

BRIDGE LIFETIME SYSTEM RELIABILITY UNDER MULTIPLE LIMIT STATES

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ABSTRACT: A system reliability approach to minimizing the life-cycle cost of a deteriorating structure offers significant advantages such as a rational assessment of the assumed risk of failure, and an understanding of the importance and contribution of individual components to the overall reliability of the structure. The reliability of a structural system as a whole is the measure of its overall performance. This measure has to include both ultimate and serviceability limit states. A system model of a structure traditionally consists of a series-parallel combination of strength-based component limit states. Serviceability limit states however, can play a tremendously important role in optimizing the inspection and repair of a deteriorating structure. This paper proposes the use of serviceability flags as a means to incorporate serviceability concerns into a strength-based reliability analysis. Using highway bridges as an example, available data sources for serviceability flags are considered. The effect of including serviceability flags in an optimum life-cycle analysis is illustrated on a specific highway bridge.

INTRODUCTION

As the 2001 American Society of Civil Engineers Report Card for America's Infrastructure (ASCE 2001) indicates, our nation needs to spend \$1.3 trillion over the next 5 years to overcome our current infrastructure deficiencies. In bridges alone, 29% have been classified as functionally deficient and it will require \$10.6 billion per year over the next 10 years to remedy the situation. As infrastructure deteriorates over time, there is an increased risk to society and a limited amount of money to solve the problem. The risk of failure of any structure can never be totally eliminated. The risk can be reduced, but eventually a point of diminished marginal returns is reached where minor reductions in risk require unjustified costs. Ideally, engineers want to spend the money most efficiently on the projects that pose the greatest degree of risk. Because acceptance of risk requires that uncertainty be quantified, reliability methods are useful and appropriate. Reliability methods can be used to optimize the life-cycle inspection and repair of critical structures (Estes 1997).

While many reliability analyses focus on a specific component or a specific limit state, there are distinct advantages to analyzing a structure from a system reliability perspective. In a system analysis, all strength-based limit states are identified and combined into a series-parallel model. A series system fails when any one of its components fails, while a parallel system fails only after every component fails. The system reliability is a function of the series-parallel model, the individual reliabilities of the members, and the correlation between the failure modes (Estes and Frangopol 1998). A system reliability approach allows the engineer to identify the importance of an individual component or failure mode with respect to the overall performance of the system.

Estes and Frangopol (1999) outlined a methodology for using system reliability to optimize the lifetime inspection and repair of a deteriorating structure. The approach was illustrated on Bridge E-17-AH, a specific highway bridge (Figs. 1 and 2) located in the metro-Denver area of Colorado. The bridge sys-

tem reliability model considered 16 failure modes that included moment failure of the concrete deck, shear and moment failures of the girders, and various failure modes of the substructure. The analysis considered 24 random variables that included material strengths, modeling uncertainties, loads, and load effects. Probabilistic deterioration models were used to model the corrosion of girders and the penetration of chlorides through the concrete that ultimately corroded the reinforcing bars. Five possible repair options, shown in Table 1, were examined and their associated costs were computed. Based on a 2% discount rate of money and the requirement to make a repair anytime the system reliability index β_{system} fell below 2.0, all possible combinations of the available repair options, as shown in Fig. 3, were considered. Table 2 shows that the optimum reliability-based repair strategy is dependent on the desired useful life of the structure.

The system reliability analysis revealed that the component with the lowest reliability was not always the component that most needed to be repaired. It was possible for the reliability index of a component in the parallel portion of the system model to fall below 2.0 without causing the reliability of the system to fall below its threshold value of $\beta_{\text{min}} = 2.0$. As time passed and different components deteriorated at different rates, the most critical component early in the life of the structure is not necessarily the most critical component later on. Such analysis is only possible when the structure is observed as a system of components.

