Vertical restraints and horizontal control

Robert Innes*

and

Stephen F. Hamilton**

This article considers vertical restraints in a setting in which duopoly retailers each sell more than one manufactured good. Vertical restraints by a dominant manufacturer enable the firm to acquire horizontal control over a competitively supplied retail good. The equilibrium contracts produce symptoms that are consistent with a variety of observed retail practices, including slotting fees paid to retailers by competitive suppliers, loss leadership, and predatory accommodation with below-cost manufacturer pricing for the dominant brand(s). Applications are developed for supermarket retailing, where the manufacturer of a national brand seeks to control the retail pricing of a supermarket's private label, and for convenience stores, where a gasoline provider seeks to control the retail pricing of an in-store composite consumption good.

1. Introduction

■ Vertical restraints by manufacturers on the retailers of their products continue to be a source of legal and policy debate. Indeed, in June of 2007, the U.S. Supreme Court reversed course on the legal treatment of resale price maintenance (RPM), overturning its earlier decision on the per se illegality of the practice in favor of a reasoned approach.[1] This ruling is in line with the traditional explanation for vertical restraints that the practice serves to align private incentives between manufacturers and retailers in the sale of manufactured goods. Absent restraint, intensive price competition among retailers can lead to an inadequate level of pre-sale retail services (Telser, 1960; Mathewson and Winter, 1984; Marvel and McCafferty, 1984; Rey and Tirole, 1986; Klein and Murphy, 1988; Winter, 1993) or promote excessive post-sale quality differentiation (Bolton and Bonanno, 1988). Vertical restraints can correct these distortions. Doing so generally produces efficiency benefits, a point that has been argued by many economists following Bork (1966) and Posner (1976).[2]

*University of Arizona

**Cal Poly San Luis Obispo

We thank two anonymous referees for very helpful comments and suggestions. We also thank Steve Cassou, Jason Lepore, Bruce Marion, Jeff Perloff, Tim Richards, and seminar participants at the University of Toulouse for helpful comments and discussions. We thank the Food System Research Group at the University of Wisconsin for financial support.

[1] See *Dr. Miles Medical Co. v. John D. Park and Sons*, 220 U.S. 373 (1911) and *Leegin Creative Leather Products, Inc. v. PSKS, Inc.* Slip Op. no. 06–480 (decided June 28, 2007).

[2] There are two main counterpoints to the pro-competitive view. Under uncertainty, conflicts can arise when a manufacturer must balance private incentives in supply with the need to provide insurance to retailers (Rey and Tirole,

This article considers vertical restraints in a multi-product retail environment. In this setting, we identify more pernicious effects: a vertical restraint on a manufacturer's own good serves as a mechanism to control the retail pricing of another ("rival") manufactured good.

We consider a successive oligopoly market structure with two manufacturing industries, two retailers, and two goods. In the upstream market, one good is produced by a monopolist (or differentiated-product duopolists) and the second good is produced by a competitive fringe. In the downstream market, the two goods are bundled together in the sense that each retailer carries both goods and each consumer buys both goods from one of the two retailers. Examples of such a vertical structure include supermarkets that carry both a national brand and a store brand (private label), convenience stores that sell gasoline and in-store consumption goods, and computer retailers that bundle essential components (such as processors and operating systems) with a set of commoditized components (such as DRAM, hard drives, and flat-panel displays).

Our analysis builds on several recent papers. Winter (1993) considers a single manufacturer that imposes vertical restraints on duopoly retailers to elicit the optimal mix of priced and non-priced retail services. Absent contracts, the retailers compete excessively in prices and fail to provide a sufficient level of service; RPM combined with a wholesale price elevated above marginal cost simultaneously corrects both distortions. Rey and Vergé (2004) consider how RPM can be used by duopoly manufacturers to control the retail pricing of duopoly retailers. The manufacturers' use of vertical restraints frees their wholesale prices to be set at marginal cost, and this circumvents retail-manufacturer contract externalities that would otherwise result in disadvantageous price competition.[3]

In the present setting, vertical restraints likewise serve to resolve retail market externalities; however, the essential difference is our focus on the joint pricing decisions of *multi-product* retailers who sell both a dominant manufacturer's good and a product produced by a competitive fringe. In this regard, our arguments are related to the substantial literature on the extension of monopoly power to other products through the use of tying arrangements (e.g., Whinston, 1990; Carbajo, de Meza, and Seidmann, 1990; Shaffer, 1991b) or commodity bundling (e.g., Nalebuff, 2004; Mathewson and Winter, 1997; DeGraba and Mohammed, 1999; McAfee, McMillan, and Whinston, 1989). Innes and Hamilton (2006), for example, show how a dominant firm can use explicit cross-market controls on retailers' contracts to achieve an integrated multi-good monopoly outcome; the monopoly manufacturer dictates that suppliers of the rival good pay lump-sum transfers that serve to elevate the good's wholesale price and thereby prompt retailers to charge monopoly prices. In practice, such explicit cross-market controls are likely to be infeasible, whether due to proscriptions of antitrust law or due to an inherent inability of the monopolist to observe and verify retailers' cross-market conduct. Here we focus on how a vertical restraint imposed on a manufacturer's *own product* can be used to extract rents from the market for another product, without stipulating any cross-market tying, bundling, pricing, or contract terms.

Vertical contracts that extract cross-market rents produce several notable symptoms. Irrespective of the relationship between the products in utility (complements, substitutes, or independent goods), vertical restraints on the monopolist's product induce retailers to sign contracts with suppliers in the competitive fringe that involve fixed fees paid to the retailer ("slotting allowances"). Slotting allowances are prevalent in practice (Federal Trade Commission, 2003), and highly controversial. We show that their effects are often anti-competitive in a multi-product context.

For weakly substitutable, independent, or weakly complementary goods, vertical restraints result in negative retail margins on the dominant manufactured good. Hence, our analysis offers a

1986). Under oligopoly, vertical restraints can be used by retailers in their contracts with manufacturers to dampen downstream competition (Shaffer, 1991a). For an excellent review, see Mathewson and Winter (1998).

[3] See also Marx and Shaffer (2004) and Rey, Thal, and Vergé (2006), who study the structure of vertical contracts between a single manufacturer and differentiated duopoly retailers of the manufacturer's product.

new explanation for loss-leader retail pricing that does not rely on coordination failures (Bagwell and Ramey, 1994), imperfect information (Lal and Matutes, 1994), heterogeneous consumers (DeGraba, 2006), or product complementarities in multi-product monopoly markets (Bliss, 1988). For strongly substitutable goods, the dominant manufacturer selects a wholesale price below marginal cost, producing a type of "predatory accommodation" similar to that derived by Marx and Shaffer (1999), albeit for different reasons.

In Marx and Shaffer (1999), below-cost wholesale prices arise because a monopoly retailer can thereby extract rents from competing suppliers. In our setting, a dominant manufacturer distorts her wholesale price from marginal cost to counter the retailers' incentive to discount the rival retail good. For independent goods, a dominant manufacturer sets a wholesale price above the (maintained) retail price to decrease retailers' per-customer rents. Retailer losses on the dominant good, in turn, temper interretailer incentives to attract customers from rivals by discounting the price of the competitively supplied good. For substitute goods, raising the wholesale price above marginal cost has two effects on the retailers' pricing incentives. Narrowing the retailers' margin on the dominant good under the restraint decreases the return to attracting customers (favoring a higher retail price for the rival good), but also reduces the opportunity cost of shifting consumption from the dominant good to the competitively supplied substitute good (favoring a lower retail price for the rival good). With strong substitutes, the second effect dominates and a below-cost wholesale price by the dominant manufacturer is needed to prompt the retailers to raise the retail price of the rival good.

The remainder of the article is organized as follows. In Sections 2–5, we frame our analysis around duopoly retailers who each sell a monopoly-produced good and a second, competitively supplied good. Section 2 develops the model and Section 3 compares the collective optimum that maximizes joint manufacturer-retailer profits to the outcome without vertical restraints. In Sections 4 and 5, we consider how vertical restraints can reconcile these outcomes in an environment without and with retailer-fringe contracts, respectively. We develop applications to gasoline stations/convenience stores and supermarket retailing, and derive some implications for antitrust policy.

In Section 6, we consider retail formats without product bundling and extend our observations to oligopoly settings in which multiple manufacturers impose vertical restraints on retailers. Under oligopoly, each manufacturer ignores the profits of rival manufacturers when selecting a target retail price and, as a result, incentives for predatory accommodation are particularly harmful. Indeed, in the case of highly substitutable goods, vertical restraints lead to higher equilibrium retail prices than those which would emerge in a multi-product monopoly market. The reason is that each manufacturer selects a below-cost wholesale price to control the retail pricing of the fringe good, and this motivates her rival to solicit higher retail prices (both for her own product and for the fringe good) to capitalize on the retailers' ability to acquire artificially high rents on sales of the rival manufacturer's good.

2. The model

■ Consider a "$2 \times 2 \times 2$" successive oligopoly structure with two goods, two manufacturers, and two retailers. Good 1 is a "name brand" produced by a monopoly manufacturer and good 2 is a "generic brand" supplied to the retailers by a competitive fringe. Production of each good involves constant unit cost, denoted c^1 and c^2, and for simplicity, retail costs are suppressed. The retailers engage in pure intermediation, procuring goods from manufacturers at wholesale prices w^1 and w^2 in the upstream market and selling goods to consumers in the downstream market.

Due to economies of "one-stop" shopping that pervade retail environments (Bliss, 1988), each consumer purchases a consumption bundle (y^1, y^2) from a single retailer. Given her choice of retailer $j \in \{1,2\}$ and consumption bundle, a consumer obtains the utility

$$u(y^1, y^2) - \sum_{i=1,2} p^i_j y^i, \tag{1}$$

where y^i is the quantity of good i purchased, and p^i_j is the price of good i at retail location j.

We assume $u(.)$ is increasing and concave with bounded first derivatives and that own product effects dominate cross-effects, $| \mathrm{dln}u_i \,/\, \mathrm{dln}y^i | > | \mathrm{dln}u_i \,/\, \mathrm{dln}y^j |$ for $j \neq i$, where $u_i \equiv \partial u(.)/\partial y^i$. The products can be substitutes, $u_{12} \equiv \partial^2 u(.)/\partial y^1 \partial y^2 < 0$, complements $u_{12} > 0$, or independent goods, $u_{12} = 0$. The optimal consumption choice at retailer j yields the indirect utility

$$u^*_j \equiv u^*\left(p^1_j, p^2_j\right) = \max_{\{y^1, y^2\}} u(y^1, y^2) - \sum_{i=1,2} p^i_j y^i. \tag{2}$$

A consumer's choice between retailers is determined in a standard Hotelling framework, with the two retailers located at either end of the unit interval.[4] Consumers are uniformly distributed on this interval and incur preference (travel) cost of t per unit distance. The location $\theta \in [0,1]$ represents a consumer's distance from retailer 1, and $(1 - \theta)$ her distance from retailer 2. This, a θ -type consumer, obtains net utility $u^*_1 - t\theta$ if purchasing from retailer 1 and $u^*_2 - t(1 - \theta)$ if purchasing from retailer 2. Given retail prices for each good at each retailer, a consumer of type

$$\theta^* \left(u^*_1, u^*_2\right) = (1/2) + \left[\left(u^*_1 - u^*_2\right)/(2t)\right]$$

is indifferent between the retailers, and the market is partitioned into consumer types $\theta \leq \theta^*$ (u^*_1, u^*_2), who purchase both goods from retailer 1, and consumer types $\theta > \theta^*$ (u^*_1, u^*_2), who purchase both goods from retailer 2.

