Senior Project: Global Position Determination from Observed Relative Position of Celestial Bodies

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Senior Project: Global Position Determination from Observed Relative Position of Celestial Bodies

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A method was developed to determine the latitude and longitude of an observer based on the observed position of the Moon and several other celestial bodies. The basic principal developed dealt with the proximity of the Moon. Its relative displacement from calculated values was measured using photography by comparison with stars near the Moon. Photographs were taken from a location in San Luis Obispo at Longitude 120°35.9' and Latitude 35°13.3'. The analysis method has determined the location of the observer to a Longitude of 117°43.8'. An additional method located the observer to 36°38.7'N Latitude and 114°47.6'W Longitude.

Nomenclature

A = angle measured between stars 1 and 2, degrees
a = distance measured between stars 1 and 2, pixels
a_i = ambiguous polynomial coefficient
B = angle measured between star 1 and the Moon, degrees
b = distance measured between star 1 and the Moon, pixels
b_i = ambiguous location coordinate
C = angle measured between star 2 and the Moon, degrees
c = distance measured between star 2 and the Moon, pixels
Dec = angle measured away from the plane created by the vernal equinox line and the ecliptic plane – Declination, 00°00.0min
d = distance between the Earth and the Moon, pc
P = time from zero hour, hours
p = parallax angle, arcseconds
p_i = vector in pixel frame where i represents the direction index
Q = coordinate transformation matrix
RA = angle measured from the vernal equinox line lying in the plane perpendicular to the ecliptic plane – Right Ascension, 00°00.0min
α = angle opposite A and a, degrees
β = angle opposite B and b, degrees
γ = angle opposite C and c, degrees
θ = angle of rotation used to relate pixel space to RA and Dec space, degrees

I. Introduction

In order to accurately determine location on the surface of the Earth, constellations of positioning satellites have been launched. Global Positioning Satellites send signals which are received and analyzed and which require tens of satellites to achieve accurate global coverage. While this system is unmatchable in its accuracy to determine the longitude, latitude, and altitude of an observer, it is quite costly. This alternative system relies solely on natural celestial bodies and therefore does not require any satellite construction.

Latitude and longitude are the two primary coordinates used to determine position on the surface of the Earth. Longitude is measured around the equator starting at the Prime Meridian, and is measured in degrees, minutes, and seconds. One degree translates to 60 minutes. Latitude measures the angle above or below the equator. On the surface of a purely spherical body, these two coordinates are enough to fully define an observer's position.

Latitude and longitude are used in reference to a spherical body. A similar system is used to determine the position of stars and other distant bodies. Right ascension and declination are used to determine where to look in the sky in order to find a celestial body. Right ascension measures the angle between a plane including the vernal equinox line and perpendicular to the ecliptic plane. It is measured in hours, minutes, and seconds. Declination then measures the angle between the vernal equinox line within the Earth's orbital plane. It is measured in degrees, minutes, and seconds. The vernal equinox line is the vector defined by the line formed on the vernal equinox where the ecliptic and Earth equatorial planes intersect.

Due to their extreme distance, the drift motion of stars is not apparent when viewed from Earth. That is, even though stars move at speeds of millions of miles per hour through the galaxy, the effect on their observed position is much more affected by Earth's rotation than the stars own motion. For this reason, Stars are considered fixed points in space. Closer bodies, the moon and planets, are seen to move from the perspective of Earth. Their proximity shows them to move through the sky relative to the stars behind them.

