earth's rotation also sped it up when the plane flew westward, but slowed it down when the plane flew eastward.[(1)(2)]

Experimental observation of small particles shows a large degree of special relativistic time dilation. Particles with very small mass can undergo extreme time dilation because their speed is often close to that of light. One example is the pion which decays into a muon and a neutrino in a mean lifetime of 2.6×10^{-8}s. When these particles are created in the upper atmosphere, they have so much energy that they are travelling at almost the speed of light. Even at this speed, they should only be able to travel 7.8 meters before they decay, but we see them travel tens of kilometers because their time is ticking much more slowly than ours.[(3)]

In contrast, enormous celestial bodies hardly ever move fast enough to experience significant relativistic time dilation, but because of their enormous mass, they can experience extreme gravitational time dilation. For example, at the event horizon of a black hole, gravity is so strong that even light cannot escape. If we were to watch an object be sucked into a black hole, we would see it fall closer and closer, until we would see it (and it's time) appear to stop on the event horizon.[(4)]

The effects of special relativistic time dilation on such large objects are negligible, but can be calculated. The surface of the earth at the equator has a speed of 465.1 m/s relative to the center of the earth. This results in us experiencing 10^{-7} seconds less than the center of the earth per day, or 1 second less every 27,000 years. Comparing us on the surface of the earth to the center of the sun, an average relative velocity of 29,800 m/s, we experience a "loss" of 4.3×10^{-4} seconds per day, or one second every 6.42 years. The effect becomes more noticeable when we compare our sun to the center of the Milky Way, which has a relative velocity of 220 km/s. We experience 2.32×10^{-2} seconds less than the center of the Milky Way each day, or one second less every 43.04 days.

The special relativistic time dilation people have experienced relative to one another is even less. Sergei Krikalev has spent 803 days in space at an orbit velocity of about 7350m/s making him experience about 1/50th of a second less than us due to special relativistic time dilation. Relative to the surface of the earth, ordinary motion by plane, train, or automobile is going to be significantly less.[(5)]

Sergei also has been affected by gravitational time dilation less than the rest of us have. Living at an altitude of 350 kilometers for 803 days means he has experienced 1/500th of a second more than us due to the difference in our gravitational time dilations.

In our everyday lives, these effects happen, but on such a small scale that they are usually ignored. It is the goal of this project to design a program which can apply these principles to ordinary everyday travel, and show the effects of time dilation that we experience, relative to a stationary position on the surface of the earth.

Theory:

Due to special relativity, a stationary observer will measure a moving clock tick slower by the Lorentz factor of:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad Eq.1$$

where v is the velocity relative to the observer and c is the speed of light, 3x10^{8} m/s. In other words the observer will see that it takes the moving clock

$$\Delta t = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma \Delta t' \qquad Eq.2$$

to tick, where $\Delta t'$ is the proper time, or amount of time elapsed in the moving clock's reference frame.[6] By finding the difference between $\Delta t'$ and Δt, we can find how much time is "lost" by the moving clock. If v is extremely small however, $\Delta t'$ is the same as Δt to about 18 decimal places, making it difficult to compare numerically in real time. At the fastest this app would be used, (in an airplane, at approximately 250m/s) $\Delta t'$ is the same as Δt for 12 decimal places. Because of this, we are going to approximate γ by using the first two terms of the binomial series expansion about v=0.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \cdots \approx 1 + \frac{v^2}{c^2} = \gamma \pm 10^{-24} \qquad Eq.3$$

For v=250 m/s, the third term of the series expansion is on the order of 10^{-25}, while the second term is on the order of 10^{-13}. Because the third term is only 1 ten trillionth of the second term, it is safe to ignore it in this application. The small size of the third term also means that the series representation using only the first two terms is accurate to the actual γ within 10^{-24}, as long as the speed remains under 250 m/s. Using this approximation for gamma, we can rewrite equation 2 as

$$\Delta t = \gamma \Delta t' \approx \Delta t' \left(1 + \frac{v^2}{2c^2}\right) \rightarrow \Delta t = \Delta t' + \frac{\Delta t'}{2}\frac{v^2}{c^2} \qquad Eq.4$$

In these equations, Δt is the time measured by a stationary point on the surface of the earth, and $\Delta t'$ is the time measured by the phone. Now the special relativistic difference in the two clocks' times is just

$$\Delta t_{SR} = \Delta t - \Delta t' = \frac{\Delta t'}{2}\frac{v^2}{c^2} \qquad Eq.5$$

To report this time to the user, all we need is a small time interval and a velocity over that time interval. To do this in a smartphone, we will gather gps coordinate data, find distance traveled per time interval, and the sampling time will be our $\Delta t'$.