3. Collective optimum and no contract outcomes

■ In this section, we examine how the retail pricing outcome departs from the collective optimum in an environment without contracts. We then characterize the role of vertical restraints in aligning incentives between a dominant manufacturer and the retailers of her product. We assume throughout the article that retailers cannot implement exclusive territories that would split the consumers between them, void any competition for customers, and thereby lead trivially to multi-product monopoly prices.[5] For most examples in practice, exclusive territories are infeasible because consumers cannot be compelled to buy from a given retailer.

A vertically integrated monopolist solves:

$$\max_{p^1, p^2} \sum_{i=1,2} (p^i - c^i) y^i(p^1, p^2) \equiv \Pi(p^1, p^2; c^1, c^2) \Rightarrow \{p^{1*}, p^{2*}\}, \tag{3}$$

where $y^i(.) \equiv \mathrm{argmax}\, \{u(y^1, y^2) - \sum_{i=1,2} p^i y^i\}$.[6] The solution to (3) yields the maximum profit available in the market, $\Pi^* \equiv \Pi(p^{1*}, p^{2*}; c^1, c^2)$.

Next consider the choice problem of retailer 1.[7] Absent contracts, the dominant firm selects a wholesale price w^1 and the competitive fringe prices at cost, $w^2 = c^2$. Given these wholesale

[4] We suppress retailers' choices of location because these choices are long run in nature, and are therefore likely to precede the contractual decisions of interest in this article. To a large extent, these decisions are based on considerations outside of our model, such as rents and the size and location of consumer markets.

[5] We therefore also assume that all consumers are served in this retail market, with t not so large as to foreclose the midpoint $(\theta = 1/2)$ consumer.

[6] We assume $\Pi()$ is concave for a relevant range of (p^1, p^2). This holds when $u()$ is concave with third-order derivatives that are sufficiently small relative to second-order derivatives.

[7] Choices of retailer 2 are symmetric and thus omitted.

prices, the duopoly retailers compete in retail prices. Retailer 1 solves

$$\max_{p_1^1,p_1^2} \pi_1\left(p_1^1, p_1^2; \bar{u}_2, w^1, w^2\right) \equiv \sum_{i=1,2} \left(p_1^i - w^i\right) y^i\left(p_1^1, p_1^2\right)\phi\left(p_1^1, p_1^2; \bar{u}_2\right)$$

$$= \phi\left(p_1^1, p_1^2; \bar{u}_2\right)\left[\Pi\left(p_1^1, p_1^2; c^1, c^2\right) - \sum_{i=1,2}\left(w^i - c^i\right) y^i\left(p_1^1, p_1^2\right)\right], \tag{4}$$

where $\Pi(.)$ is defined in (3). Normalizing the number of consumers to one, $\phi(p_1^1, p_1^2; \bar{u}_2) = \theta^*(u^*(p_1^1, p_1^2), \bar{u}_2)$ is the market demand for retailer 1, given the prices set by retailer 2 and the attendant utility level $\bar{u}_2$. The first-order necessary conditions for a solution to (4) are

$$\frac{\partial \pi_1}{\partial p_1^1} = \phi\left(\frac{\partial \Pi}{\partial p_1^1}\right) + \Pi\left(\frac{\partial \phi}{\partial p_1^1}\right) - \sum_{i=1,2}\left(w^i - c^i\right)\left[\phi\left(\frac{\partial y^i}{\partial p_1^1}\right) + y^i\left(\frac{\partial \phi}{\partial p_1^1}\right)\right] = 0, \tag{5}$$

$$\frac{\partial \pi_1}{\partial p_1^2} = \phi\left(\frac{\partial \Pi}{\partial p_1^2}\right) + \Pi\left(\frac{\partial \phi}{\partial p_1^2}\right) - \sum_{i=1,2}\left(w^i - c^i\right)\left[\phi\left(\frac{\partial y^i}{\partial p_1^2}\right) + y^i\left(\frac{\partial \phi}{\partial p_1^2}\right)\right] = 0, \tag{6}$$

where, using Roy's identity,

$$\partial \phi / \partial p_1^i = \left(\partial u^* / \partial p_1^i\right)/2t = -y^i\left(p_1^1, p_1^2\right)/2t < 0. \tag{7}$$

Notice that the collective optimum (p^{1*}, p^{2*}) is achieved when the first term in each of equations (5) and (6) is equal to zero. Hence, the individual incentives of a retailer are compatible with the collective interest only when the remaining terms sum to zero. These terms correspond to two distortions. First, on the *inter*retailer margin, higher prices by retailer 1 prompt consumers to switch to the rival retailer (the business-stealing effect). This loss of store traffic is costly to the retailer, but of no concern to the vertically integrated chain. The business-stealing effect provides the retailer with an incentive to set each retail price below the level which maximizes joint profits. Second is an effect on the *intra*retailer margin. To the extent that the retailer pays above-cost wholesale prices to its suppliers ($w^i > c^i$), retail price effects on demand have a smaller impact on retailer profit than on the profit of the vertically integrated chain, which faces true cost c^i. This "double-marginalization" effect generally induces the retailer to set prices above the level which maximizes joint profits.

Now, following Winter (1993) (and recalling that $w^2 = c^2$), suppose that the wholesale price of good 1 is selected so that the business-stealing and double-marginalization effects exactly offset *for the good 1 retail price*,

$$w^1 - c^1 = \frac{\Pi(.)\left(\partial \phi / \partial p_1^1\right)}{\left(\phi\left(\partial y^1 / \partial p_1^1\right) + y^1\left(\partial \phi / \partial p_1^1\right)\right)} > 0. \tag{8}$$

With the wholesale prices set as in (8), the last terms in (5) vanish. Nevertheless, the retailers compete for customers by jointly selecting prices for *both* goods. In general, the wholesale price in (8) does not also elicit the collectively optimal price of the fringe good 2 in (6). Formally, with w^1 set as in (8), evaluating equation (6) when p^2 equals its integrated optimum, p^{2*}, yields

$$\left.\frac{\partial \pi_1(p^{1*}, p^{2*}; \bar{u}_2, w^1, c^2)}{\partial p_1^2}\right|_{\text{eq.(8)}} = \frac{\phi \Pi^*\left[\left(\partial \phi / \partial p_1^2\right)\left(\partial y^1 / \partial p_1^1\right) - \left(\partial \phi / \partial p_1^1\right)\left(\partial y^1 / \partial p_1^2\right)\right]}{\phi\left(\partial y^1 / \partial p_1^1\right) + y^1\left(\partial \phi / \partial p_1^1\right)} < 0.^{8} \tag{9}$$

The inequality in (9) implies that the retailer discounts the price of good 2 below p^{2*} in order to attract customers. Hence, absent contracts, a monopoly manufacturer cannot set her wholesale price to induce her retailers to set collectively optimal retail prices for *both* goods, (p^{1*}, p^{2*}).

[8] With $\partial y^1/\partial p_1^1 < 0$, $\partial \phi/\partial p_1^i < 0$, $\Pi^* > 0$, and $\phi > 0$, the inequality holds by inspection for independent goods and substitutes ($\partial y^1/\partial p_1^2 \geq 0$). For complementary goods ($\partial y^1/\partial p_1^2 < 0$), the inequality follows from our assumption, | $d\ln u_i$ / $d\ln y^i$ | > | $d\ln u_i$ / $d\ln y^j$ | for $j \neq i$.

4. Monopoly-retailer contracts

■ **Analysis.** We now characterize contracts between the monopoly manufacturer and her retailers that elicit collectively optimal retail prices. This task would be trivial if contracts could stipulate the retail price for the fringe product ($p^2 = p^{2*}$) and punish any defections from this price. But such cross-product restraints would run afoul of prevailing antitrust law in most industrial countries, for instance violating both the "tying doctrine" and proscriptions against price fixing in U.S. case law irrespective of whether a "rule of reason" is applied.[9] Another direct cross-market control is a contract that requires retailers to levy a "slotting fee" on the rival manufactured good. Such a contract, recently considered by Innes and Hamilton (2006), would be difficult to enforce due to the dominant firm's inability to observe and verify retail contracts with fringe suppliers. We rule out such direct cross-product restraints as being either overtly anti-competitive or unenforceable,[10] and consider monopoly-retailer contracts that instead have only three terms: resale price maintenance (RPM) for the dominant manufacturer's good (requiring $p^1 = p^{1R} = p^{1*}$), a wholesale price (w^1), and a fixed tariff (f^1) to redistribute rents.[11]

We assume that these contract terms are determined by bargaining (see, for example, Macleod and Malcomson, 1995). Because the issue of interest here is the contract form that achieves the collective optimum, we do not describe the precise structure of the bargaining game. Instead, we assume that the game has a unique subgame-perfect bargaining equilibrium that splits collective gains from contract implementation according to a known rule (as in Rubinstein, 1982; Shaked, 1987; and others).

We also assume, for now, that the retailers are unable to sign contracts with good 2 manufacturers. For instance, each retailer may be vertically integrated with a good 2 manufacturer, as in the case of a supermarket in-store bakery. If the monopoly manufacturer imposes a vertical restraint on her retail price of $p^{1R} = p^{1*}$, then her retailers no longer optimize over the good 1 price, and the integrated optimum can be attained if a wholesale price, w^1, can be found to induce the duopoly retailers to select $p^2 = p^{2*}$ per equation (6). By inspection (and with $w^2 = c^2$ by construction), the wholesale price that achieves this integrated optimum is

[9] Although cross-market price controls differ from tied product sales, they represent a cross-market tie that transparently fixes prices. Prevailing case law in the United States proscribes such conduct and this is likely why, in practice, firms may use less transparent means to exercise cross-market control. For example, in the Northern Pacific Railway decision (*Northern Pac. Ry. v. United States*, 356 U.S. 1, 1958), the Supreme Court affirmed that "among the practices which the courts have heretofore deemed unlawful in and of themselves are price fixing . . . and tying arrangements." Under the rule of reason (as endorsed by Justice Sandra Day O'Connor), a cross-market tying restriction is judged to run afoul of antitrust law if the restriction has anti-competitive effects in the tied market, as is the case with a cross-market restraint that supports an elevated market price for the rival good.

[10] In doing so, we abstract from the potential ability of retailers to make *implicit* cross-market commitments in a repeated game. Three observations are relevant on this subject. First, for the explicit contracts (on verifiable outcomes) that we study, it is not difficult to construct bonding provisions that make the contracts self-enforcing. With implicit commitments, in contrast, any bonds would themselves become the object of dispute in the event of "commitment breach," arguably voiding their use as punishment. Even in a repeated relationship with perfect information, it is not clear how or whether a retailer's self-interested departure from an implicit cross-market pricing commitment can be deterred by a rational (subgame-perfect) punishment. Second, assuming rational punishments can render implicit commitments incentive compatible, such commitments would presumably be tougher to enforce than the simple vertical contracts that we characterize here. Hence, because the contracts we consider support the collectively optimal outcome, the possibility of implicit commitments does not void the motives we identify for vertical restraints. Third, a word of caution. For legal restrictions on vertical contracts, such as antitrust policies that ban RPM (see below in Section 4), the potential for implicit commitments is crucial. If such commitments are possible, but not verifiable, then antitrust policy governing vertical restraints would be completely ineffectual.