II. Apparatus and Procedure

The position of stars are considered fixed over time. Their coordinates are updated annually as the vernal equinox line changes. This assumption is valid because the stars' distance from Earth makes their motion undetectable without specialized equipment. Their motion is so slow that fixed telescopes may use any number of stars to calibrate themselves. The Moon, Luna, moves much faster through the sky. It has a synodic period of about 27.3 days. That is, its relative position as viewed from the Earth is cyclic and repeats every 27.3 days. This means that it can move many degrees across the sky in a single night. Because of its relatively fast motion across the sky, Luna's position can be expressed using a polynomial equation. The polynomial coefficients are updated daily. The position of Luna is then found using

$$b = a_0 + a_1 P + a_2 P^2 + a_3 P^3 + a_4 P^4 + a_5 P^5 \tag{1}$$

where b represents the coordinate of interest, a_i represents the coefficients, and P is the time expressed in fractions of a day. All time is taken at UT. This is the time as seen at the Prime Meridian. For 2010, the Lunar polynomials are found in reference 1. An example of these polynomial coefficients is shown in table 1. The coefficients shown are for May 22, 2010 at 0h. *RA* and *Dec* information for stars was obtained from reference 2.

Table 1: Lunar Polynomials for May 22 2010 at 0h

	a_0	a_1	a_2	a_3	a_4	a_5
RA	163.6807567	12.8203458	-0.0708784	0.0423153	0.0000957	-0.0003299
Dec	2.0339217	-5.9703821	-0.0395830	0.0521861	-0.0003348	0.0000469
HP	0.9866246	-0.0021989	-0.0006635	-0.0000047	-0.0000017	

Because of Luna's proximity, there will be an error associated with both observed coordinates. One error term comes from that time is measured as constant throughout each time zone even though the longitude changes as the observer moves east to west. Therefore the actual time the observer sees is not the measured time. As there are 24 time zones and 24 hours in a day, each time zone measures about one hour across. This is an approximation that only counts as an initial guess and correction factor later.

Latitude error is caused by a change in viewing angle from the equatorial. As the observer moves above the equator, Luna will appear more southerly in the sky. Likewise the opposite is true. Viewing from more below the equator will cause Luna to appear more northerly. Both of these error functions can be combined to determine the location of the observer.

Data will be acquired using a digital camera. Several cameras may be used to compare the quality.

III. Analysis

Data was collected using two digital cameras. The camera used in the first set of pictures captured a very narrow area of the sky. Figure 1 shows a 30 second exposure using this camera. It is well focused, but collected many stars which were not visible to the observer's eye. For this reason, these pictures cannot yet be used to determine observer position. Further research in underway to identify the dim lights exposed. This picture was taken on April 23, 2010.

Figure 1: A 30 second exposure taken at (time). The picture shows the Moon and several celestial objects. The picture was taken in Atascadero, CA just outside of San Luis Obispo along Permufo Canyon Road. Some objects are believed to have brightness ratings as dim as 14.8.

Due to the lack of knowledge presented in the previous set of pictures, more pictures were taken using a different camera. This camera used a much wider capture angle and was able to span over 35º across its long axis. Figure 2 is a 10 second exposure using this camera. Notice that there are fewer stars seen in this picture. This camera was less proficient at capturing dim objects, but the large capture angle provided much more convenient results. Spica and Denebola may be clearly identified in the picture. They have been circled and labeled. There are other potential objects in the picture which could be identified and used, but these two are the brightest after Luna and will suffice for analysis.

