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Ultrasonic Beam Propagation in Turbulent Flow

F.J. Weber
Mechanical Engineering Department
Worcester Polytechnic Institute
Worcester, MA 01609
Email: fjw@wpi.edu

W.W. Durgin
Mechanical Engineering Department
Worcester Polytechnic Institute
Worcester, MA 01609
Email: wwdurgin@wpi.edu

H. Johari
Mechanical Engineering Department
Worcester Polytechnic Institute
Worcester, MA 01609
Email: hjohari@wpi.edu

Abstract

A study was conducted to examine how a coherent sound burst, such as those used in an ultrasonic flowmeter, crosses a turbulent flow field. Many ultrasonic flowmeters use a time of flight method to determine the mean flow. This measurement method sends sound waves at some angle across a velocity field between two transducers, and the time required for the sound burst to cross this distance is measured on the order of nanoseconds. The system then reverses so that the sound wave burst is sent in the opposite direction. Knowing the distance traveled, the local average sound speed, and the back and forth travel times, the integrated velocity along the sound path can be computed. By using average values for distance and time, an average flowrate may be determined. To perform this analysis, data from several velocity fields were used to determine how a coherent sound burst would travel across the field. Comparisons of the infinitely thin case and the finite thickness case are performed. The study focused on using a modified ray trace method on a velocity field data set obtained using PIV.

1 Introduction

By sending a coherent sound wave front across a flow field and measuring the time lapse for the sound to cross a known distance, an average fluid velocity may be determined. Ultimately, this flow measurement using the time of travel across a flow field was developed into the ultrasonic flowmeter technology in use today. Using ultrasonic waves, one may construct a sound beam with precisely known characteristics, that will allow accurate measurement of the travel time between two known points. The ultrasonic flowmeter can be built into, or clamped onto, a pipeline and used to measure the flow of a fluid, within a flow system.

Ultrasonic flowmeters have several unique characteristics, which make them ideal for industrial applications. The first being the head loss of the meter is equivalent to just the pipe loss of a pipe in which the meter is located, as there are no protrusions into the flow. This compares to a variable area flowmeter, which can have a loss of as much as 50% of the velocity head of the flow. [Streeter] A second characteristic of ultrasonic flowmeters is the large turn down ratio of the meters. This compares to a variable area flowmeter for which there are computed ideal ranges. If the flow goes above or below this ideal measurement area, the error becomes larger due to the V^2 relationships in these types of meters. Finally, ultrasonic flowmeters may be used in other than ideal circumstances much as flow contaminated by solids, without the worry of blockage.

Industry has widely accepted the ultrasonic flowmeter as an acceptable method to obtain flow measurements in a process. In addition to the aforementioned characteristics, the appeal of this type of flowmeter includes low maintenance, ease of use, and supposed high accuracy. To gain accuracy, ultrasonic flowmeters have become increasingly complex and, therefore, costly in terms of actual dollar cost, as well as the ability to perform routine maintenance. In addition, in order to increase the accuracy, a substantial calibration period may be required, which then depending on the modeling conditions may be invalidated in the final installation. The National Institute of Standards and Technology (NIST) currently has a program to determine a standard for applying ultrasonic technology into flow measurement installations.[Mattingly, Yeh] In order to institute such a standard, the principles of operation of ultrasonic flowmeters must be well understood. Principles to be better understood include how sound waves interact with a turbulent flow field, and how a temperature profile affects an ultrasonic meter reading.

2 Background

The principle behind the ultrasonic flowmeter is that the sound wave produced by the ultrasonic transducer moves with the fluid in which is traveling. In order for this device to work, the travel time between a transmitter and receiver must be accurately measured using an electronic circuit. These flowmeters, in general, use various averaging techniques to eliminate data scatter, as it is perceived to be unwanted electronic noise and/or random or systemic noise resulting from timing and signal measurement inaccuracies. When using an ultrasonic flowmeter, the measured data scatter may not be solely due to electronic noise. It seems to be caused, in part, by flow noise resulting from turbulence in a flow field.