[11] With these simple contracts, we will show that the monopolist can achieve the collective optimum. Hence, there is no loss in generality from restricting contracts to this form. Equivalently, the vertical restraint could involve a good 1 quantity provision (e.g., at the level $y^1 = y^1(p^{1*}, p^{2*})/2$ in the symmetric two-retailer case) in place of RPM (see Reiffen, 1999). For more on the equivalence between various forms of vertical restraints in a deterministic setting, see Mathewson and Winter (1984). For simplicity, we assume that contracts are observable to both retailers. For complications from unobservable contracts, see Crémer and Riordan (1987) and O'Brien and Shaffer (1992).

$$w^1 - c^1 = \frac{\Pi(.)\left(\partial\phi/\partial p_1^2\right)}{\left(\phi\left(\partial y^1/\partial p_1^2\right) + y^1\left(\partial\phi/\partial p_1^2\right)\right)}. \tag{10}$$

In a symmetric retail market equilibrium (where $\phi = 1/2$), equation (10) reduces to

$$w^1 - c^1 = \Pi^* y^{2*}/\delta, \tag{11}$$

where $y^{i*} = y^i(p^{1*}, p^{2*})$, $i = 1, 2$, is the equilibrium quantity of brand i sold by each retailer in the collective optimum; $\Pi^* = \Pi(p^{1*}, p^{2*}; c^1, c^2)$ is the maximum profit level that solves problem (3); and $\delta \equiv y^{1*}y^{2*} - t(\partial y^{1*}/\partial p^2)$. Notice that δ is positive in the case of independent and complementary goods ($\partial y^{1*}/\partial p^2 \leq 0$), but can be negative for highly substitutable goods.

Definition: Substitute goods 1 and 2 are *weak substitutes* when $\delta = y^{1*}y^{2*} - t(\partial y^{1*}/\partial p^2) > 0$ and *strong substitutes* when $\delta < 0$.

In the case of independent goods, $\partial y^1/\partial p^2 = 0$, notice that the wholesale price in (11) is selected so that $(w^1 - c^1)\, y^{1*} = \Pi^*$. This is an intuitive result. Because the wholesale price of the fringe product is set at marginal cost, $w^2 = c^2$, the retailers depart from the collective optimum due only to the business-stealing incentive. Each retailer wishes to discount the price of good 2 to attract customers from his rival, and this incentive is eliminated when the dominant firm selects her wholesale price to fully extract variable per-customer profit from her retailers. To do so, sales of product 1 are made below invoice ($w^1 > p^{1*}$)—a "loss leader" outcome—so that each retailer's loss on good 1 exactly offsets his gain on sales of good 2 at the integrated price p^{2*}.

When the retail goods are not independent, the wholesale price must also correct for the incentive of retailers to shift consumers between the two goods on the intraretailer margin. Here the sign of δ is crucial, reflecting a tension between two effects in the retailers' choice of p^2. To see this, consider the outcome for substitute goods when the dominant manufacturer raises w^1 above c^1. As w^1 rises, the resulting reduction in per-customer rents under the restraint $p^{1R} = p^{1*}$ dampens retailers' business-stealing incentives, and this favors a higher p^2. But a smaller retail margin on good 1 also reduces the opportunity cost of siphoning off sales of good 1 by offering a price discount on good 2, and this "siphoning effect" favors a lower p^2.

In the case of *strong substitutes*, the siphoning effect dominates the business-stealing effect, and the contract must combine a vertical restraint with a *lower* w^1 (below c^1) to induce retailers to raise the price of good 2. Accordingly, with $w^2 = c^2$ and $w^1 < c^1$, per-customer retail profit is positive, $\Pi = \Pi(p^{1*}, p^{2*}; w^1, c^2) > 0$. In the case of *weak substitutes*, the business-stealing effect dominates and the monopolist raises w^1 above c^1 to stimulate her retailers to raise p^2. With w^1 raised above c^1, a higher p^2 provides smaller rents on the intraretailer margin from switching consumption to good 1 than it does for the integrated firm. Consequently, as long as the retailers have an incentive to steal customers, they select $p^2 < p^{2*}$. Only if retailers instead have incentives to rid themselves of customers (i.e., $\Pi < 0$) can integrated pricing incentives be restored.[12]

Note that the sign and magnitude of δ depends upon both the strength of the cross-price effect on demand, $\partial y^1/\partial p^2$, and on retailer differentiation (as measured by the travel cost t). When t is high, implying little interretailer rivalry, the goods are *strong* substitutes ($\delta < 0$). Conversely, when t is low, the goods are *weak* substitutes ($\delta > 0$). Put differently, t acts as an implicit weight on the relative strength of the siphoning (versus business-stealing) effect. In a perfectly contestable retail market ($t = 0$), the business-stealing effect is all that matters and profit per customer must be completely eliminated to attain monopoly pricing. As t rises, business-stealing incentives decline, reducing the retailers' incentive to discount the price of good 2. This lowers

[12] If the goods are complements, the "siphoning effect" also favors a higher p^2. Hence, setting w^1 above c^1 unambiguously prompts the retailer to elevate p^2, thereby countering the underpricing that otherwise results from business-stealing incentives. In the monopolist's optimum, w^1 is elevated just far enough to exactly offset the *positive* business-stealing incentives that result from *positive* per-customer profit ($\Pi > 0$).

TABLE 1 Summary of Contract Outcomes

Retail Goods Are:	Wholesale Price	Variable Retail Profit
Complements	$w^{1*} > c^1$	$\Pi > 0$
Independent	$w^{1*} > p^{1*} > c^1$	$\Pi = 0$
Weak substitutes	$w^{1*} > p^{1*} > c^1$	$\Pi < 0$
Strong substitutes	$w^{1*} < c^1$	$\Pi > 0$

the necessary deviation of wholesale price from cost to restore the integrated outcome.[13]

In the event that $\delta = 0$, equation (11) has no bounded solution. For this case, the business-stealing and siphoning effects exactly offset, so that changes in the wholesale price w^1 do not alter the retailers' pricing decision for good 2. For now, we assume that δ is bounded away from zero. We relax this assumption in Section 5 when we consider retailer contracts with fringe suppliers. In this setting, a dominant manufacturer can use RPM and two-part tariffs to acquire horizontal control over the rival good for all values of δ (including $\delta = 0$).

In summary, we have the following outcomes (see Table 1):

Proposition 1. Suppose each retailer is vertically integrated with a fringe manufacturer so that $w^2 = c^2$. Then the dominant firm can achieve the collective optimum ($p^1 = p^{1*}$, $p^2 = p^{2*}$) using a contract that imposes a vertical restraint ($p^{1R} = p^{1*}$) and sets: (i) $w^1 < c^1$ when the goods are strong substitutes ($\delta < 0$), and (ii) $w^1 > c^1$ when the goods are weak substitutes, complements, or independent ($\delta > 0$). With strong substitutes or complements, per-customer retail profit is positive in equilibrium ($\Pi > 0$); with independent goods, it is zero ($\Pi = 0$); and with weak substitutes, it is negative ($\Pi < 0$). Negative, zero, and small positive per-customer retail profit is achieved in each case with loss-leader pricing ($w^1 > p^1 = p^{1*}$).

□ **Loss-leader example: gasoline stations and convenience stores.** Consider the case of an oil company that distributes gasoline through dealer-operated stations that contain convenience stores ("quick stops"). The stations procure in-store consumption commodities from a set of competitive industries, which we assume can be aggregated into a composite consumption good. The in-store composite good is weakly related or independent from gasoline in consumption, and economies of multi-product purchases arise that favor the joint purchase of gasoline and consumer goods.[14]

Our model predicts RPM contracts for gasoline that exhibit "loss leadership" ($w^1 > p^1$). In practice, gasoline at convenience stores is widely recognized to be a loss leader, especially in urban areas (Bulow et al., 2001).[15] Indeed, a number of state statutes in the United States explicitly prohibit below-cost gasoline pricing in light of this possibility, for instance Tennessee's Petroleum Trade Practices Act and Florida's Motor Fuel Marketing Practices Act.

There are three types of arrangements for marketing of branded gasoline: (i) company-operated stations, (ii) lessee dealerships, and (iii) dealer-owned stations. The prevalence of these different arrangements varies by region. For example, on the U.S. West Coast, lessee-dealer sales represent almost 50% of the market (Meyer and Fischer, 2004). In some states, company-operated dealerships are prohibited by "divorcement" statutes.

[13] Formally, $d(w^1 - c^1)/dt\,|_{\text{eq.(11)}}\; q \overset{s}{=} \partial y^1/\partial p^2$, which is negative for complements (where $w^1 - c^1 > 0$) and positive for strong substitutes (where $w^1 - c^1 < 0$). For weak substitutes, ($w^1 - c^1$) rises with t. The reason is that, as t rises, the extent of underpricing falls, but the effectiveness of an increased wholesale price (w^1), in spurring a higher p^2, also falls. The second effect dominates, requiring a higher wholesale price deviation in order to fully correct underpricing.

[14] Per-customer gasoline demand is determined by considerations such as automobile size that are unlikely to be strongly related to a consumer's demand for convenience products.

[15] Note that urban gasoline stations have significant repeat custom and, hence (consistent with our model), customers with good information about store prices.

Our analysis is most relevant to lessee dealerships and arguably branded dealer-owned stations that also contract with the central marketing company. Lessee dealer contracts typically involve two types of fixed payments to the brand company, a fixed purchase price for the franchise and periodic lease payments. In addition, contracts stipulate the dealer tank wagon (DTW) wholesale price for gasoline purchases from the company, and involve minimum volume requirements (Meyer and Fischer, 2004). Minimum purchase requirements are also common in contracts with dealer-owned stations (www.state.hi.us/lrb/rpts95/petro/pet4.html).

In these contracts, the minimum volume requirements play the same role as the RPM modelled in this article when the resale price (p^1) is below the wholesale (DTW) price (w^1). In particular, the joint use of wholesale (DTW) pricing and minimum volume requirements can compel the station to make gasoline a loss leader and compensate with profitable convenience store sales, and this tempers the return to stealing business from other stations of the branded marketer by discounting the prices of in-store convenience items.

If gasoline and convenience goods are independent in consumption, our model predicts either no fixed transfers or fixed payments from the branded monopolist to lessee dealers. There are two reasons why observed fixed payments may go in the other direction. First, if the goods are weak complements, our model predicts loss leadership and positive per-customer profit that, if the oil company has the preponderance of bargaining power, would largely be rebated by dealers to the brand company. Second, with interbrand competition (e.g., Exxon versus Texaco), a given brand-name oligopolist would temper dealers' business-stealing incentives (to avoid intrabrand business stealing), but not eliminate them (to encourage interbrand business stealing). In this case, even with independent goods, positive per-customer retail profit would exist that would be rebated back to the brand company, with loss leading to reduce intrabrand business stealing.