The bridge reliability analysis focused entirely on strength-based limit states. It did not include serviceability concerns such as potholes in the concrete deck, excessive deflections, or spalling on the columns that may necessitate a bridge repair



FIG. 1. Bridge E-17-AH—Elevation

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FIG. 2. Bridge E-17-AH-Roadway and Railings

TABLE 1. Repair Options and Associated Repair Costs for Bridge E-17-AH

Option No.	Repair option	Repair cost (\$)
0	Do nothing	0
1	Replace deck	225,600
2	Replace exterior girders	229,200
3	Replace exterior girders and deck	341,800
4	Replace superstructure	487,100
5	Replace bridge	659,900

but which will not cause the bridge to collapse. It is extremely difficult to incorporate these problems into a system reliability analysis because the level of concern over serviceability issues is not as high because the consequences of failure are not as great. Society is willing to accept greater risk when the consequences are driver discomfort, aesthetics, or public concern, rather than collapse of the bridge.

The minimum system reliability index for this bridge example was $\beta_{min} = 2.0$, which equates to a notional probability of failure of 0.022 (i.e., about one chance in 50) under the most extreme load condition during the service life. With potholes in a deck, one may be willing to accept a 10% chance of occurrence or even a 50-50 chance before making the repair. Such disparity is reflected in current design procedures where load and resistance factors are applied to strength-based limit concerns but not to serviceability limit states. There is no effective way to insert a serviceability limit state into a strength-based series-parallel system model when the acceptable risk level of that serviceability component is different from that of the strength-based components.

Consider a hypothetical series system for a girder consisting of components relating to failure by shear, moment, and excessive deflection, as shown in Fig. 4. If the level of concern were different for these three failure modes there would be no

TABLE 2. Optimum Lifetime Repair Strategy for Bridge E-17-AH Based on Strength-Based Series-Parallel System Model

Expected life (years)	Optimum Repair Strategy	
	Option No. ^a (year)	Cost (\$)
0-50	Do nothing	0
50-94	1 (50)	83,813
94-106	1 (50), 1 (94)	118,881
106-108	1 (50), 3 (94)	136,945
>108	1 (50), 5 (94)	186,393

^aSee Table 1.

acceptable target system reliability index. A target system reliability index cannot be prescribed when strength and serviceability limit states are combined. A high β_{system} value would overly constrain the possibility of excessive deflections, and a low β_{system} value would allow the probabilities of shear and moment failure modes to become unacceptably high.

This paper proposes the use of serviceability flags to account for serviceability concerns in a system reliability analysis. The definition of serviceability flags, types of serviceability limit states, and available data for estimating their values are discussed. Finally, the relevant serviceability flags are added to the strength-based reliability analysis of Bridge E-17-AH and the results compared.

SERVICEABILITY FLAGS

A serviceability flag is a means of overriding the strength-based reliability analysis. It adds another constraint to the problem and can only make the result more conservative. An engineer inserts a serviceability flag to accommodate any additional concern on a structure that is not addressed in the strength-based limit-state equations. For example, if the engineer believes that a concrete deck will have to be replaced

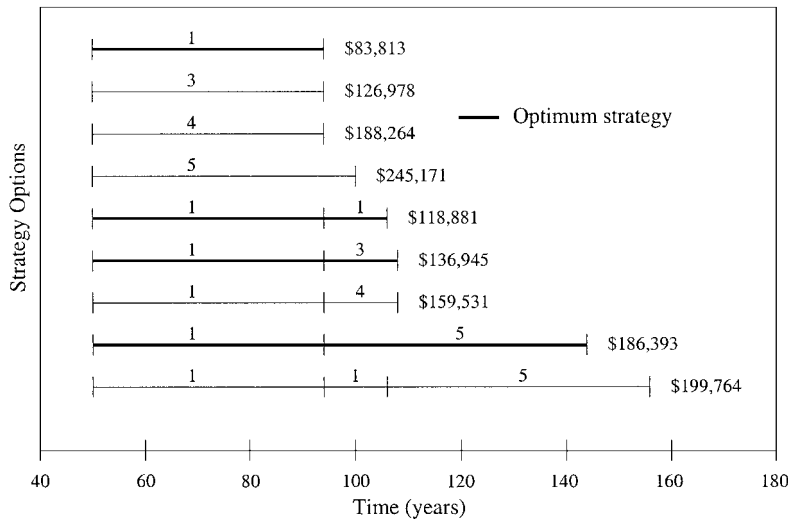


FIG. 3. All Feasible Repair Options for Bridge E-17-AH Using Series-Parallel Model Requiring Failure of Three Adjacent Girders