Flow noise as it is being used here, is the inherent turbulence in the flow. The effect of turbulence on the propagation of the sound wave is a key to understanding the data scatter encountered in ultrasonic flowmeter applications, as well as the ultimate accuracy of an ultrasonic flowmeter device. Mattingly and Yeh reported that by not accounting for the effect of the velocity profile in a pipe, the measured flow could be as much as 5% off.[Mattingly] Mattingly and Yeh described the effect a velocity profile in a pipe has on the bending of the actual sound travel path in a pipe flow. They then compared the flow measurements obtained in this method to those obtained if a straight sound path was used. Additionally, they examined the effect different average pipe velocity profiles had on the sound path and flow measurement. The error in flow computation proved to be significant, particularly compared to manufacturers' claims of 0.25%. However, a detailed analysis of the way an individual sound wave front propagates through a turbulent field was not performed.

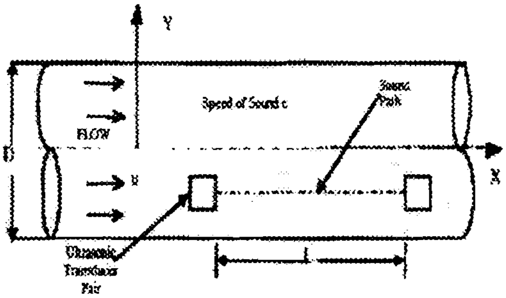


Figure 1 Setup for a Basic Derivation of Sound Propagation Equations for a Measurement Volume Parallel to the Axial Flow

The research reported herein, sought to determine the effect of turbulence on sound crossing a turbulent flow field. To study this effect a simulated sound wave was followed through a flow field as measured by Particle Imaging Velocitometry (PIV) for a channel flow using a software program developed for this purpose. Since this analytic technique does not have any electronic noise, it was possible to determine the effect of turbulence on the passage of a sound wave directly. Repeated calculations of the time differences for upstream and downstream propagating waves, ΔT, as the turbulent field evolved, showed considerable scatter. Statistical analysis indicated the effects and methodology of averaging to obtain a particular accuracy within a certain confidence interval. This accuracy can be added to the electronic noise of an ultrasonic flowmeter thus determining the ultimate accuracy of the meter.

3 Ultrasonic Flowmeter Operation

The basic ultrasonic flowmeter equations were reviewed by Hemp in 1998.[Hemp] In this formulation, time of flight across a flow is determined from:

$$\frac{dx}{dt} = c + u, \tag{1}$$

where x is the coordinate along the sound propagation path, u is the corresponding fluid velocity component along the sound path, and c is the local speed of sound as depicted in Figure 1. Separating the variables and expanding for u/c<< 1 one obtains:

$$t_{12} \approx \int_0^L \left(\frac{1}{c}\left(1-\frac{u}{c^2}\right)\right)dx. \tag{2}$$

so that:

$$t_{12} \approx \frac{L}{c} - \frac{\bar{u}L}{c^2}. \tag{3}$$

where $\bar{u} = \frac{1}{L}\int_0^L u\,dx$ is the average velocity.

In this derivation, path length and speed of sound are assumed constant, and a mean velocity profile, which only varies along the radius of a pipe or channel, not along the axis, is used. A similar derivation can be performed using sound crossing in an opposite direction using:

$$\frac{dx}{dt} = c - u. \qquad (4)$$

Using these relations one may find the mean expected time differences between the upstream and downstream transits of an ultrasonic sound wave in a flow.

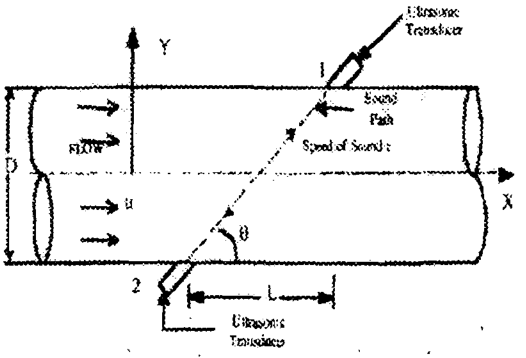


Figure 2 Setup of a Typical Transit Time Ultrasonic Flowmeter

Starting once again with Equation 1, but assuming c, and $\bar{u}$ are constant averages, the downstream travel time can be found using:

$$\int_1^2 dt = \int_0^l \frac{dx}{c - \bar{u}}, \qquad (5)$$

and the upstream travel time is:

$$\int_1^2 dt = \int_0^l \frac{dx}{c + \bar{u}}. \qquad (6)$$

Using the setup shown in Figure 2, and integrating one obtains:

$$t_{12} = \frac{L/\cos\theta}{c + \bar{u}}, \qquad (7)$$

and

$$t_{21} = \frac{L/\cos\theta}{c - \bar{u}} \qquad (8)$$

is obtained.