□ **Minimum versus maximum RPM.** Although antitrust policy toward resale price restraints is fluid and varies across jurisdictions, the presumed anti-competitive effects that often determine the legality of these practices are judged by whether they raise prices or not (see Comanor and Rey, 1997). Under such a criterion, *minimum* resale prices are illegal in the European Union (because they can only serve to raise prices), but *maximum* resale prices are not. In the United States, *minimum* RPM was per se illegal prior to June 2007, whereas *maximum* RPM has been judged by the less restrictive *rule of reason* since 1997.[16] In our model, equilibrium resale price restraints can take either form.

Corollary 1. The optimal contract of Proposition 1 requires *minimum* resale prices ($p^1 \geq p^{1*}$) in the case of strong substitutes and *maximum* resale prices ($p^1 \leq p^{1*}$) in all other cases.

Absent restraint, the below-cost good 1 wholesale price for strong substitutes would spur retailers to charge a good 1 retail price that is below the monopoly level p^{1*}. Conversely, loss-leader pricing for weak substitutes would lead retailers to charge a higher retail price.

Judging anti-competitive effects of the vertical contract is more complicated than the "minimum versus maximum" distinction suggests. To see this, consider the effect of banning maximum RPM in the case of weak substitutes. Absent RPM, the dominant firm selects a wholesale price (w^1) above the level that elicits the monopoly price p^{1*} in order to induce her retailers to charge a higher retail price for the fringe good. The optimal two-part contract balances the marginal cost of the good 1 price distortion ($p^1 > p^{1*}$) on profits with the marginal benefit of stimulating an increase in the price of good 2 ($p^2 < p^{2*}$). Hence, relative to the optimal RPM contract that supports (p^{1*}, p^{2*}), the good 1 price rises and the good 2 price falls. Banning *maximum* RPM can raise or lower economic welfare depending upon which effect dominates.

[16] Minimum RPM was deemed per se illegal in *Dr. Miles Medical Co. v. John D. Park and Sons* (220 U.S. 373, 1911); in 1968, maximum RPM was also judged to be a per se violation of the Sherman Act (*Albrecht v. Herald Co.*, 390 U.S. 145, 1968). In two more recent decisions, however, the Supreme Court has ruled that both forms of RPM should be judged by a "rule of reason," in 1997 for maximum RPM (*State Oil v. Khan*, 522 U.S. 3, 1997), and in 2007 for minimum RPM (*Leegin Creative Leather Products, Inc. v. PSKS, Inc.,* Slip Op. no. 06–480).

For strong substitutes, in contrast, both effects work in the same direction. When faced with a dominant siphoning effect, the monopoly manufacturer sets her wholesale price (w^1) *below* the level that elicits the monopoly price p^{1*} in order to spur a higher good 2 price. Again, the optimal two-part contract balances the marginal cost of the good 1 price distortion ($p^1 < p^{1*}$) with the marginal benefit of raising the good 2 price ($p^2 < p^{2*}$); however, this tradeoff now leads to lower prices for both goods. Banning *minimum* RPM is thus pro-competitive. In sum[17]:

Corollary 2. Banning *minimum* RPM lowers both retail prices ($p^1 < p^{1*}, p^2 < p^{2*}$) and thus raises economic welfare. In contrast, banning *maximum* RPM can lead to a higher retail price for one good and a lower retail price for the other good.

5. Retailer-fringe contracts

■ **Analysis.** We now turn to the possibility of retailer contracting with suppliers of the fringe good. Vertical separation from a competitive manufacturing sector is generally desirable for a retailer because it permits the design of contracts that soften downstream price competition (Shaffer, 1991a). A similar incentive exists for separation between retailers and fringe suppliers in a multi-product retail environment, but with one important caveat: unlike monopoly-retail contracts, which are designed to control retail pricing for mutual advantage, retail-fringe contracts essentially involve a retailer's attempt to regulate his own pricing behavior. With repeated contracts over time, the ability to do so may be limited; that is, a retailer may recognize the average cost of fringe supply (true marginal cost) even when the contracted wholesale price is above cost. This would be the case, for example, if the retailer cannot commit to an exclusive supply arrangement, in which case no fringe supplier would be willing to pay the upfront slotting fees characterized below. Often, however, we expect relatively long term exclusive supply contracts to be possible (due in part to the fixed costs of establishing a vertical relationship). Consistent with the literature, we make this assumption here.

Consider the following four-stage game. First, the monopoly manufacturer selects a contract with each retailer that stipulates a wholesale price (w^1), fixed fee (f^1), and RPM ($p^{1R} = p^{1*}$), as before. Second, each retailer signs an independent contract with a fringe supplier that stipulates a wholesale price (w^2) and a fixed tariff (f^2).[18] Third, given observable wholesale prices from the first two stages, retailers compete in fringe good retail prices p^2 (with good 1 prices determined by contract). Finally, production and exchange occur.[19]

The analytical challenge is to show that, given a vertical restraint ($p^{1R} = p^{1*}$), there is a wholesale price w^1 that prompts the duopoly retailers to sign two-part contracts with fringe suppliers which in turn yield the collective optimum (p^{1*}, p^{2*}). Vertical separation occurs in this setting whenever retailers choose contracts with $w^2 \neq c^2$.

When contracting with fringe suppliers, a retailer can require a lump-sum payment of f_i^2, let suppliers compete in wholesale prices (w_i^2) to acquire the contract, and select among suppliers with the lowest prices on offer. Hence, in equilibrium, the terms of the contract will satisfy the

[17] Additional (plausible) regularity conditions are needed for proof of this result (see the Appendix). Note, in addition, that our claimed effects of antitrust policy rely on our premise that *implicit* cross-market controls cannot be implemented by monopoly manufacturers (see note 10). If implicit commitments to cross-market retail prices can be made, then they will supplant any proscribed vertical restraint and continue to produce monopoly outcomes.

[18] The restriction to two-part retailer-fringe contracts comes at no cost in generality; as of the second stage, a retailer can mimic outcomes from any nonlinear fringe supply contract using a two-part equivalent. As is well known, this equivalence breaks down when there is uncertainty and asymmetric information that are absent in our model (see, for example, Mathewson and Winter, 1984; Martimort, 1996; Kühn, 1997).

[19] In principle, different orders of play are possible, for example simultaneous contracting between (i) retailers and the dominant firm, and (ii) retailers and fringe suppliers. The qualitative results derived below are robust to such alternative games. Note, however, that different orders of play will generally lead to lower integrated profit (because the monopoly manufacturer takes fringe wholesale markups as given and excludes these margins from joint profit maximization). As a result, in an expanded game wherein retailers choose the order of contracting, the order that we assume (monopoly-retailer contracting first) is an equilibrium outcome.

(competitive) zero-profit condition,

$$\left(w_i^2 - c^2\right) y^2\left(p_i^1, p_i^2\right) \phi(.) = f_i^2. \tag{12}$$

Note that (12) applies whether the retail contract stipulates a fixed payment from the supplier to the retailer, $f_i^2 > 0$, or vice versa; hence, we impose no restriction on the sign of f_i^2.

Next, define the per-customer retail profit function as

$$\Pi(p^1, p^2; w^1, w^2) = \sum_{i=1,2} (p^i - w^i) y^i(p^1, p^2).$$

Given retailer 1's contracted wholesale price with the supplier of good 2 (w_1^2), and the contractually predetermined wholesale and retail prices of good 1 (w^1 and $p^1 = p^{1*}$), the retailer's optimal price for good 2 (p_1^2) is determined as follows:

$$\max_{p_1^2} \Pi\left(p^{1*}, p_1^2; w^1, w_1^2\right) \phi\left(p^{1*}, p_1^2; \bar{u} = u^*\left(p^{1*}, p_2^2\right)\right) \Rightarrow p^2\left(w^1, w_1^2; p_2^2\right), \tag{13}$$

where p_2^2 is the rival retailer's price selection. Proceeding similarly with retailer 2, and equating the reaction functions, gives the equilibrium retail prices,

$$p_1^{2e} = p^{2e}\left(w^1, w_1^2; w_2^2\right), \quad p_2^{2e} = p^{2e}\left(w1, w_2^2; w_1^2\right), \tag{14}$$

where w_2^2 is the good 2 wholesale price faced by retailer 2. In order for the equilibrium in (14) to be locally stable at the integrated optimum, the following regularity restriction must hold:

Assumption 1.

At $p^2(.) = p^{2*}$, $\partial p^2\left(w^1, w_1^2; p_2^2 = p^{2*}\right) / \partial p_2^2 < 1$.[20]

Turning to the contract stage, each retailer chooses the fringe wholesale price w_i^2 to maximize profit subject to the subsequent price responses in (14). Given that supplier profits are rebated to the retailer in the retailer-fringe contract (12), retailer 1's problem is

$$\max_{w_1^2} \mathrm{J} = \Pi\left(p^{1*}, p^{2e}\left(w^1, w_1^2; w_2^2\right); w^1, c^2\right) \phi\left(p^{1*}, p^{2e}\left(w^1, w_1^2; w_2^2\right)\right); \bar{u} = u^*\left(p^{1*}, p^{2e}\left(w^1, w_2^2; w_1^2\right)\right). \tag{15}$$

The symmetric contract equilibrium solves (15), with $w_1^2 = w_2^2 = w^2$.

Now consider the problem of the monopoly manufacturer. Her challenge is to select a wholesale price w^1 such that, with the resulting equilibrium w^2 from the retail contracts solving (15), retailers set good 2 retail prices to maximize integrated profit, $p^2 = p^{2*}$. To characterize this solution, we seek a wholesale price pair (w^1, w^2) that simultaneously satisfies two conditions in the symmetric retail equilibrium: (i) $w_1^2 = w^2$ solves (15) when $w_2^2 = w^2$, and (ii) the resulting $p_1^2 = p^{2*}$ in the pricing stage solves (13) when $p_2^2 = p^{2*}$. Assuming the requisite second-order conditions hold, differentiating (13) with respect to p_1^2 and evaluating at $p_1^2 = p_2^2 = p^{2*}$ gives

$$\mathrm{F}_1(w^1, w^2) = -\Pi(p^{1*}, p^{2*}; w^1, w^2) y^{2*} - t \sum_{i=1,2} (w^i - c^i) \partial y^{i*} / \partial p^2 = 0. \tag{16}$$

When $\delta \neq 0$, equation (16) has the closed-form solution $w^1(w^2)$ that yields the collectively optimal price selection for good 2, $p^2 = p^2\ (w^1, w^2\ ; p_2^2 = p^{2*}) = p^{2*}$.