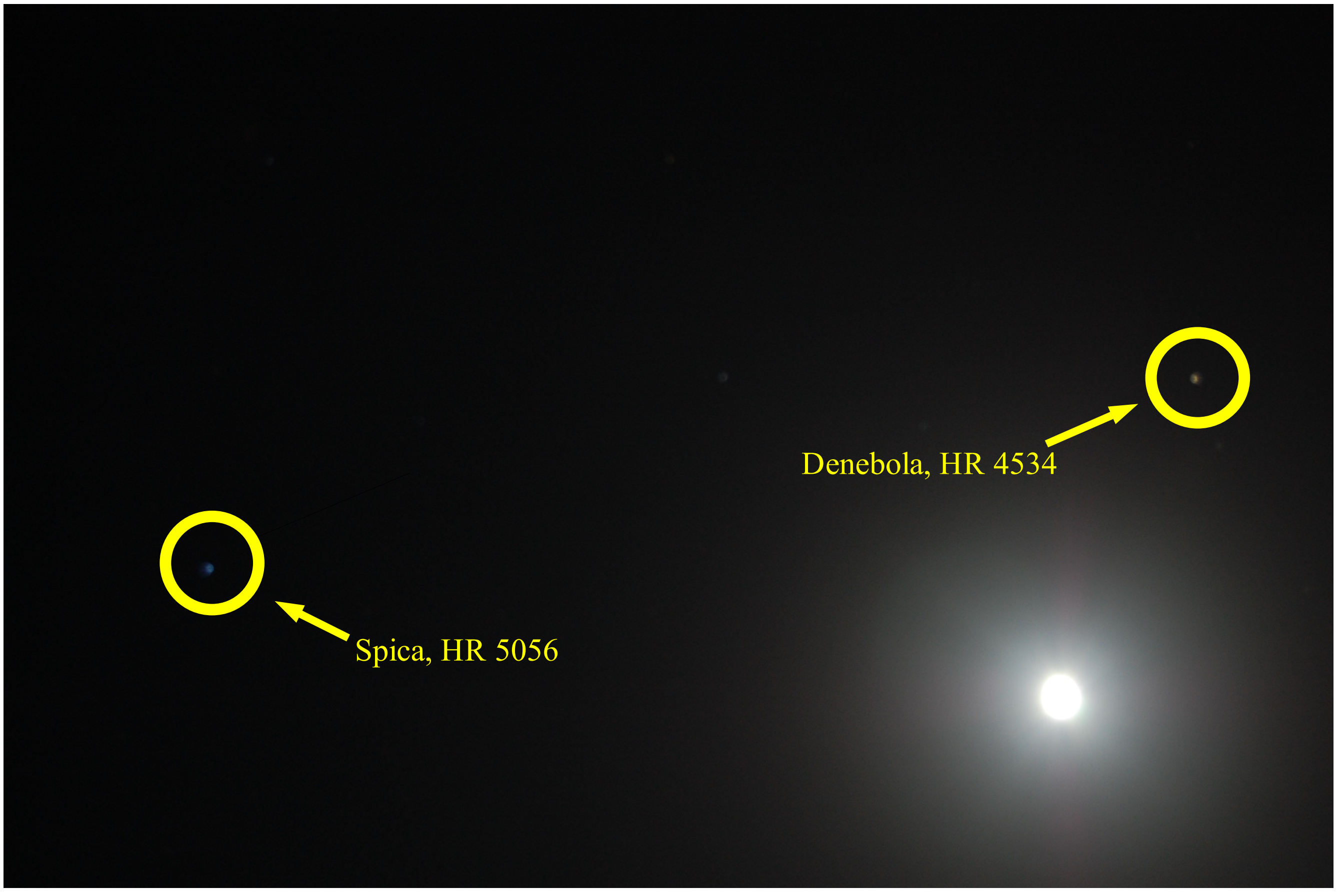

Figure 2: A 10 second exposure taken at 22:46:00 local time on May 22, 2010. The Picture was taken on Tank Farm Road in San Luis Obispo. Spica and Denebola are clearly visible.

The first step in the analysis was to load the picture into Matlab. Using the image toolbox, the areas containing each star was identified. This method highlighted the region of interest and blacked out the rest. The center pixel for the star was then identified by searching the region for the brightest spot. Since there is some drift over time as the Earth rotates, the star will not remain in a fixed location on the image. It will drift, but over short periods of time, the drift will cause one or a few pixels to intensify. These intensified pixels represent the star's location half way between the shutter opening and closing. Identifying the center of Luna required a similar process. For Luna, there was no single brightest pixel. Over the duration of capture, many pixels reached maximum intensity. These pixels' locations were then averaged to find their central pixel. This method may require more fine tuning, but for now it results in only tenths of a degree offset in calculated observer position.