Solving Equations 7 and 8 for a constant speed of sound, c:

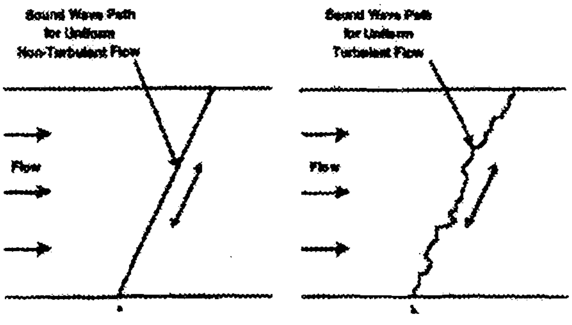
$$\overline{V} = \frac{L}{2\cos\theta}\left[\frac{1}{t_{21}} - \frac{1}{t_{12}}\right] \qquad (9)$$

is obtained. This equation assumes the axial distance between the transducers and the angle with respect to the axis the sound waves are launched is accurately known.

In this equation, the axial length and the diametrical length between the ultrasonic transducers must be well known. A second method starts similarly to the previous method, but after getting to Equation 8, it is assumed that $c = \frac{L}{t_{21}} = \frac{L}{t_{12}}$, therefore, arriving at:

$$\overline{V} = \frac{t_{21} - t_{12}}{2D}c^2. \qquad (10)$$

It may be noted that Equations 9 and 10 have several common features. They all assume constant path length, L, constant speed of sound, c, and some mean velocity, V. To determine flow in a system from this mean velocity, computed in 9 and 10, a velocity profile must be assumed for the system. This velocity profile assumption is a weak point in the theory of ultrasonic flowmeters. By examining and understanding the velocity field characteristics one may make better decisions about the characteristics of a velocity profile of the flow field allowing higher accuracy of the flow measurement. Most research on the subject of ultrasonic flowmeter accuracy and method of employment, as it turns out, has centered on these using the aforementioned values as a constant or average. Little work has been done to date using instantaneous turbulent profiles to predict the effect on the time of travel of a sound wave in a turbulent field. For instance the sketch in Figure 3a



a. b.

Figure 3 Magnified Examples of a Ray Trace in a Flow:

- **a. Non-Turbulent or Average Velocity Used;**
- **b. b. Instantaneous Velocities Used**

is the typical assumed travel path for sound in a turbulent uniform flow field; however, if the turbulence is accounted for the path becomes more like that shown in Figure 3b.

4 Ray Trace Method

The ray trace method is a standard analysis tool used to study how sound crosses a velocity field. By breaking the path of the sound wave into small segments, it would be possible to include the effects of local variations in temperature, and fluid velocity on the sound wave propagation. The segments do not necessarily need to be infinitesimal, but should be on the order of the size of the Kolomogrov scaled eddies. In the standard ray trace method, the ray is an infinitely thin line representing the sound velocity. Using standard vector analysis, the sound wave vector and the local flow velocity vector are combined obtaining a resultant. In this way, the path of a sound wave may be determined. The problem in analyzing the effect of turbulence on a sound wave is that since the ray representing the sound is infinitely small, the path will only be affected by flow velocities along the line of the ray. Since a sound beam in an ultrasonic flowmeter is formed as a coherent front of finite size, sketch Figure 4, the ray trace method does not provide insight as to how the fluid structure affects the sound front. In order to determine the evolution of the beam front, multiple ray traces were used in a computer simulation to represent locations across the front. These rays are close enough together to assume a straight line could connect each ray at a specific time step. In this work, five rays were used, thus the sound front was divided into quarters. This method proved successful in the study of the interaction of fluid mechanic structures with sound wave fronts of varying sizes. This method is termed the "Modified Ray Trace Method."