Similarly, to solve (15), we take the first-order condition, use (13) and (14) to expand terms, and evaluate when $w_1^2 = w_2^2 = w^2$ and $p^{2e}() = p^{2*}$ (by (16)) and $\phi = 1/2$ (a symmetric

[20] Sufficient conditions for Assumption 1 are: $\partial^2 \Pi(p^{1*}, p^{2*}; w^1(w^2), w^2) / \partial (p^2)^2 < 0$ and $p^{2*} y^{2*} + t(dlny^{2*}/dln\, p^2) \geq 0$, where $w^1(w^2)$ solves equation (16) below.

equilibrium)[21]:

$$\begin{aligned} \mathrm{F}_2(w^1, w^2) = {} & \Pi(p^{1*}, p^{2*}; w^1, w^2) y^{2*}\big(\partial p^2()/\partial p_2^2\big) \\ & + (w^2 - c^2)\big\{\big[t\partial y^{2*}/\partial p^2\big] - (y^{2*})^2\big(1 - \big(\partial p^2()/\partial p_2^2\big)\big)\big\} = 0, \end{aligned} \tag{17}$$

where $\partial p^2\ ()/\partial p_2^2 = \partial p^2\ (w^1, w^2\ ; p_2^2 = p^{2*})/\partial p_2^2$.

Inspection of conditions (16) and (17) results in the following:

Proposition 2. If the two goods are independent in consumption ($\partial y^{1*}/\partial p^2 = 0$), then the collective optimum is supported by $w^2 = c^2$ and $w^1 > p^{1*}$ such that $\Pi(p^{1*}, p^{2*}; w^1, c^2) = 0$.

For independent goods, our results replicate those derived in Section 4. The monopoly manufacturer writes a loss-leading vertical contract ($w^1 > p^{1*}$) with her retailers that eliminates the retailers' variable profit per customer ($\Pi = 0$). With zero variable profit, the retailers gain no advantage by signing contracts with fringe suppliers: although a contract with a fringe supplier can be used to steal business from the rival retailer, shifting customers no longer shifts rent. Commensurately, retailer-fringe contracts do not arise.

The above logic does not extend to goods that are not independent in consumption ($\partial y^1()/\partial p^2 \neq 0$). For these cases, we have:

Proposition 3. When $\partial y^{1*}/\partial p^2 \neq 0$, there is a $w^{2*} > c^2$ such that $(w^{1*}, w^{2*}) = (w^1(w^{2*}), w^{2*})$ solves equations (16) and (17). Hence, the collective optimum can be achieved by vertical restraints on the retailers of the dominant manufacturer's own good. The optimal contract prompts the retailers to set positive tariffs ($f^{2*} > 0$) on fringe manufacturers.

Under the monopolist's optimal contract, we demonstrate below that a retailer's profit per customer (Π) can be either positive or negative in equilibrium (much as in Proposition 1). The novel aspect of Proposition 3 is that, *in either case*, retailers sign contracts with fringe suppliers that stipulate above-cost wholesale prices for good 2, $w^2 > c^2$. When per-customer profit is positive in the retail market ($\Pi > 0$), the elevated wholesale price implicitly commits the retailer to charge a higher good 2 retail price, which is advantageous to the retailer because his rival responds with a higher good 2 price (Shaffer's [1991a] insight). However, the retailers also find an elevated wholesale price to be advantageous when per-customer profit is negative in the retail market ($\Pi < 0$). This is because the rival now responds with a *lower* retail price, which rids the contracting retailer of costly customers on the interretailer margin.[22] By (12), the above-cost wholesale price is supported by positive tariffs on fringe suppliers ($f^2 > 0$).

Understanding these retailer-fringe contractual incentives, the monopolist selects her wholesale price (w^1) so as to elicit integrated good 2 pricing. Specifically, we define the retailer's profit per customer under the optimal contract as $\Pi^{**} = \Pi(p^{1*}, p^{2*}; w^{1*}, w^{2*})$, and make use of the parameter δ defined in (11),[23]

$$\delta = y^{1*}y^{2*} - t\partial y^{1*}/\partial p^2 \overset{s}{=} -dw^1(w^2)/dw^2 \overset{s}{=} d\Pi(p^{1*}, p^{2*}; w^1(w^2), w^2)/dw^2. \tag{18}$$

---

[21] From (2) and the definition of ϕ in (4), $[\partial\phi()/\partial u][\partial u^*(p^{1*}, p^2)/\partial p^2] = y^2()/2t$. With $\Pi(.; w^1, c^2) = \Pi(.; w^1, w^2) + (w^2 - c^2)\, y^2(p^{1*}, p^2)$, we have from (13), $(d/dp^2)[\Pi(.; w^1, c^2)\phi()] = (w^2 - c^2)\ \{\partial y^2()/\partial p^2\phi + y^2()(\partial\phi/\partial p^2)\}$, where $\partial\phi/\partial p^2 = -y^2()/2t$. Differentiating (14), $\partial p^{2e}(w^1, w_2^2; w^2)/\partial w^2 = [\partial p^{2e}(w^1, w^2; w_2^2)/\partial w^2](\partial p^2(w^1,w^2; p_2^2)/\partial p_2^2)$ when $w^2 = w_2^2$. Substituting into the first-order condition for (15) when $w^2 = w_2^2$ and $p^{2e}() = p^{2*}$ gives (17).

[22] The intuition for this rival retailer response is rather subtle. Speaking somewhat loosely, when $\Pi < 0$, retailers set p^2 at a high level that trades off two effects on retail profit: (i) the marginal benefit of a higher p^2 in ridding the retailer of costly customers, and (ii) the marginal cost of higher p^2 in lowering per-customer profit Π. Now, when the opposing retailer raises its price (due to an elevated wholesale price), the second effect—the marginal cost of a higher p^2—becomes larger because the retailer's market share rises. Because the first (customer-shedding) benefit of higher p^2 is unchanged, the retailer responds with a lower price.

[23] The first sign equality in (18) follows from differentiation of (16). The second follows from substitution for $(dw^1(w^2)/dw^2)$ in the equation, $d\Pi()/dw^2 = -y^1(dw^1(w^2)/dw^2) - y^2$, and appeals to our initial assumption that $|\, d\ln u_1/d\ln y^1 \,| > |\, d\ln u_1/d\ln y^2 \,|$.

Recall that $\delta > 0$ in the case of complementary goods, but that, for substitute goods, $\delta > 0$ when the goods are *weak substitutes* and $\delta < 0$ when the goods are *strong substitutes*.

Proposition 4. (i) When the retail goods are complements ($\partial y^{1*}/\partial p^2 < 0$), $\Pi^{**} > 0$ and $w^{1*} > c^1$; (ii) when the retail goods are weak substitutes ($\partial y^{1*}/\partial p^2 > 0$), $\Pi^{**} < 0$ and $w^{1*} > p^{1*} > c^1$; and (iii) when the retail goods are strong substitutes ($\partial y^{1*}/\partial p^2 > 0$), $\Pi^{**} > 0$ and $w^{1*} < c^1$.

The intuition for Proposition 4 is straightforward. Consider the starting point of marginal cost good 1 pricing, $w^1 = c^1$. When retailers raise their good 2 wholesale prices with fringe contracts ($w^2 > c^2$), they reduce—but do not eliminate—their incentive to discount good 2. This is because fringe contracts create two effects: (i) on the inter retailer margin, customers are now less profitable, which decreases incentives to discount p^2; and (ii) on the intraretailer margin, the opportunity cost of reducing good 2 sales by elevating p^2 falls. As a result, retailer-fringe contracts temper both the business-stealing and siphoning effects, which reduces, but does not eliminate, the manufacturer's incentive to distort the good 1 wholesale price from marginal cost.

Corollary 3. Retailer-fringe contracting reduces the extent to which good 1 wholesale prices depart from cost, $|w^1 - c^1|$, implying a higher wholesale price when the goods are strong substitutes, and a lower wholesale price when the goods are weak substitutes or complements.

The preceding results are derived under the premise of a non-zero δ. Unlike the case of vertically integrated retailers considered in Section 4, however, the $\delta \neq 0$ restriction is not necessary for the exercise of horizontal control when the retailers are vertically separated from their good 2 suppliers. Even when $\delta = 0$, there is an above-cost good 2 wholesale price, $\hat{w}^2 > c^2$, that elicits optimal good 2 retail pricing, $p^2 = p^{2*}$. Formally, evaluating (16) when $\delta = 0$ yields

$$\hat{w}^2 - c^2 = y^*\Pi^*/\gamma > 0, \tag{19}$$

where $\gamma \equiv (y^{2*})^2 - t(\partial y^{2*}/\partial p^2) > 0$ is the good 2 counterpart to δ. This above-cost wholesale price can be supported by endogenous retailer-selected slotting fees that, in turn, are influenced by the dominant manufacturer's choice of the good 1 wholesale price. To see this, suppose w^1 is set equal to unit cost ($w^1 = c^1$). The retailers then set positive slotting fees on good 2 for the strategic reasons observed by Shaffer (1991a). Due to business-stealing incentives, however, these fees do not fully counter interretailer competition, and the resulting wholesale prices are set below the level necessary to induce monopoly pricing of good 2 ($w^2 < \hat{w}^2$). The dominant manufacturer responds by lowering her wholesale price below cost ($w^1 < c^1$), which raises per-customer retail profit and thereby increases retailers' incentives to raise slotting fees. The strategic benefit to setting a higher wholesale price (w^2) is now greater for retailers because the resulting increase in the rival retailer's retail price (p^2)—and the associated shift in custom—is now more profitable.

Proposition 5. When $\delta = 0$, the collective optimum can be achieved by a vertical restraint on the dominant manufacturer's good ($p^{1R} = p^{1*}$), and a below-cost wholesale price, $w^1 \in (-\infty, c^1)$, that supports the good 2 wholesale price, $\hat{w}^2 > c^2$, defined in equation (19).

For cases in which δ is close or equal to zero, Proposition 5 provides a new explanation for vertical separation between retailers and competitive suppliers as a mechanism to facilitate horizontal control of the marketplace.

□ **Application to supermarkets.** Supermarkets, drug chains, and mass merchandisers frequently offer private-label ("store-brand") products that closely substitute for national brands.[24]

[24] In general, the term "private label" refers to any product in which a manufacturer enters into a relationship with a buyer to use the buyer's name on its product. Under this definition, private labels are sold in a wide variety of product categories including wine, credit cards, medical equipment, electronics, software, and website content (both graphics and text). Here we choose to use the terms "private label" and "store brand" synonymously and focus on the case of supermarket private labels that are close substitutes for a national brand.

Private-label products in the supermarket industry generate a significant share of total retail sales (22% in Europe and 16% in North America in 2002) and, in the United States, have a greater market share than the leading manufactured brand in roughly 30% of all product categories.

Private labels are supplied to retailers in three different ways: (i) direct production by the retailer (e.g., an in-store bakery); (ii) by the manufacturer of a national brand (e.g., Coca-Cola produces ASDA Cola in the United Kingdom); and (iii) by contract manufacturers specializing in private-label production (e.g., Ralcorp produces various ready-to-eat cereals and crackers). In the first case, horizontal control can be achieved directly by vertical restraints without third-party contracts (as in Proposition 1). In the second case, a single manufacturer can maximize collective rents over the national brand and private-label, without the use of a vertical restraint, by jointly selecting wholesale prices. However, the third case is the most common, as the private label market is dominated by small, independent suppliers (*Supermarket News*, 1995).

Private-label procurement at supermarkets typically occurs through the use of an in-house broker (IHB). IHBs assist supermarkets with their private-label programs by selecting among potential private-label suppliers and by providing services such as procurement, category management, quality control, label design, retail pricing, and merchandising. Nearly 80% of private-label purchases by U.S. supermarkets are brokered through IHBs at a cost ranging from 1% to 6% of sales (Marion, 1998).

