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DETERMINATION OF STATISTICAL CHARACTERISTICS OF ISOTROPIC TURBULENCE

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ABSTRACT

Ultrasonic flow measurement technology has the potential to significantly improve flow measurement accuracy beyond that presently achieved. However, current applications fail to achieve theoretical accuracies because of turbulence effects on ultrasonic wave transit time. Herein, the ultrasonic flowmeter equation is reconsidered, where the effects of turbulent velocity and sound speed fluctuations are included. The result is an integral equation for the corresponding correlation functions. In this paper experimental velocity data are used to solve this integral equation analytically. As a result, some statistical characteristics of the turbulent flow are developed and can explain the limitations of measurement accuracy observed in applications.

NOMENCLATURE

$\overline{c}$	Mean speed of sound
c'	Speed of sound fluctuations
$\overline{u}$	Mean velocity component in $x-$direction
u'	Fluctuations of velocity component in $x-$direction
s	Sound wave travel path
Δt	Sound wave travel time
N	Number of experiments
$K_{\Delta t}(s,s')$	Spatial correlation function of travel time Δt
$K_{u'}(s,s')$	Spatial correlation function of $x-$component velocity fluctuations
$K_{c'}(s,s')$	Spatial correlation function of sound speed fluctuations
$\psi_i(x)$, $\psi_j(x')$	Shape functions

INTRODUCTION

Technical advances in ultrasonics for the past twenty years have resulted in the development of electronic instrumentation capable of measuring very small time differences associated with changes in the ultrasound wave propagation time (Lynnworth, 1989). These capabilities are used effectively in oblique path flowmeters. The ability of ultrasound to measure noninvasively, nondestructively and rapidly is clearly desirable (Yeh and Mattingly, 1998). Nevertheless, the accuracy of these

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devices has not improved very much, certainly not proportional to improvements in the measurement technology. We believe the explanation lies in the effect of turbulence on the ultrasonic wave propagation.

Our objective in this research is an attempt to find analytical solution of the ultrasonic flowmeter equation; namely an integral equation for the corresponding correlation functions derived from general ultrasonic flowmeter equation. The paper focuses on construction of statistical characteristics of the turbulent flow based on experimental data.

Our approach in this paper is:

1. Develop statistical characteristics of the ultrasonic wave travel time using an ensemble of experiments to construct a spatial correlation function of the pulse propagation time period.
2. To solve an inverse problem of finding analytical spatial correlation functions of turbulent velocity and sound speed variations based on the experimentally obtained spatial correlation function of time.

METHOD OF APPROACH

The method utilizes ultrasonic pulses that travel in straight paths. Figure 1 illustrates the ultrasonic flowmeter in a duct. The upper and lower transducer locations and signal path are shown. In the limiting case, the mean flow should not affect the transit time of the signal.

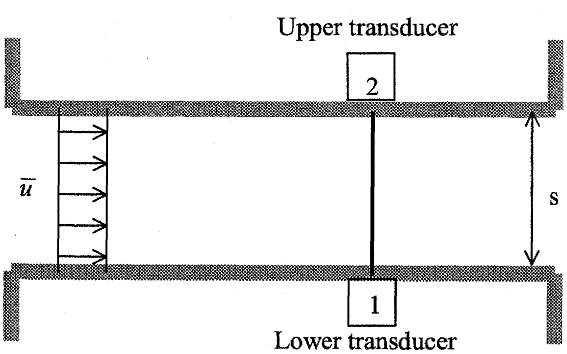

Figure 1. Sketch of wind-tunnel test section with ultrasonic flowmeter.

The equation for the ultrasonic flowmeter is:

$$\int_1^2 dt = \int_1^2 \frac{ds}{U+C} \qquad (1).$$

Sound pulses travel with a velocity that is the vector sum of the local flow velocity, turbulent velocity, sound speed along the propagation direction and sound speed fluctuations as a result of temperature variance. Hence, velocity U and sound speed C in the expression (1) can be decomposed into mean and fluctuating components as following:

$$\begin{aligned} C &= \bar{c} + c' \\ U &= \bar{u} + u' \end{aligned} \qquad (2).$$