For the general relativistic time dilation, the procedure is very similar. We start with the equation for time dilation at a distance r from the center of the massive body:

$$\Delta t' = \Delta t \sqrt{1 - \frac{2GM}{rc^2}} \qquad Eq.6$$

Where G is the gravitational constant, M is the mass, and r is the distance between the center of the mass and the observer. Rearranging this we get something remarkably similar to the special relativistic equation:

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{2GM}{rc^2}}} \qquad Eq.7$$

We need to use this equation once to figure out the relative time dilation from the center of the earth to the surface of the earth, and then again to find the time dilation from the surface of the earth to our device. For the surface of the earth, using the average radius R = 6367.5km (with Δt_{CE} being the center of the earth, Δt being the surface of the earth, and $\Delta t'$

being the time as measured by the phone):

$$\Delta t_{CE} = \frac{\Delta t}{\sqrt{1 - \frac{2GM}{Rc^2}}} = \Delta t\,(1 + 6.95335 * 10^{-10}) \qquad Eq.8$$

(For these equations, t_s denotes the time at the surface of the earth, t' denotes the time at an observer a height h above the surface of the earth, and t denotes the time at the center of the

earth.) So then the difference between a clock on the surface of the earth and a clock at the center of the earth is:

$$\Delta t - \Delta t_{CE} = \Delta t_{CE} * 6.95335 * 10^{-10} \qquad Eq.9$$

That is, the clock at the surface will be leading by about seven parts in ten billion.

To find the dilation at an arbitrary distance from the center R+h, we will use the same binomial expansion as in the special relativistic case. It is important to note that in this instance there will be more error than previously, due to expanding about R+h = 0, but evaluating at points always with R+h > R.

$$\Delta t' = \frac{\Delta t_{CE}}{\sqrt{1 - \frac{2GM}{(R+h)c^2}}} \approx \Delta t_{CE}\left(1 + \frac{GM}{(R+h)c^2} + \frac{3GM}{4(R+h)c^4} + \cdots\right)$$

$$\approx \Delta t_{CE} + \Delta t_{CE}\frac{GM}{(R+h)c^2} \qquad Eq.10$$

And using a further binomial expansion for $\left(1 + \frac{h}{R}\right)^{-1}$, the difference in the clock at height h and the clock at the center of the earth is:

$$\Delta t' - \Delta t_{CE} = \Delta t_{CE}\frac{GM}{(R+h)c^2} = \Delta t_{CE}\frac{GM}{\left(1 + \frac{h}{R}\right)Rc^2} = \Delta t_{CE}\frac{GM}{Rc^2} * \left(1 - \frac{h}{R}\right) \qquad Eq.11$$

I mentioned that the error would be more than was introduced previously, but if we use h=0 in equation 11, and compare to the result of equation nine, we find agreement in the first 8 significant figures, which means the error is still extremely small. Another concern is that both

equation eleven and equation 5 rely on Δt, the time at the earth's center instead of $\Delta t'$, the time of the phone. Because the two values are so similar, this also causes negligible error.

To find the difference between the clock at h and the clock on the surface (Δt_{GR}), we merely subtract equation nine from equation eleven:

$$\Delta t_{GR} = \Delta t' - \Delta t_{CE} - (\Delta t - \Delta t_{CE}) = \Delta t' - \Delta t$$

$$= \Delta t \left(\frac{GM}{Rc^2} * \left(1 - \frac{h}{R}\right) - 6.95335 * 10^{-10} \right) \qquad Eq.12$$

To find the difference in times created by gravitational time dilation, all we need is a height above the surface of the earth, and an amount of time spent at that height.

Implementation:

Initially the idea was to only calculate special relativistic time dilation effects by using the built in accelerometer of a smartphone to find its velocity. There was some trouble with this because the accelerometer, (despite its name) measures forces applied to the phone, not accelerations. This gives an opportunity to talk about an interesting corollary of general relativity, the equivalence principle.

The equivalence principle says that if everything else is the same, it is impossible to tell if a force is due to an acceleration or due to gravity. Because the phone only measures forces, it always registers a total force of g when not accelerating. If the orientation of the phone never were to change, this could be accounted for, since gravity could just be assumed in one

direction. When the orientation of the device changes however, the measured forces change, giving the illusion of an acceleration. This means the accelerometer cannot be used reliably to find the velocity of the device.