Consider a national brand manufacturer who imposes a vertical restraint to control the retail pricing of a supermarket private label. For strong substitutes, our model predicts the emergence of retailer contracts for private labels that involve lump-sum payments to supermarkets in exchange for elevated wholesale prices ($w^2 > c^2$). Indeed, evidence suggests that IHBs rebate a significant share of their brokerage commission to supermarket retailers. This is done either through direct cash payment from IHBs to their retailers or through "in-kind" rebates, for instance by placing supermarket employees on their payrolls and acquiring retail service functions previously performed by supermarket personnel. Marion (1998) estimates that up to 80%–95% of the brokerage commission collected from private-label suppliers by IHBs is rebated through lump-sum transfers to retailer accounts. To the extent that brokerage commissions on private-label sales pass through to wholesale prices, this practice raises the wholesale price of private-label products, $w^2 > c^2$.

Regarding contracts between national brand manufacturers and supermarket retail chains, our model predicts *minimum* resale price contracts that are per se illegal in Europe and, until 2007, in the United States (see Comanor and Rey, 1997). If RPM were allowed, we would expect to see below-cost wholesale pricing of the national brand ($w^1 < c^1$), and a tariff paid by retailers to the manufacturer of the national brand. However, we expect the proscription of RPM to instead spur legal two-part contracts such that the national brand wholesale price (w^1) is above cost, but lower than the level that would otherwise prompt monopoly pricing (p^{1*}). The lower wholesale price is advantageous to the manufacturer of the national brand because it yields a higher retail price for the private label; however, due to the profit costs of lowering the wholesale price of the national brand, the contract would not go so far as to elicit a monopoly price for the private label. A ban on minimum RPM would therefore result in strictly lower retail prices for both goods.

Given the illegality of RPM, our model also indicates that supermarket-fringe contracts for slotting allowances are anti-competitive. The direct effect of a retailer-fringe contract is to raise the retail price of good 2; however, there is also an indirect effect on good 1. The retailer-fringe contract reduces the incentive of manufacturer 1 to lower w^1 to achieve a higher p^2, and this raises the retail price of good 1 (see Corollary 3). Slotting allowances on suppliers in a competitive fringe can thus lead to higher retail prices for *both* goods.

6. Extensions

■ **Oligopoly.** We have assumed that good 1 is produced by a monopolist. Suppose instead that the upstream market for good 1 is populated by differentiated duopolists. We then have a three-good model that is otherwise the same as before. The three goods are the two duopoly-supplied products (close substitutes that we will denote by 1A and 1B) and the fringe good. We assume that the first two goods enter consumer utility $u(y^{1A}, y^{1B}, y^2)$ in a symmetric way.

The duopoly competition is similar to that studied by Rey and Vergé (2004). The key difference here is that we have a fringe market which the dominant manufacturers seek to control. In Rey and Vergé (2004), an above-cost wholesale price by one duopolist induces the rival firm to support a retail price that is discounted below the monopoly level because the duopolist ignores the cost of the discounting to the rival's margin. Using RPM to control retail pricing frees the manufacturers to set wholesale prices equal to marginal cost, thereby aligning vertical incentives. In our model, however, RPM does not void the duopolists' interest in distorting their wholesale prices because these prices are used to control retailer pricing of the fringe good. Consequently, retail prices, in equilibrium, are driven away from monopoly levels.

Consider the case in which retailers are integrated with a fringe supplier (as in Section 4). The duopolists negotiate observable three-part contracts (maintained retail price, wholesale price, and fixed transfer) with the two retailers. Denoting the associated retail and wholesale price pairs by (p^{1A},w^{1A}) and (p^{1B},w^{1B}), and assuming common manufacturer unit costs c for all three goods (for symmetry and to avoid clutter), the retailers solve

$$\max_{p^2} \left[\Pi^*(p^{1A}, p^{1B}, p^2) - \sum_{i=1A,1B} (w^i - c) y^i(p^{1A}, p^{1B}, p^2) \right] \phi \left(p^{1A}, p^{1B}, p^2; \bar{u} \right), \qquad (20)$$

where $\Pi^*() = \sum_{i=1A,1B,2} (p^i - c) y^i(p^{1A}, p^{1B}, p^2)$ is fully integrated profit, $\bar{u}$ is consumer utility at the rival retailer, and $\phi = (1/2) + [(u^*(p^{1A}, p^{1B}, p^2) - \bar{u})/(2t)]$ is the retailer's market share.

We envision two simultaneous bargaining games, in each of which the two retailers negotiate with one of the duopoly manufacturers, taking the contracts with the rival manufacturer as given.[25] As before, we abstract from particulars of the bargaining games that determine how collective gains are split between the contracting parties, and assume instead that the parties maximize joint rents and share their gains using fixed transfers. Contracts between manufacturer 1A and her retailers thus solve the joint profit maximization problem,

$$\max_{p^{1A},p^2} \Pi^{**}(p^{1A}, p^{1B}, p^2; w^{1B}) \equiv \Pi^*(p^{1A}, p^{1B}, p^2) - (w^{1B} - c) y^{1B}(p^{1A}, p^{1B}, p^2), \qquad (21)$$

which involves maximizing integrated profit less manufacturer 1B's margin.

Under plausible regularity restrictions, the duopoly retail price that solves problem (21) (p^{1A}) declines with the wholesale price w^{1B}.[26] A higher good 1A retail price, by stimulating demand for the substitute good 1B, yields profit gains to manufacturer 1B that rise with the wholesale price w^{1B}. Because these gains are ignored by manufacturer 1A and her retail contracting partners, the contracting manufacturer counters a rise in w^{1B} with a decrease in p^{1A}.

In a symmetric contract (and pricing) equilibrium, we have the retail prices from (21),[27]

$$p^{1A} = p^{1B} = p^{1**}(w), \quad p^2 = p^{2**}(w), \qquad (22)$$

and the wholesale price $w = w^{1A} = w^{1B}$ that yields the latter fringe retail price from the retailers' solution to (20); that is, w solves the first-order condition for problem (20),

$$-\Pi^{**}() y^2 + (w - c)\{y^{1A} y^2 - t(\partial y^{1A}/\partial p^2)\} = 0, \qquad (23)$$

[25] We thereby sidestep potentially complex issues on the use of one duopolist's contract to extract rents from the other duopolist's contracting game.

[26] Sufficient conditions for this property are (i) consistent with second-order conditions, $|\Pi^{**}_{2,2}| > |\Pi^{**}_{1A,2}|$, where $\Pi^{**}_{i,j} = \partial^2 \Pi^{**}/\partial p^i \partial p^j$, and (ii) 1A and 1B are "stronger" substitutes than goods 1B and 2 in the sense that $\partial y^{1B}/\partial p^{1A} \geq |\partial y^{1B}/\partial p^2|$. Then (using second-order conditions): $\partial p^{1A}/\partial w^{1B}|_{\text{eq.(20)}} \stackrel{s}{=} (\partial y^{1B}/\partial p^{1A})\, \Pi^{**}_{2,2} - (\partial y^{1B}/\partial p^2)\, \Pi^{**}_{1A,2} < 0$.

[27] Problem (21) yields solutions $(p^{1A+}(w^{1B}, p^{1B}), p^{2+}(w^{1B}, p^{1B}))$. By symmetry, we can define the equilibrium price functions for common $w = w^{1A} = w^{1B}$: $p^{1**}(w) = p^{1A}$: $p^{1A} = p^{1A+}(w, p^{1A})$, $p^{2**}(w) = p^{2+}(w, p^{1**}(w))$. For stability of the retail pricing equilibrium, we assume that $\partial p^{1A+}/\partial p^{1B} < 1$ (c.f., Assumption 1).

where all functions are evaluated at equilibrium prices. Equation (23) has two implications. First is the analog to equation (11)[28]:

$$w - c = \Pi^{**}()y^2/\delta^{\mathrm{A}} \stackrel{s}{=} \delta^{\mathrm{A}}, \tag{24}$$

where $\delta^{\mathrm{A}} \equiv y^{1\mathrm{A}}y^2 - t(\partial y^{1\mathrm{A}}/\partial p^2)$. Second is the equilibrium retail profit per customer:

$$\Pi^{***} \equiv \Pi^{**} - (w - c)y^{1\mathrm{A}} = -t(w - c)(\partial y^{1\mathrm{A}}/\partial p^2)/y^2. \tag{25}$$

Together, (24) and (25) directly imply the key qualitative results of Proposition 1.

The important distinction here is that the equilibrium retail prices do not maximize collective profit, an unfortunate consequence of the duopolists' lack of coordination in their contracting problems with retailers. Ordinarily, this coordination failure has favorable implications for consumer prices relative to the monopoly outcome; however, this is not necessarily the case here. Consider, for example, the case in which all three goods are strong, symmetric substitutes. In this case, a symmetric price equilibrium arises, $p^{1\mathrm{A}} = p^{1\mathrm{B}} = p^2 = p^{1**}(w)$, with $\mathrm{d}p^{1**}(w)/\mathrm{d}w < 0$.[29] From problem (21), retail prices fall with the rival duopolist's wholesale price w because the benefit of higher retail prices on sales to customers who switch to the rival duopoly good is ignored. Hence, with $w < c$ in equation (24), all retail prices are higher than would be charged by a fully integrated monopolist. Manufacturers charge retailers below-cost wholesale prices in order to spur them to raise their prices on the fringe good. The low wholesale price of the rival duopoly product, in turn, inflates the retail margin for that good, which raises the perceived rent on sales of the rival good to an artificially high level from the perspective of the contracting manufacturer. As a result, oligopolistic competition leads to an equilibrium outcome that is socially inferior to that which would emerge under monopoly.

Of course, this is not a general conclusion. With weak substitutes, for example, the wholesale price w is *raised* above cost in order to spur a higher good 2 retail price (problem (20)). The elevated wholesale price in turn lowers the optimal good 1 retail price for the contracting parties below the monopoly level (with $\mathrm{d}p^{1**}(w)/\mathrm{d}w < 0$). Under plausible conditions, this also lowers their optimal good 2 retail price ($\mathrm{d}p^{2**}(w)/\mathrm{d}w < 0$). In sum, we have:

Proposition 6. Duopoly production of good 1 leads to (i) below-cost wholesale prices ($w < c$) when the fringe good is a strong substitute ($\delta^{\mathrm{A}} < 0$), (ii) above-cost wholesale prices ($w > c$) otherwise, (iii) loss-leader pricing ($w > p^{1**}$) for independent goods, weak substitutes, and weak complements, and (iv) retail prices that depart from the collective integrated industry optimum ($p^{1*} = p^{1**}(c), p^{2*} = p^{2**}(c)$). For independent goods, $p^{1**} < p^{1*}, p^{2**} = p^{2*}$; for strong substitutes, $p^{1**} > p^{1*}, p^{2**} > p^{2*}$; for weak substitutes, $p^{1**} < p^{1*}, p^{2**} < p^{2*}$; and for complements, $p^{1**} < p^{1*}$, $p^{2**} > p^{2*}$.