The data from experiments performed for a finite number of different lengths s and s' are collected. For each of those lengths a value $\Delta t(s)\Delta t(s')$ is calculated. Averaging the latter through the entire ensemble (Bendat and Piersol, 1986) results in a space correlation function of travel time Δt as follows

$$K_{\Delta t}(s,s') = \frac{\sum_N \Delta t(s)\Delta t(s')}{N} \qquad (3).$$

Our objective here, then, is to construct spatial correlation functions for turbulent velocity u' and sound speed fluctuations c' (Pugachev, 1960). Perturbation analysis applied to (1) leads to the following expression:

$$\Delta t(s) = \int_s \frac{dx}{\bar{u}+u'+\bar{c}+c'} = \frac{1}{\bar{c}}\int_s \frac{dx}{1+\frac{\bar{u}}{\bar{c}}+\frac{u'}{\bar{c}}+\frac{c'}{\bar{c}}} \cong \frac{1}{\bar{c}}\int_s \left(1-\frac{u'}{\bar{c}}-\frac{c'}{\bar{c}}\right)dx = \frac{s}{\bar{c}} - \frac{1}{\bar{c}}\int_s (u'+c')\,dx \qquad (4).$$

Here we assumed that $\bar{u} << \bar{c}$ and terms $u'/\bar{c}$ and $c'/\bar{c}$ are the same order infinitesimal compare to unity. In order to construct correlation function we introduce new variable:

$$\Delta^0 t(s) = \Delta t(s) - \langle \Delta t(s) \rangle \qquad (5).$$

Consequently,

$$\begin{aligned} \Delta^0 t(s) &= -\frac{1}{\bar{c}}\int_s (u'+c')\,dx \\ \Delta^0 t(s') &= -\frac{1}{\bar{c}}\int_{s'} (u'+c')\,dx' \end{aligned} \qquad (6).$$

Then,

$$\Delta^0 t(s)\Delta^0 t(s') = \frac{1}{\bar{c}^2}\left[\int_s\int_{s'}\{u'(x)u'(x')+c'(x)c'(x')\}dxdx'\right]+ \left[\int_s\int_{s'}\{c'(x)u'(x')+u'(x)c'(x')\}dxdx'\right] \quad (7).$$

For the velocity measurement used here, the that data were collected in the isotropic region of flow, so that u' and c' are not correlated (Yeh and Van Atta, 1973). Consequently, space correlation function of time can be defined as

$$\left\langle \Delta^0 t(s)\Delta^0 t(s')\right\rangle = K_{\Delta t}(s,s'_m) = \frac{1}{\bar{c}^2}\left[\int_s\int_{s'}\left(K_{u'}(x,x')+K_{c'}(x,x')\right)dxdx'\right] \quad (8).$$

It is important to emphasize that space correlation function $K_{u'}(x,x')$ alone can be defined based on data from room temperature experiments. Then, data from heated air experiments can be used to identify $K_{c'}(x,x')$, knowing $K_{u'}(x,x')$ and $K_{\Delta t}(s,s')$.

DISCUSSION OF RESULTS

We are developing apparatus to demonstrate this methodology using data from ultrasonic measurements of turbulence with dual-transducers employed in the travel-time technique. However, in order to demonstrate the technique, we use turbulent velocity data obtained from particle image velocimetry (PIV). In this way, we can compute the wave transit times for wave passage through the measured field and construct the statistics of the distribution.

Equation (8), with left hand side known, has to be solved with respect to correlation functions. However, for room temperature experiment (temperature is constant throughout the channel) the correlation function of the sound speed velocity fluctuations can be neglected. As a result, correlation function of velocity fluctuations may be obtained directly from (8). Derivation of an exact analytical solution of equation (8) does seem to be evident in the case when significant number of points is taken from an experiment. The travel-time correlation function has to be extrapolated so, that it could be written in analytical way. Then, differentiating the latter with respect to s and s' gives an exact analytical expression for correlation function of velocity fluctuations. However, in the case of insufficient experimental data, and, consequently, small number of points this approach does not proved to be efficient, because of extrapolation accuracy. Meanwhile, an approximate analytical solution in a form of a series may be obtained. To demonstrate this approach, the correlation function of velocity fluctuations may be expanded into a series

$$K_{u'}(x,x') = \sum_i^N\sum_j^M \lambda_{ij}\psi_i(x)\psi_j(x') \quad (9),$$

where $(s_* - s_*^{'})$ and $\psi_j(x')$ are shape functions.

To express results in explicit simple way, the shape functions were chosen in triangular form, as demonstrated in Figure 2, with maximum height equal to unity, length equal to $2s/N = 2\Delta x$, for $x_{i-1} \le x \le x_{i+1}$ and zero otherwise.