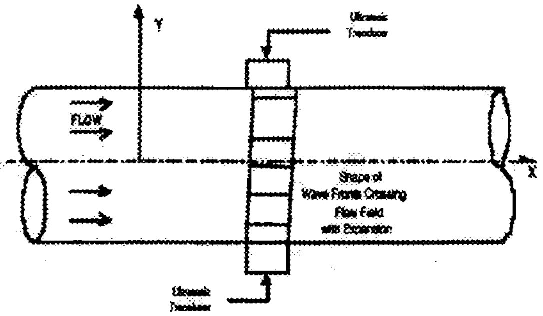


Figure 4 Sketch of a Wave Front Crossing a Uniform Velocity Field; Note as the Wave Front Crosses the Field, it is Convected Downstream by the Flow and Spreads Slightly.(not to scale)

5 Analysis Using a PIV Determined Flow Field

A modified ray trace program was developed so that the travel time for a sound wave across a velocity field could be analyzed. After proving the program using several simple flow fields, the program was adapted to allow the use of PIV data. The PIV technique typically uses an asynchronous laser sheet and cameras to determine the magnitude and direction of a velocity flow field. The technique produces an instantaneous two-dimensional grid of velocity vectors for a flow. In this way a flow field can be frozen in time and various analyses can be performed on the field. Typically, PIV systems will take data at rates nearing 15 Hz for as long as the computers storage systems can keep up with the data rate. Due to the size of the flow field and the speed of sound, a single crossing of the sound wave takes on the order of 50 microseconds. Since the PIV data was taken at 15 Hz, each individual data set was used for a single set of back and forth crossings of sound waves. By performing the analysis in this method a frozen flow is assumed. This assumption is reasonable since the fluid will move less than 25 micrometers while the beam itself is on the order of 0.3 to 5 millimeters, which is 12 to 200 times larger than the movement of the fluid particles.

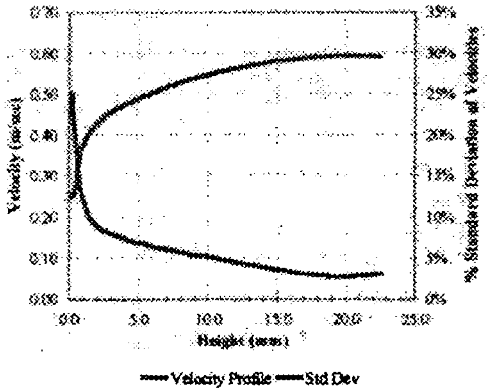


Figure 5 Velocity Profile versus Radius and Standard Deviation versus Radius as Computed using a 1-D Analysis Similar to an Ultrasonic Flowmeter.

PIV data for a channel flow was obtained from measurements provided by Dr. Kenneth Kiger of the University of Maryland. [Kiger] The channel used to obtain the data was 4 cm wide by 36 cm high channel, 4.87 m long. The test location was approximately 420 cm downstream of the entrance to the channel. The Reynolds number based on channel width of the flow was approximately 23,000. The velocity fields were measured at 15 Hz for approximately 51 sec thereby obtaining approximately 768 velocity fields. A grid of 122 points spanwise by 67 points streamwise was used in obtaining the velocity data. To produce as dense a grid as possible within the limitations to the PIV system, the grid was overlaid over slightly more than 1/2 the channel and was sized at 2.2 cm by 2.2 cm, giving a grid spacing of approximately 0.18 mm spanwise and 0.32 mm streamwise.

The PIV velocity data were read into the ray-tracing program and provided the velocity field information used in the program. Due to the size of the channel and the data rate of the PIV system, the data were assumed to be *frozen* at fixed times so that the flow field remained stationary as the sound wave traveled across the channel. This is not an unreasonable assumption considering the relative magnitudes of the flow velocities were small compared to the sound velocities in water. Since the velocity was discretized, the ray-tracing program performed a two-dimensional interpolation of the data to obtain the resolution needed to examine the ΔT's between the forward and reverse propagating acoustic waves. The interpolation allowed for the small time steps, while retaining the fundamental nature of the measured flow. Since the expected time differences were so small, the time step used had to be less than the expected time difference. In this case, the time differences would be on the order of 1.0 ns. Therefore, the time steps had to be as small as 10 picoseconds. In addition, since the PIV data for the flow was only measured over half the channel width, the analysis was only performed over that portion of the channel.