All three objects lie in two coordinate systems within the picture. There is one set by the image which is measured in pixels. There is another which is set by *RA* and *Dec* measurements. *RA* and *Dec* measurements are important because they provide a standard that can be used throughout the world. The other is only useful to the observer at the time and place of image capture. Therefore it is necessary to convert the pixel space image into *RA-Dec* space. This can only be accomplished if there are at least two known objects in the picture. Figure 2 has two objects: Spica and Denebola. This is not necessarily enough, but it is assumed that there minimal no warping within the picture from environmental effects. The atmosphere can distort an image, but this effect is proportional to the distance above the horizon. Viewing closer to the horizon means looking through more atmosphere. Atmosphere can bend light. This causes distortion. The assumption was made that this effect would cause minimal distortion in the analysis because the conversion is based on relative data withing the picture and there is little difference in the distance off the horizon between all three objects.

To transform coordinate systems, a transformation matrix, Q, is required. Transformation matrices take a vector in one coordinate system and express it in terms of another. They do this by rotating each component into an expression in the second system.

$$\begin{bmatrix} RA \\ Dec \end{bmatrix} = Q \begin{bmatrix} p_x \\ p_y \end{bmatrix} \tag{2}$$

In this case, Q rotates about a z-axis and can be expressed in terms of its x- and y-components.

$$Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{3}$$

where θ is the rotation factor between the two coordinate frames. θ is found by taking the inverse cosine of the dot product of the unit vector connecting the two stars. Any known vector within the picture can be used, but in this particular picture, this is only vector that is well defined.

There is also a scaling factor applied in the x-direction (first coordinate) in pixel-space. This is because the picture was take at an angle relative to the horizon. This factor is related to the angle above the horizon. The scaling factor is found by taking the cosine of this angle.

After defining the rotation into the *RA-Dec* frame from the pixel frame, a scaling factor is applied. The scaling factor comes from dividing the angle between the stars by the number of pixels connecting the stars. This gives a value for the change in angle per pixel in the picture.

Once *RA-Dec* frame is defined, the Moon is located within it. Each pixel corresponds to a fractional step in the *RA* and *Dec* directions. By counting the pixels from one star to the Moon, its *RA* and *Dec* are found. These are the measured RA and Dec. They must be compared to the calculated values in order to generate an error function and find latitude and longitude information.

Longitude is found by taking the time difference between data acquisition and a time determined by iterating the error function. Longitude lines run in 360º around the equator of the Earth. This means that as the Earth rotates, different longitude lines are exposed to the Moon. Time zones are distributed almost evenly across the equator to reflect hour lines. There are 24 hour lines which means that each time zone contains a constant time one hour separate from its neighbors on either side. Longitude is a continuum and therefore will display errors in the observed position of the Moon when compared to the timezone time. This error has a direct correlation to the observer's longitude. By iterating the error function to minimize the error in calculated and observed *RA* and *Dec*, a change in time is found. This is then added to the observer's recorded time and converted into degrees. This value is the observer's longitude.

Latitude makes up the rest of the error. Unfortunately, Latitude only complicates matters. Latitude and Longitude cannot be identified separately in this way. Therefore, a new method was pursued to analyze the coupled motion between latitude and longitude in order to account for the final source of error.

The previous model only received information on longitude and this would only work for small errors due to the time sensitive nature of the Moon's passage. A method involving Lunar Parallax was then investigated. Lunar Parallax is the offset angle which can be observed in the Moon's position. This is based on the distance to the Moon and the radius of Earth. At any moment, there is a point on the surface of the Earth with no error. This point lies directly between the Earth and the Moon. Any error in observation measurement is due to a displacement of the observer away from this point.

The Moon sees a Parallax angle of roughly one degree. This means that anyone viewing the Moon while it is directly on their horizon would observe almost a one degree difference than someone viewing the Moon from directly between the Moon and the Earth. Smaller errors reflect smaller displacement from this center line. The change in angle is directly related to the observer's radius from the centerline. Therefore, a ratio can be made between the maximum Lunar Parallax and the observed offset. By comparing that to the Earth's radius, a circle can be drawn from the zero-error centerline containing the observer.