FIG. 4. Hypothetical Series System Model of Typical Girder

every 30 years due to excessive potholes that do not significantly affect the moment strength of the slab but still present unacceptable driving conditions, then a serviceability flag is created. In this case, the slab would be repaired every 30 years or whenever the strength-based solution dictates, whichever is sooner.

The engineer decides which serviceability flags to insert. In the analysis of Bridge E-17-AH, there were three concerns that needed to be addressed. The concrete bridge deck may need to be replaced due to potholes and spalls prior to the point where the penetration of chloride salts in concrete and reinforcement deteriorated the strength-based reliability of the structure. Second, the only portion of the substructure for which a deterioration model was deemed appropriate was the pier cap. Clearly, the pier columns, pier footings, and abutments would all deteriorate at some point, even if there were no available probabilistic models to describe the rate or process of deterioration. Finally, the steel hand railing was deteriorating, but the failure of the hand railing would not cause catastrophic failure of the bridge and was not included in the strength-based analysis. Therefore, serviceability flags are inserted for the concrete deck, the hand railings, and the substructure, as a whole. It was concluded that the strength-based analysis on the girders was sufficient and no serviceability flag was added.

The source of information for serviceability flags was historical data. With bridge inspection programs and bridge management systems widely used, many studies are available that obtained historical inspection data from many states and developed prediction models. Hearn et al. (1995) provided a summary of many of these studies and their accompanying results. Most of these studies and models describe how existing bridges have progressed through prescribed condition states that provide a general description of a bridge's deterioration over time. The reasons or mechanisms that caused the deterioration are not addressed in these models. The models merely reflect how a large number of bridges have behaved over time. These models are used for serviceability flags because they incorporate the non-strength-based intangibles that have not or cannot be quantified.

The potential pitfall with using these models is that the unique structure under consideration and its environment may be very different from the majority of the structures from which the data was taken. For example, Bridge E-17-AH is constructed over railroad tracks. Most bridges are built over water where the substructure is subjected to scour or over highways where cars and trucks passing underneath expose the substructure to splashed water and pollutants. The substructure of Bridge E-17-AH could reasonably be assumed to last longer than what is indicated by the data for the average bridge. Unless a compelling reason exists to the contrary, the most appropriate available bridge model is used.

CONDITION STATES

Many bridge deterioration studies are based on the national bridge inventory (NBI) condition ratings. As part of the national bridge inspection program, states are required to inspect their bridges every 2 years and report the results to the Federal Highway Administration (FHWA) in a standardized format of condition ratings. The ratings, as listed in Table 3, range from a high score of 9, indicating a bridge in excellent condition to a low of 0, indicating a bridge that has already failed (FHWA 1988). This rating information comprises the national bridge inventory from which many studies find their data. Some stud-

TABLE 3. National Bridge Inventory Condition Ratings (FHWA 1988)

NBI rating	Description	Repair action
9	Excellent condition	None
8	Very good condition	None
7	Good condition	Minor maintenance
6	Satisfactory condition	Major maintenance
5	Fair condition	Minor repair
4	Poor condition	Major repair
3	Serious condition	Rehabilitate
2	Critical condition	Replace
1	Imminent failure condition	Close bridge and evacuate
0	Failed condition	Beyond corrective action

TABLE 4. PONTIS (1995) CS Ratings for Unprotected Concrete Deck with Asphalt Concrete Overlay