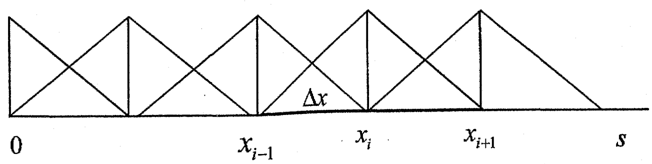

Figure 2 Triangular shape functions.

Evaluating integrals of the shape functions yields

$$\int_0^s \psi_i(x)dx = \begin{cases} \dfrac{s}{N}, i=1,2,..,N-1 \\ \dfrac{s}{2N}, i=0,N \end{cases}$$
$$\int_0^{s'} \psi_j(x')dx' = \begin{cases} \dfrac{s'}{M}, i=1,2,..,M-1 \\ \dfrac{s'}{2M}, i=0,M \end{cases} \quad (10)$$

Substituting (10), (9) into (8), coefficients λ_{ij} are to be determined for known left hand side as

$$\lambda_{ij} = \frac{K_{\Delta t}(s_*, s_*^{'})}{\int_0^{s_*}\psi_i(x)dx\int_0^{s_*^{'}}\psi_j(x^{'})dx^{'}} \quad (11).$$

These derivations will result in approximate solution to equation (8), satisfied at all given points $s_*, s_*^{'}$ and expressed in form (10) for room temperature. Once this correlation function is derived, correlation function of sound speed fluctuations may be obtained similarly by conducting an experiment at different temperature. Values of the travel-time correlation function are given in Table 1 for different s_* and s_*', and $T=150^\circ F$. From the Table 1 it is seen, that the correlation function of travel time does not depend on the difference $\left(s_* - s_*'\right)$. Figure 3 illustrates correlation function $K_{\Delta t}\left(s_*, s_*'\right)$ for a fixed distance $s_* = 1$.

s_* \ s_*'	1	1.00382	1.01815	1.03528	1.06418
	$K_{\Delta t}(s_*, s_*')$				
1		0.9518	0.756361	0.48868	0.274516
1.00382			0.875781	0.626053	0.329049
1.01815				0.847812	0.597737

Table 1 Value of normalized correlation function of travel time $K_{\Delta t}\left(s_*, s_*'\right)$ for $T=150^\circ F$ as a function of nondimensional distance s_* and s_*'.

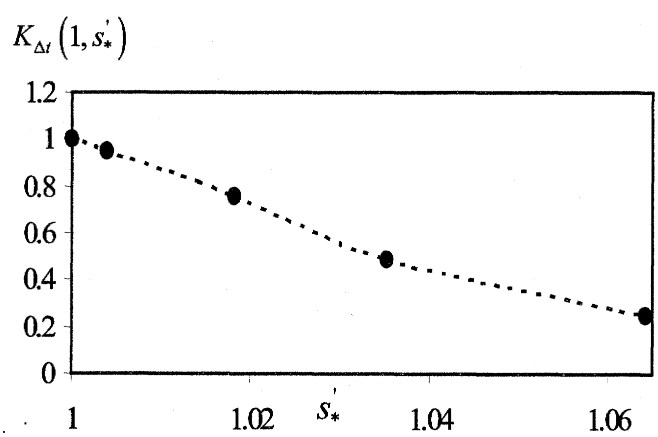

Figure 3 Normalized correlation function for a fixed $s_* = 1$ versus s_*'

CONCLUSIONS

The basic ultrasonic flowmeter equation including the effect of velocity and sound speed fluctuations was considered. The integral equation relating the travel time correlation function with the correlation functions of turbulent velocity and sound speed variations was derived. By assuming that influence of the correlation function of sound speed variation is negligible when an experiment is conducted at constant room temperature, an integral equation for unknown correlation function of turbulent velocity was obtained. An approximate analytical solution for the latter was constructed using weighted residual method. The resulting spatial correlation represents turbulent flow characteristics that can be obtained by analyzing the time series of transit times for ultrasonic waves across isotropic turbulent flow.

It will, of course, be possible to construct similar correlations from transit time measurements during flowmeter operation. These correlations will reflect the nature of the turbulent flowfield through which the ultrasonic waves passed and may form a basis for correcting flow indications to achieve greater accuracy. The effect of temperature fluctuations is expected to be analogous but is presently unverified. Data from heated-grid experiments and the additional assumption that the correlation function of turbulent velocity is temperature independent, will form the basis for an approximate solution for correlation function of sound-speed fluctuations in a similar way.

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