□ **Retail formats without product bundling.** Until now we have considered a spatial downstream market that involves multi-product transactions between consumers and retailers. This bundling of consumer purchases in a single retail transaction is a natural property in many, but not all, retail markets.

Is this bundling important for our results on horizontal control? For the case of independent goods (where $\partial y^1(.)/\partial p^2 = \partial^2 u(.)/\partial y^l \partial y^2 = 0$), the answer is "yes." However, for all other cases, the answer is "no." Consider a generalized statement of our model, with product k $\in \{1,2\}$ demands at retailer $i \neq j$, $(i,j) \in \{1,2\}$, denoted by $\mathrm{D}^{\mathrm{k}}(p_i^1, p_i^2, p_j^1, p_j^2)$. Then three-part monopoly-retailer contracts (with maintained price $p^{1\mathrm{R}}$, wholesale price w^1, and fixed transfer) yield the

[28] The sign equality in (24) follows from $\Pi^{**}() > 0$ in a symmetric equilibrium in order for the duopoly manufacturers to earn nonnegative profit. ($\Pi^{**} \leq 0$ implies the contradiction of $w > c$ and zero manufacturer profit.)

[29] Here, by symmetry and second-order conditions, we have $0 < -\Pi^{**}_{2,2} = -\Pi^{**}_{1A,1A} > |\Pi^{**}_{1A,2}|$ and $\partial y^{1\mathrm{B}}/\partial p^{1\mathrm{A}} = \partial y^{1\mathrm{B}}/\partial p^2 > 0$, implying that $\partial p^{1\mathrm{A}+}/\partial w^{1\mathrm{B}}|_{(20)} < 0$ (see note 26). Hence, with $\partial p^{1\mathrm{A}+}/\partial p^{1\mathrm{B}} < 1$ (note 27), $\mathrm{d}p^{1**}(w)/\mathrm{d}w = [\partial p^{1\mathrm{A}+}/\partial w]/[1 - (\partial p^{1\mathrm{A}+}/\partial p^{1\mathrm{B}})] < 0$.

following retailer i choice for the fringe good price p_i^2, assuming the retailer has integrated good 2 production (for simplicity):

$$\max_{p_i^2}(p^{1R} - w^1)D^1\left(p^{1R}, p_i^2, p^{1R}, p_j^2\right) + \left(p_i^2 - c^2\right)D^2\left(p^{1R}, p_i^2, p^{1R}, p_j^2\right). \qquad (26)$$

Compare this choice to that for the fully integrated industry:

$$\max_{p^1,p^2} \sum_{k=1,2} (p^k - c^k)D^k(p^1, p^2; p^1, p^2) \rightarrow (p^{1*}, p^{2*}). \qquad (27)$$

Examining the respective first-order conditions, the solution to problem (26) is supported by contracts that set $p^{1R} = p^{1*}$ and w^1 that satisfy

$$-(w^1 - c^1)\left(\partial D^1(.)/\partial p_i^2\right) = \sum_{k=1,2} (p^k - c^k)\left(\partial D^k(.)/\partial p_j^2\right). \qquad (28)$$

Now suppose that consumers do not have multi-product purchase economies and therefore can buy one good from one retailer and the other good from the other retailer, without cost. If the two goods are also independent in consumption, the retailers' good 1 demands are completely invariant to good 2 prices, $\partial D^1(.)/\partial p_i^2 = \partial D^1(.)/\partial p_j^2 = 0$. Because good 2 demands depend on good 2 prices, $\partial D^2(.)/\partial p_j^2 > 0$, there is no wholesale price (w^1) that can satisfy condition (28) for this case. When the two goods are completely independent in *both* consumption and the shopping process, retail pricing of the monopoly good 1 and the fringe good 2 are also independent; hence, the monopolist cannot control the latter with her contract on the former. In this case, there is a motive for cross-market control, but these controls must be explicit, for instance through the use of tying arrangements, commodity bundling, or cross-market slotting fees of the form described by Innes and Hamilton (2006).

If the two goods are *not* independent in consumption, then $\partial D^1(.)/\partial p_i^2 \neq 0$, and equation (28) has a solution. Our main conclusions then stand. Hence, any jointness between products in either consumption or shopping enables the monopolist to exert horizontal control using a three-part vertical contract with no explicit cross-market terms. Moreover, with $\partial D^k(.)/\partial p_j^2 > 0$, the wholesale cost distortion is the same as described in Proposition 1, $(w^1 - c^1) \overset{s}{=} \delta \equiv -\partial D^1(.)/\partial p_i^2$.

7. Conclusion

■ In this article, we study how a vertical restraint by a manufacturer of one good can be used to simultaneously control the retail pricing of another good, resulting in the extension of monopoly power to a second market. The central elements required for this to occur are: (i) a multi-product retail market in which oligopoly retailers compete in common goods; (ii) a monopoly (or oligopoly) manufacturing industry in the upstream market for one of the goods; and (iii) an element of jointness between the goods in consumer demand. Jointness can arise either in consumption (so that a consumer's demand for one good is affected by price of the other good) or in "shopping" (so that economies exist in buying both products at once). Because such jointness exists in a wide range of economic settings, our analysis suggests that antitrust scrutiny is warranted for vertical restraints, even when direct mechanisms for cross-market control, for instance explicit tying and price-fixing arrangements, are not employed.

We identify several symptoms of vertical contracts used to exert horizontal control, including predatory (below-cost) wholesale pricing for strong substitutes, retailer-driven "slotting fees" for competitive suppliers of the rival good, and loss-leader retail pricing for weak substitutes, weak complements, and independent goods. Some of these practices are commonplace; for example, transfers from contract manufacturers to retailers in the form of discounted loans, technology, and demonstration equipment are common in many retail settings. For the case of supermarket retailing, moreover, the available evidence indicates that direct cash transfers occur through rebates paid to retailers by in-house brokers of their private labels.[30]

All of these symptoms, and the vertical restraints that underpin them, are the subject of the ongoing antitrust policy debate. For example, statutes prohibit below-cost retail pricing in a number of European countries and for gasoline in a number of U.S. states (Allain and Chambolle, 2005). Antitrust law also proscribes predatory pricing. As elucidated in the case of *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.* (92-466, 509 U.S. 209, 1993), a prerequisite for below-cost pricing to be deemed predatory—and thus illegal under U.S. law—is that the pricing firm has a reasonable prospect of recouping losses. This condition is satisfied in the vertical contracts characterized in this article (even though it was not satisfied in the *Brooke* case), because losses are recouped with fixed transfers between the parties.

Are legal proscriptions of below-cost pricing beneficial to society? In our model, we distinguish between below-cost wholesale pricing ("predatory accommodation" associated with strong substitutes) and below-cost retail pricing ("loss leadership" for weakly related goods). A proscription against predation at the wholesale level prevents the monopolist from lowering her wholesale price below cost in order to spur a higher retail price for the rival good. Hence, if RPM is allowed while the wholesale price is constrained to be no lower than unit cost, the rival retail price would be set below the multi-product monopoly level. If retail prices are strategic substitutes, moreover, in the sense that a higher price for one good leads retailers to charge a lower price for the other good, the (maintained) retail price on the monopoly good would also be set below the monopoly level to induce a higher retail price on the fringe good. Similarly, a proscription against retail predation (loss leadership) would prevent the monopolist from raising her wholesale price above the (maintained) retail price. Such regulation would result in a wholesale price set equal to the retail price, prompting retailers to set the retail price of the fringe good below the multi-product monopoly level. In both cases, antitrust regulations that prohibit below-cost pricing are pro-competitive and lead to a reduction in both retail prices.

Antitrust law also limits vertical restraints directly (Comanor and Rey, 1997). Is a flexible antitrust approach to vertical restraints, as ruled by the U.S. Supreme Court in *Leegan Creative Leather Products, Inc. v. PSKS, Inc.* (Slip Op. no. 06–480, 2007), in the public interest? In our analysis, the vertical restraint enables contracting parties to support the integrated monopoly outcome. Nevertheless, the welfare implications of the practice depend upon the baseline selected for comparison. If no contracts are allowed at all, so that all products are supplied by wholesale pricing, double-marginalization of the monopoly good results in a retail price that is higher than the monopoly level. Vertical contracts can have favorable welfare properties in this case. If two-part contracts are allowed, then our analysis indicates that *also* allowing for minimum RPM provisions reduces welfare; however, maximum RPM can produce pro-competitive effects even when the design of the vertical restraint is to achieve horizontal control of the marketplace. Broadly speaking, this logic supports a flexible "rule of reason" for judging the legality of vertical restraints, but also argues for careful scrutiny of the impact of vertical restraints on cross-market competition in multi-product retail environments.

Appendix

■ *Proof of Proposition 1.* Properties (i) and (ii) follow from equation (11). For equilibrium retail profit per customer, we turn to the first-order condition for a retailer's choice of p^2:

$$\begin{aligned} &\mathrm{F}_1(w^1, c^2) = -\Pi(p^{1*}, p^{2*}; w^1, c^2) y^2(p^{1*}, p^{2*}) - t(w^1 - c^1)\partial y^1(p^{1*}, p^{2*})/\partial p^2 = 0 \\ &\Rightarrow \quad \Pi() = -t(w^1 - c^1)(\partial y^1()/\partial p^2)/y^2. \qquad \text{(A1)} \end{aligned}$$

The claimed signs follow from (A1) and properties (i) and (ii). *Q.E.D.*

[30] The related practice of charging slotting allowances to suppliers has drawn recent regulatory attention in the United States (Federal Trade Commission, 2003), although no explicit linkage has been made to the use of vertical restraints.

Proof of Corollary 1. Substituting (6) into (5) (evaluated at (p^{1*}, p^{2*})) gives

$$\partial\pi_1/\partial\pi_1 = (2y^2)^{-1}\{(w^1 - c^1)\mathrm{A}^1 - (w^2 - c^2)\mathrm{A}^2\},$$

where $\mathrm{A}^i \equiv y^i\ (\partial y^i\ ()/\partial p^j) - y^j\ (\partial y^i\ ()/\partial p^i)$, $j \neq i$, and $\mathrm{A}^i > 0$ for $i \in \{1,2\}$ by our initial assumption that $|\,\mathrm{dln}u_i\ /\ \mathrm{dln}y^i\,| > |\,\mathrm{dln}u_i\ /\ \mathrm{dln}y^j\,|$ for $j \neq i$. Hence, at $p^1 = p^{1*}$, with $w^2 = c^2$, $\partial\pi_1/\partial p^1 < 0$ for strong substitutes and $\partial\pi_1/\partial p^1 > 0$ otherwise. *Q.E.D.*

Proof of Corollary 2. Preliminaries. Define $(p^1(w), p^2(w))$ that solve equations (5) and (6) in the symmetric equilibrium (where $w = w^1$ and $w^2 = c^2$). Further define the wholesale prices that solve equations (8) and (10), respectively,

$$w_{(8)} : p^1(w) = p^{1*},\ w_{(10)} : p^2(w) = p^{2*},$$

where $w_{(8)} > c^1$ (from equation (8)) and $w_{(10)} < (>)\ c^1$ for strong substitutes (weak substitutes, independent goods, and complements) (from equation (10)). Note from equation (9),

$$p^2(w_{(8)}) < p^{2*}. \tag{A2}$$

Similarly, evaluating equation (5) with $w^1 = w_{(10)}$, we have

$$\partial\pi_1(p^{1*}, p^{2*}; \bar{u}_2, w^1, c^2)/\partial p_1^1 = \varphi\Pi^* \left\{\left(\partial\varphi/\partial p_1^1\right)\left(\partial y^1/\partial p_1^2\right) - \left(\partial\varphi/\partial p_2^2\right)\left(\partial y^1/\partial p_1^1\right)\right\} / \left\{\varphi\left(\partial y^1/\partial p_1^2\right) + y^1\left(\partial\varphi/\partial p_1^2\right)\right\}.$$

Hence, in a symmetric equilibrium,

$$p^1(w_{(10)}) < (>) p^{1*} \text{ for strong substitutes (other goods).} \tag{A3}$$

Constrained to two-part contracts, the monopoly manufacturer's choice problem is

$$\max_w \Pi(p^1(w), p^2(w); c^1, c^2). \tag{A4}$$

Assumption. For a relevant range of w, $p^1(w)$ and $p^2(w)$ are monotone (A1), and $\partial^2\Pi/\partial p^1\partial p^2 \overset{s}{=} \partial y^1/\partial p^2$ at $(p^1(w), p^2(w))$ by (A2), where $\partial^2\Pi/\partial p^1\partial p^2 = 2(\partial y^1/\partial p^2) + \sum_{i=1,2}(p^i - c^i)(\partial^2 y^i/\partial p^1\partial p^2)$.