CS	Description
1	The surfacing of the deck has no repaired areas and there are no potholes in this surfacing.
2	Repaired areas and/or potholes or impending potholes exist. Their combined area is <2% of the deck area.
3	Repaired areas and/or potholes or impending potholes exist. Their combined area is <10% of the deck area.
4	Repaired areas and/or potholes or impending potholes exist. Their combined area is >10% but <25% of the deck area.
5	Repaired areas and/or potholes or impending potholes exist. Their combined area is <25% of the deck area.

ies lump all bridges together, while others attempt to separate them by location, type of bridge, traffic volume, and environment.

As bridge management systems have progressed, many states have developed programs to include much more information than the minimum required by the federal government. Attempts to study how different bridge components behave over time have been made for railings, joints, bearings, and all types of decks, girders, and substructures. In many cases, the states have developed their own condition states with more precise descriptions for every bridge element that gets inspected. These individual condition states are then converted to the NBI scale for federal reporting requirements.

In Colorado, for example, which uses the PONTIS bridge management system, an asphalt concrete deck is rated according to one of five condition states listed in Table 4 (PONTIS 1995). Such reporting has allowed the data for specific bridge elements to be collected, studied, and modeled.

CONDITION STATE DETERIORATION MODELS

Some available models were considered in developing serviceability flags for the deck, railings, and substructure for Bridge E-17-AH. Many of the models are based on a linear deterioration of condition states where the deterioration rate can be expressed in terms of condition rating loss per year (CR/year), where the condition rating at any time t can be computed. Looking specifically at reinforced concrete (RC) decks, railings, and RC substructures, the results of several studies based on linear models, as described in Hearn et al. (1995) are shown in Table 5. The source of the study, and whether it was based on data or the opinion of experts, is included. The number of years required to reach NBI condition state 4 (poor condition) and condition state 3 (serious condition) is indicated based on the given condition rating loss per year.

Some of the studies became more specific regarding traffic volume and location. For example, Chen and Johnston (1987) indicated 39 years for reaching condition state 4 for the RC decks, where the average daily traffic was >4,000, rather than the 41 years for all RC decks. The average daily traffic for

TABLE 5. Linear Condition State Deterioration Models for RC Decks, Railings, and RC Substructures (Hearn et al. 1995)

Element	Source	Basis	Time to NBI = 4 (years)	Time to NBI = 3 (years)	Deterioration rate (CR/year)
RC deck	James et al. (1993)	Data	24	29	0.210
RC deck	Stukhart et al. (1991)	Expert	33	39	0.152
RC deck	Chen and Johnston (1987)	Data	41	49	0.123
RC deck	Morrow and Johnston (1994)	Data	45	54	0.111
RC deck	Al Rahim and Johnston (1991)	Data	48	58	0.104
Steel rail	Morrow and Johnston (1994)	Data	37	44	0.135
RC substructure	James et al. (1993)	Data	23	27	0.219
RC substructure	Stukhart et al. (1991)	Expert	35	42	0.143
RC substructure	Chen and Johnston (1987)	Data	44	53	0.114
RC substructure	Morrow and Johnston (1994)	Data	42	50	0.119
RC substructure	Al Rahim and Johnston (1991)	Data	42	50	0.119

Bridge E-17-AH is 8,500. Similarly, James et al. (1993) found that the condition state deterioration rate for RC decks on state highways in the western region of the United States was 0.176 rather than 0.210 for all RC decks, which equates to 28 years to reach condition state 4 and 34 years to reach condition state 3.