Initially the PIV data set was used to examine the flow in a stream-wise direction using a 1 dimensional method of summing the sound velocity and the x direction flow velocities along a single row in each direction. Each time step was then averaged along this same row producing a single velocity at each point. This process was done for each point from the channel wall to the last row of the PIV data near the center of the channel. Figure 5 is a plot of this data along with the standard deviation of the data at each point. The mean velocity profile graph starts at approximately 0.22 m/sec and rises to approximately 0.60 m/sec, these values are read on the right sides. This plot represents data as if it were taken using a long time constant velocity measurement device at a series of points in flow field. This graph in fact matches the typical shape of a turbulent flow profile. The second set of data in Figure 5 is the standard deviation of the velocity measurement data. This data can be interpreted similarly to the root mean square of the turbulence in a pipe or channel. The only difference is that the scatter does not include a near wall decrease. This however is due to the PIV data not being close enough to the wall to observe this effect.

Figures 6 and 7 present data depicting the instantaneous ΔT versus time using the same 1-D analysis technique. Each figure is at a different location in the channel. These plots show not only the fluctuations with time but also that the average ΔT and therefore velocities do vary as a function of distance from the wall. The temporal variation, of course, represents the expected output of a theoretical ultrasonic flowmeter with a very fast time constant and a narrow beam parallel to the flow.

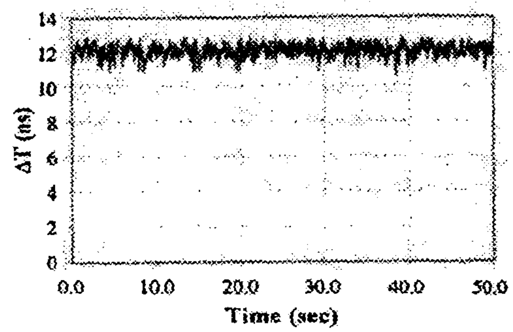


Figure 6 Delta T vs Time Calculated from the PIV Data; Measurement Volume Parallel to Flow, 1-D, Near Center Wall

While performing a 1-D operation on what amounts to a thin line provides interesting information as to the shape of the velocity profile and the standard deviation as a function of radius in a flow, the information obtained cannot be compared to data from an ultrasonic flowmeter. The reason for this is that "infinitely" thin line only reacts to the velocities it encounters as it travels between the transducers. An actual ultrasonic sound pulse has a finite width, which effectively averages the sound beam as it travels across a velocity field as shown in Figure 4.

By making the simulated beam a finite width, and turning it to an angle to the flow field a better simulation of the ultrasonic flowmeter was obtained. The modified ray trace program described previously was used for these calculations. By placing the flow measurement volume at an angle to the flow field, either of the equations 9 or 10 can be used to determine the integrated velocity of the flow.

Figure 8 is a plot of the computed velocity averaged along the ultrasonic path as computed by the modified ray trace program versus time. This path-averaged velocity is the instantaneous ultrasonic flowmeter output and is equivalent to ΔT. The sound travel path was set for this computation to be 60°. Three pieces of information are plotted in the graph. First is the computed instantaneous path average velocity, which are the individual points plotted with the dashed lines. The second set of data are the arithmetic mean and the upper and lower standard deviations represented by the 3 straight lines. Finally, a 2 second running average is plotted with a heavy line. It may be noted that this 2 second average is not a straight line but represents less scatter than the instantaneous as would be expected. Ultrasonic flowmeters typically provide this type of averaging technique. The standard deviation of the instantaneous data in this case is approximately 3.5%, whereas the deviation of the running average is on the order of 1%. So by using a standard arithmetic averaging technique the velocity measurement data scatter is reduced by over 3 times. However, the danger in using this type of averaging technique is that while it works for turbulent flow fields with little large-scale motion, if some type of structure such as a vortex street is embedded in the velocity field the averaging technique will fail. The reason for this is that this type of averaging requires independence or small spatial correlation. In addition, if the sampling rate is too fast the individual data points again may not be independent. If the individual data points are not independent, a different averaging technique may be required which allows for correlation of each point.