The only other way to find the velocity of the phone is to compare consecutive gps coordinates to find the distance traveled each time interval. Once the method of gps was established, a new avenue to find general relativistic time dilation was opened, because gps reports the altitude of the device. General relativistic time dilation depends only on the mass and distance between two objects, which means the only variable in the gravitational time dilation we experience from earth is our distance from the center of it. A sort of irony of using this method is that the satellites that enable gps have to be corrected for the time dilation they experience, both because of the reduced gravity in orbit, and the tremendous speed of their orbit.

The first thing the program is going to do is read in data of altitude, longitude, and latitude. Using these data, it will convert the spherical points to Cartesian coordinates to prepare to calculate distance between consecutive points. Converting to Cartesian coordinates was chosen over using the haversine formula for great circle distances because there can be movement outside of the surface of the sphere as well, which the geodesic of the great circle cannot account for. Another method which was rejected was treating the two points as vectors from the center of the earth and finding the vector that connected their endpoints. The problem with this method was using an arcsine to find the extremely small angle between the vectors. The arcsine was fairly inaccurate, and these inaccuracies became compounded after using the law of cosines. Converting to Cartesian coordinates and finding distance requires fewer transcendental equations and is far more accurate.

After evaluating the distance, the program simply divides the distance traveled by the time between calculations ($1/10^{th}$ of a second) to find speed. Using this speed and a time of $1/10^{th}$ of a second in equation five, the program calculates the time “lost” relative to a stationary point on the surface of the earth.

Similarly, equation twelve only contains variables for height above the surface of the earth and a time interval. Using the altitude reported from gps and the same time interval, equation twelve reports the time “gained” with respect to a stationary point on the surface of the earth.

Summary:

The application uses equation twelve and equation five to calculate the time difference between the observer and a stationary spot on the earth’s surface. The finished simulation program is available in flash at http://www.swfcabin.com/open/1287722492. Using this simulation program, the user is able to see how the time experienced by an observer might be different than the time experienced at rest on the earth’s surface, due both to special relativistic time dilation, and gravitational time dilation.

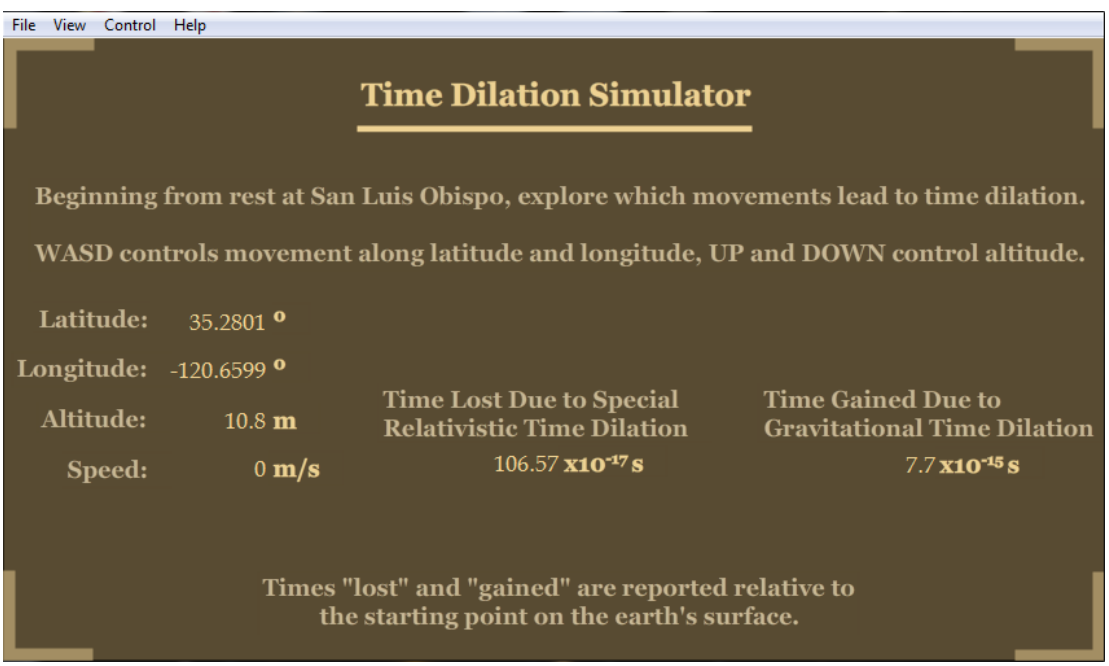

Figure 1: The simulator application in action on Windows.

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