Similar detail could be added to the substructure estimates as well. The study of Chen and Johnston (1987) actually listed three condition state deterioration rates (0.102, 0.119, and 0.114) for the coastal, mountain, and piedmont regions of North Carolina, respectively. A Pennsylvania study determined the expected service life of a deck with uncoated reinforcement to be 25 years and a substructure to be 100 years (Hearn et al. 1995). Jiang and Sinha (1989) developed the following polynomial model for a concrete bridge superstructure:

$$CS(t) = 9.0 - 0.28877329t + 0.0093685t^2 - 0.00008877t^3 \quad (1)$$

where $CS(t)$ = condition rating of the bridge at time t , where t is the age of the bridge in years, which translates to 71 years to condition state 4. Weyers et al. (1988) computed an average condition state deterioration rate for replacing a substructure of 0.077 CR/year, which indicates 65 years to condition state 4 and 78 years to condition state 3. There is no agreement between these studies and the result is an average deterioration rate. When dispersion data are available, the condition rating itself can become a random variable and a probabilistic approach can be used. Ayyub et al. (1998) applied this approach to the assessment of hydropower equipment. Markov chains provide another probabilistic approach [e.g., PONTIS (1995)].

MARKOV CHAIN MODELS

Markov chains can be used to model NBI condition ratings based on the data from large numbers of bridges using transitional probabilities. Jiang and Sinha (1989) used Markov chains to model the condition of bridge substructures in Indiana. Table 6 shows the transitional probabilities for concrete bridge substructures. In this case the transitional probabilities change as the bridge ages.

The value p_9 indicates the probability that a bridge that is currently in condition state 9 will remain in condition state 9 for the next year. For a new bridge that is only 0–6 years old, this probability is $p_9 = 0.705$. Assuming that a bridge can only change one condition state in a given year, the probability that the bridge will fall to condition state 8 is $1 - p_9$, which for the new bridge is $1 - 0.705 = 0.295$. Once this new bridge (i.e., 0–6 years old) has transitioned to condition state 8, the probability that it will remain in condition state 8 is $p_8 = 0.818$, and so forth. Using Table 6, the time-dependent bridge condition can easily be modeled.

Similarly, Cesare et al. (1992) used Markov chains to model many bridge elements in New York State using a database of

850 bridges and 2,000 individual spans. The New York condition ratings range from 7 (high) to 1 (low).

Based on these New York condition ratings, Cesare et al. (1992) developed stationary transition probabilities for numerous bridge elements. For a structural cast-in-place bridge deck with uncoated bars, the transitional probabilities are $p_7 = 0.937$, $p_6 = 0.940$, $p_5 = 0.971$, $p_4 = 0.974$, $p_3 = 0.977$, and p_2

TABLE 6. Transition Probabilities for Concrete Bridge Substructures Using Markov Chains (Jiang and Sinha 1989)

Bridge age (years)	Transitional Probabilities					
	p_9	p_8	p_7	p_6	p_5	p_4
0–6	0.705	0.818	0.810	0.802	0.801	0.800
7–12	0.980	0.709	0.711	0.980	0.980	0.856
13–18	0.638	0.639	0.748	0.980	0.980	0.980
19–24	0.798	0.791	0.788	0.980	0.870	0.824
25–30	0.794	0.810	0.773	0.980	0.980	0.980
31–36	0.815	0.794	0.787	0.980	0.980	0.737
37–42	0.800	0.798	0.815	0.980	0.850	0.980
43–48	0.800	0.800	0.309	0.938	0.980	0.050
49–54	0.800	0.800	0.800	0.711	0.707	0.768
55–60	0.800	0.800	0.800	0.050	0.050	0.505