Parallax can be calculated with the following equation.

$$p(arcsec)=\frac{1}{d(pc)} \quad (4)$$

where converting from arseconds and parsecs will yield an answer in the desired units. Also note that parallax is usually used in relation to observations on stars. This is why it uses parsecs. This also means that it is necessary to convert the base leg of the represented triangle to Earth radii instead of Earth orbital radii. To do so, divide the distance by 1AU and multiply by the radius of the Earth.

The final challenge is to identify the location of the observer on the circle drawn by parallax. A coordinate frame was made to represent longitude and latitude on the surface of the Earth. Another coordinate frame was made which would define Right Ascension and Declination in their vector space by means of the Earth-Centered Inertial frame. March 14, 2010 was the vernal equinox for the year 2010. The line drawn through the center of the Earth to the Sun at noon at zero longitude defines the first vector in this space. The second was defined as the vector normal to the ecliptic plane. The third completed the coordinate system.

The time since the vernal equinox and, the sidereal rotation rate of the Earth, and the Earth's rotational inclination define the transformation between the Earth-Centered Inertial, ECI, frame and the longitude-latitude frame. There are two rotations to transition between these two frames. The first comes from the Earth's tilt relative to its orbital plane. This is its inclination and on the vernal equinox, this tilt manifests as a rotation about the first vector in the ECI frame. The second rotation is from the spin of the Earth. Longitude rotates with the Earth so by calculating the number of rotations since noon on the vernal equinox, the final angle may be determined. This rotation occurs about the new second axis. These two rotations serve to define to relationship between the ECI and longitude-latitude frames.

The observed error in the Moon's position within the RA-Dec frame would result in the exact opposite motion of the observer relative to the zero-error centerline. This motion can be mapped as an angular motion or rotation from a given direction off the centerline onto the parallax circle found previously. Once this point has been located, the inverse transformation matrix may be used in order to convert the point into the rotating Earth frame which holds direct correlation to longitude and latitude.

IV. Results

As time progressed, the moon could clearly be seen moving across the sky. The motion was slight, but was noticeable over extended time. The most important factor was shown to be valid. The Moon moves throughout the sky in such a way that it can be used to determine the location of an observer. The method used was able to locate Longitude to within 3º of the measured value. This is greater than can be achieved using a GPS, but this system does not require the use of satellites. More development can be made to account for atmosphere and other conditions' distortion of the image.

Luna's observed position showed the observer to be at 117.7º which matched the GPS obtained by satellite to within 3º, but no information was found in regards to latitude. The second method then picked up from this point and located the observer to 36.6ºN latitude, but lost accuracy in longitude calculating 128ºW longitude. This makes identifying the center of the Moon difficult. Sources of error come from the glare seen on the camera lens. Especially given that the Moon appears full even though it was 70% full. Another source of error was the possible distortion of the image from atmospheric effects. This can be mitigated when more stars are captured and identified within the picture. This effect became more prominent with the second and final method as small scaling factors were necessary to properly define all coordinates within the picture.

V. Conclusion

The concept to determine longitude and latitude was successfully demonstrated. More detailed analysis can be done to increase fidelity of the system. This system can be developed for use in a variety of applications where the use of GPS is unavailable or impractical. The next step is to develop this system for use on satellites which have less restrictions on viewing the Moon. One of the major difficulties in testing this system was interference from clouds and fog. A satellite would not have to work around weather. It would have a more consistent view of the Moon and stars.

While the location of the observer was inaccurate, the concept was able to demonstrate possibility. A broader range of data would be necessary to further develop this concept. The most important data to collect would be pictures from multiple distant locations taken at the similar times. These pictures could then be more readily compared to find parallax information.

Once it demonstrates more accurate capability, a further step is to apply this system outside the Earth-Moon system. By replacing the Moon with the Sun, interplanetary travel can be performed with this system. More detailed development would be required to attain sufficient pointing knowledge, but the system could update its own guidance systems at any time without cumulative degradation such as is common with gyros. An interplanetary mission with this system could autonomously update its own guidance system.

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