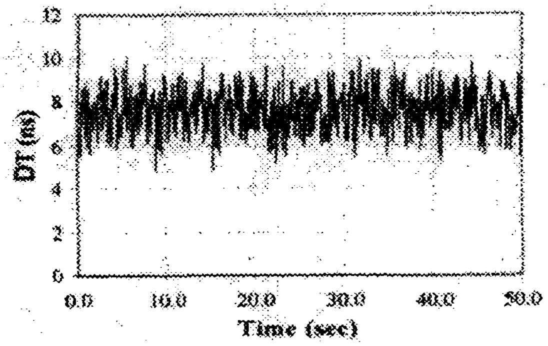


Figure 7 Delta T vs Time Calculated from the PIV Data; Measurement Volume Parallel to Flow, 1-D, Near Center

If using the standard averaging technique is the proper method for finding the average velocity for a particular flow field, a key piece of information needed to determine the average to the desired accuracy, is how many points are necessary. If the data measured are statistically independent and have a normal distribution, one may easily calculate the number of points required to find a mean velocity within a prescribed accuracy. The method to do this is:

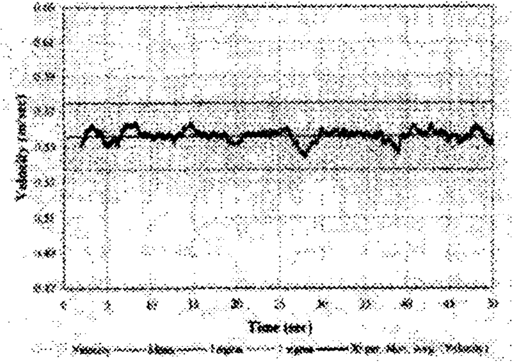


Figure 8 Velocity Computed Using the Modified Ray Trace Method Plotted vs Time

$$n = \left(\frac{\text{confidence}}{\%\text{Error}}\right)^2 = \left(\frac{2\sigma}{\bar{x} - \mu}\right)^2, \qquad (11)$$

where 2σ is the 95% confidence interval, μ is the population mean, and $\bar{x}$ is the sample mean. Using Equation 11, the number of data points required to compute the mean of a sample of data such that one is 95% confident that the mean within a certain accuracy may be computed. For example if $\sigma=2$ and $\mu = 8$, then to get a sample average within 1% accuracy requires 2500 points.

Using Equation 11, it can now be computed how many points of data are required to obtain a desired flow measurement accuracy when using an ultrasonic flowmeter. Using the data from Figure 8, to be 95% sure the computed mean velocity is within 0.25% of the of the actual velocity mean, one must obtain 775 data points. At the rate of 15 Hz, each average will take 51 seconds to obtain. Reducing the precision to 1% means only 48 points are required and approximately 3 seconds is required. These computations result from a theoretical ultrasonic flowmeter and are due to the turbulence in the flow field.

6 Conclusions

- Analyses of sound travel through a turbulent flow field can be performed using a velocity data set measured by the PIV method.
- The PIV analysis can be used to approximate an ultrasonic flowmeter by assuming a frozen flow in which the sound pulses are sent across the field in both directions simultaneously.
- Using the technique described, if the sound is launched parallel to the velocity stream and the results

are averaged with time, a plot of flow velocity profile versus position in a channel is similar to plots in literature.

- The variation or standard deviation of the average velocity when measured parallel to the flow varies from approximately 25% very close to the wall down to approximately 3% in the center of the channel as would be expected.
- When the measurement volume is turned to an angle to the flow, the average velocity can be determined, and the standard deviation of computed velocity is approximately 3.5%, which is a typical value of the standard deviation of instantaneous velocity realizations for ultrasonic flowmeters.
- Using a running velocity measurement average will reduce the standard deviation of the computed velocity; however, care must be given to ensure the velocities being measured are independent of one another.
- The number of independent data points necessary to determine an average velocity of the PIV described flow to within 1% and 0.25% of a flow are 48 and 775 respectively.
- The length of time required to find an accurate average velocity in a flow field will depend on the data scatter of the ΔT measurements by an ultrasonic flowmeter system.

Acknowledgement

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