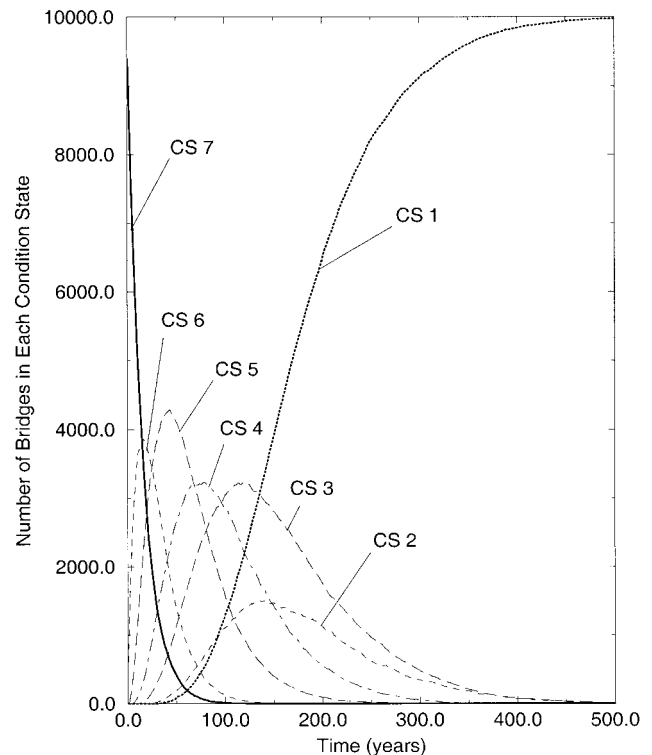


FIG. 5. Condition States (CS) for Cast-in-Place Bridge Deck over Time Using Markov Chains and New York State Condition Ratings

= 0.961. Using these probabilities and a simulation of 10,000 bridges, Fig. 5 shows the expected number of bridges being in any given condition state at any time. The probabilistic serviceability flags can be developed for any bridge element for which the Markov chain data are available.

SERVICEABILITY FLAGS FOR BRIDGE E-17-AH

It is clear that the data can differ significantly from study to study. The engineer must use the study and assumptions that best fit the structure being considered when developing serviceability flags. For the case of Bridge E-17-AH, the following serviceability flags were adopted to account for deterioration of the slab, railing, and substructure. The concrete slab will be replaced every 28 years using the James et al. (1993) study for RC slabs on state highways in the western region deteriorating to condition state 4. The railings were replaced every 37 years using the Morrow and Johnston (1994) study. Considering that only a railroad runs underneath the bridge, the substructure will be replaced every 65 years using the Weyers et al. (1988) study.

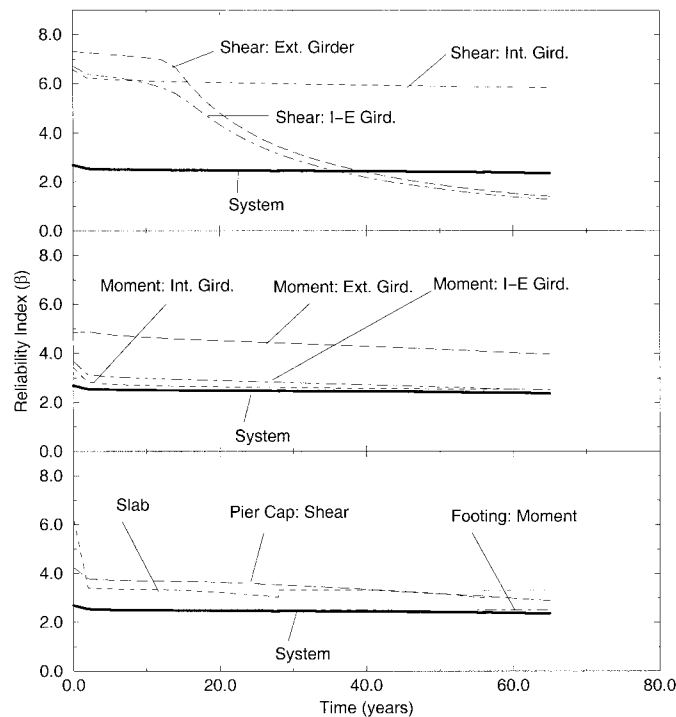


FIG. 6. Results of Repair Option 1 (Replace Deck) on Bridge E-17-AH Using Series-Parallel System Model Including Serviceability Flags

TABLE 7. Optimum Lifetime Repair Strategy for Bridge E-17-AH Based on Strength-Based Series-Parallel System Model (Serviceability Flags Included)

Expected life (years)	Optimum Repair Strategy		Cost (\$)
	Option No. ^a (year)		
0-28	Do nothing		0
28-56	1 (28)		129,579
56-65	1 (28), 1 (56)		204,006
>65	1 (28), 5 (56)		347,284

^aSee Table 1.

The results for Repair Option 1 (replace deck) (Table 1) with these serviceability flags are shown in Fig. 6 and can be compared to the same situation where serviceability flags were not in effect (Estes and Frangopol 1999). Fig. 6 shows the reliability of the bridge system over time as well as the reliabilities of the individual component limit states. The railing serviceability flag is never executed because the railing is replaced every time the slab gets replaced (every 28 years). In Fig. 6, the slab is replaced twice (years 28 and 56), which is before the strength constraint requires it. As a result, there is little effect on the system reliability from the slab replacement. Slab repair is no longer effective at year 65, where the serviceability flag requires that the substructure, and thus the bridge, be replaced.

All feasible repair options for the series-parallel bridge model, where three adjacent girders are required to fail with serviceability flags implemented, are shown in Fig. 7. These options can be compared to Fig. 3, where the serviceability flags are not used. The optimum repair strategy based on these options, including serviceability flags, is shown in Table 7. Compared to the optimum repair strategy without serviceability flags shown in Table 2, the serviceability flags result in earlier repairs, a shorter expected life of the bridge, and a more expensive optimum repair strategy. This will always be the case. At the most extreme case, where all serviceability flags are overridden by strength concerns, the optimal solution would be the strength-based case. A serviceability flag will only shorten the life of the structure.

CONCLUSIONS

Serviceability flags are a reasonable means of incorporating serviceability concerns into a strength-based reliability analysis. These flags are based on a deterioration model and need to be updated over time through inspections. It has been demonstrated how an optimum inspection plan can be developed

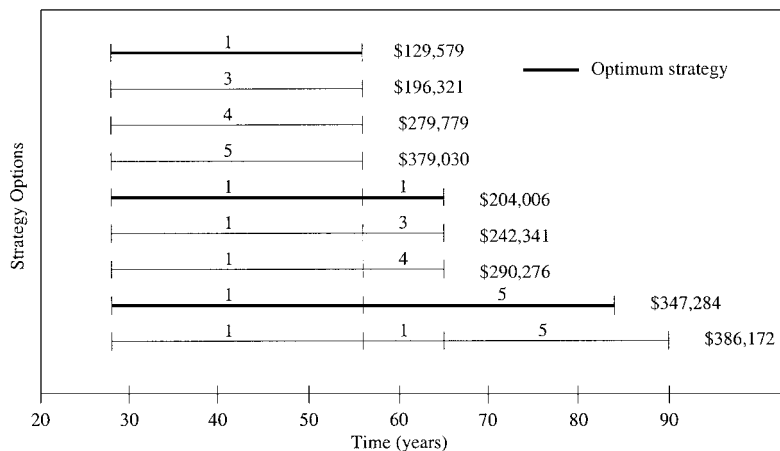


FIG. 7. All Feasible Repair Options for Bridge E-17-AH Using Series-Parallel Model Requiring Failure of Three Adjacent Girders Including Serviceability Flags

(Frangopol and Estes 1999) and updated (Estes and Frangopol 2001), based on nondestructive evaluation test results for a strength-based system reliability analysis. The data from routine visual inspections can sometimes be used to update the deterioration models and the reliability analysis (Frangopol and Estes 1997, 1999). In the case of serviceability flags, the updating is much easier. Since the condition state deterioration models were based on visual inspection data, it is straightforward to compare future visual inspection results with expected condition state transition and revise the serviceability flag accordingly. By considering serviceability in the analysis, the results are more realistic and more accurately reflect the actual decision-making process on when to repair a structure. Such analysis provides the necessary information to more efficiently allocate scarce funding resources to the projects that will yield the greatest benefits in terms of reduced risk.

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