Assumption (A1) ensures that the $p^i(w)$ functions are well behaved. Assumptions (A2) and (A3) avoid dominance of third-order effects and are satisfied if third-order derivatives of u are sufficiently small relative to second-order derivatives.

By Assumption (A1), we can define the inverse function,

$$w^1(p^1) : p^1(w^1(p^1)) = p^1, \tag{A5}$$

and the good 2 retail price mapping,

$$p^2(p^1) = p^2(w^1(p^1)). \tag{A6}$$

(A5) and (A6) imply the equivalent monopoly choice problem,

$$\max_{p1} \Pi(p^1, p^2(p^1); c^1, c^2). \tag{A7}$$

(i) *Case of strong substitutes.* By (A2) and (A3) and (A5) and (A6), we have the two points on the $p^2(p^1)$ function, namely

$$(p^{1*}, p^2(p^{1*}) = p^2(w_{(8)}) < p^{2*}) \quad \text{and} \quad (p^1(w_{(10)}) < p^{1*}, p^{2*}).$$

Hence, by Assumption (A1),

$$dp^2(p^1)/dp^1 < 0. \tag{A8}$$

For $p^1 > p^{1*}$, we have (by (A5)) $p^2(p^1) < p^2(w_{(8)}) < p^{2*}$ and, hence,

$$\begin{aligned}\Pi(p^1, p^2(p^1); c^1, c^2) = \Pi(p^{1*}, p^2(w_{(8)}); c^1, c^2) - \int_{p^2(p^1)}^{p^2(w_{(8)})}\left[\partial\Pi(p^{1*}, p^2; .)/\partial p^2\right]dp^2 + \int_{p^{1*}}^{p^1}\Bigg\{[\partial\Pi(p^1, p^{2*}; .)/\partial p^1]\\ - \int_{p^2}^{p^{2*}}[\partial^2\Pi(p^1, p^2; .)/\partial p^2\partial p^1]\,dp^2\Bigg\}dp^1 < \Pi(p^{1*}, p^2(w_{(8)}); c^1, c^2).\end{aligned} \tag{A9}$$

The inequality is due to the definition of (p^{1*}, p^{2*}), concavity of Π (note 6), $p^2(w_{(8)}) < p^{2*}$, and $[\partial^2\Pi(p^1, p^2;.)/\partial p^2\partial p^1] > 0$ for strong substitutes (A2).

Similarly, for $p^1 < p^1(w_{(10)}) < p^{1*}$, we have (by (A8)) $p^2(p^1) > p^{2*}$ and, hence,

$$\Pi(p^1, p^2(p^1); c^1, c^2) = \Pi(p^1(w_{(10)}), p^{2*}; c^1, c^2) - \int_{p^1}^{p^1(w_{(10)})} [\partial\Pi(p^1, p^{2*}; .)/\partial p^1]\mathrm{d}p^1 + \int_{p^{2*}}^{p^2(p^1)} \left\{ [\partial\Pi(p^1, p^{2*}; .)/\partial p^2] - \int_{p^1}^{p^{1*}} [\partial^2\Pi(p^1, p^2; .)/\partial p^2\partial p^1]\mathrm{d}p^1 \right\} \mathrm{d}p^2 < \Pi(p^1(w_{(10)}), p^{2*}; c^1, c^2). \tag{A10}$$

By (A9) and (A10), any solution to (A7) is a $p^1 \in [p^1(w_{(10)}), p^{1*}]$.

Differentiating (A7) and evaluating at $p^1 = p^1(w_{(10)})$, where $p^2(p^1) = p^{2*}$,

$$\mathrm{d}\Pi\left(p^1(w_{(10)}), p^2(p^1(w_{(10)})); c^1, c^2\right)/\mathrm{d}p^1 = [\partial\Pi()/\partial p^1] + [\partial\Pi()/\partial p^2][\mathrm{d}p^2/\mathrm{d}p^1] > 0, \tag{A11}$$

where the inequality follows from: (i) $p^1(w_{(10)}) < p^{1*}$, implying $[\partial\Pi(p^1, p^{2*};.)/\partial p^1] > 0$;

(ii) $\mathrm{d}p^2(p^1)/\mathrm{d}p^1 < 0$ by (A8); and (iii)

$$[\partial\Pi()/\partial p^2] = [\partial\Pi(p^{1*}, p^{2*}; .)/\partial p^2] - \int_{p^1(w_{(10)})}^{p^{1*}} [\partial^2\Pi(p^1, p^{2*}; .)/\partial p^2\partial p^1]\mathrm{d}p^1$$

$$= -\int_{p^1(w_{(10)})}^{p^{1*}} [\partial^2\Pi(p^1, p^{2*}; .)/\partial p^2\partial p^1]\mathrm{d}p^1 < 0.$$

Finally, differentiating (A7) at $p^1 = p^{1*}$, where $p^2(p^1) = p^2(w_{(8)}) < p^{2*}$,

$$\mathrm{d}\Pi(p^{1*}, p^2(p^{1*}); c^1, c^2)/\mathrm{d}p^1 < 0, \tag{A12}$$

where the inequality follows from (i)

$$[\partial\Pi()/\partial p^1] = [\partial\Pi(p^{1*}, p^{2*}; .)/\partial p^1] - \int_{p^2(w_{(8)})}^{p^{2*}} [\partial^2\Pi(p^1, p^2; .)/\partial p^2\partial p^1]\mathrm{d}p^2$$

$$= -\int_{p^2(w_{(8)})}^{p^{2*}} [\partial^2\Pi(p^1, p^2; .)/\partial p^2\partial p^1]\mathrm{d}p^2 < 0;$$

(ii) $\mathrm{d}p^2(p^1)/\mathrm{d}p^1 < 0$ by (A8); and (iii) with $p^2(w_{(8)}) < p^{2*}$, $[\partial\Pi()/\partial p^2] = [\partial\Pi(p^{1*}, p^2;.)/\partial p^2] > 0$.

Together, equations (A8)–(A12) imply that any solution to (A7) is a $p^1 \in (p^1(w_{(10)}), p^{1*})$, implying that $p^2 \in (p^2(w_{(8)}), p^{2*})$; hence, $p^1 < p^{1*}$ and $p^2 < p^{2*}$.

(ii) *Other cases*. It suffices to consider independent goods. By continuity, the same conclusions will apply to sufficiently weak substitutes and sufficiently weak complements. Here we have two points on the $p^2(p^1)$ schedule:

$$(p^{1*}, p^2(w_{(8)}) < p^{2*}) \quad \text{and} \quad (p^1(w_{(10)}) > p^{1*}, p^{2*}).$$

Hence, by Assumption A1, $\mathrm{d}\,p^2(p^1)/\mathrm{d}p^1 > 0$. Following mathematics similar to those in (A9) and (A10), we have, for $p^1 > p^1(w_{(10)})$ (and hence $p^2 > p^{2*}$),

$$\Pi(p^1, p^2(p^1); c^1, c^2) < \Pi(p^1(w_{(10)}), p^{2*}; c^1, c^2), \tag{A13}$$

and, for $p^1 > p^{1*}$ (and hence, $p^2(p^1) < p^2(w_{(8)}) < p^{2*}$),

$$\Pi(p^1, p^2(p^1); c^1, c^2) < \Pi(p^{1*}, p^2(w_{(8)}); c^1, c^2). \tag{A14}$$

Finally, differentiating (A7) at $p^1 = p^{1*}$ (where $p^2(p^1) = p^2(w_{(8)}) < p^{2*}$),

$$\mathrm{d}\Pi(p^{1*}, p^2(p^{1*}); c^1, c^2)/\mathrm{d}p^1 = [\partial\Pi()/\partial p^2][\mathrm{d}p^2/\mathrm{d}p^1] > 0, \tag{A15}$$

where the equality is due to $[\partial\Pi()/\partial p^1] = 0$ at $p^1 = p^{1*}$(by the definition of p^{1*} and the assumption of independent goods), and the inequality follows from $\mathrm{d}p^2(p^1)/\mathrm{d}p^1 > 0$ and, with $p^2(w_{(8)}) < p^{2*}$ (and the assumed concavity in Π), $[\partial\Pi()/\partial p^2] = [\partial\Pi(p^{1*}, p^2;.)/\partial p^2] > 0$. Similarly, at $p^1 = p^1(w_{(10)})$, where $p^2(p^1) = p^{2*}$,

$$\mathrm{d}\Pi(p^1(w_{(10)}), p^2(p^1(w_{(10)})); c^1, c^2)/\mathrm{d}p^1 = [\partial\Pi()/\partial p^1] < 0. \tag{A16}$$

Together, (A13)–(A16) imply that any solution to (A7) is a $p^1 \in (p^{1*}, p^1(w_{(10)}) > p^{1*})$, implying that $p^2 \in (p^2(w_{(8)}) < p^{2*}, p^{2*})$; hence, $p^1 > p^{1*}$ and $p^2 < p^{2*}$. *Q.E.D.*

Proof of Proposition 3. First note the following (given Assumption 1):

Claim 1. $\partial p^2\ \ w^1, w_1^2; p_2^2\ \ /\partial p_2^2 \overset{s}{=} \Pi\ \ p^{1*}, p_1^2; w^1, w_1^2$,where "$\overset{s}{=}$" denotes "equals in sign."

Proof of Claim 1. Differentiating the first-order condition (FOC) associated with problem (13) at $p^2 = p^2(w^1, w_1^2; p_2^2)$ and making use of the second-order condition gives

$$\begin{aligned}\partial p^2()/\partial p_2^2 &\overset{s}{=} \left(\partial \Pi()/\partial p_1^2\right)[\partial\phi()/\partial \bar{u}]\left[\partial \mathrm{u}^*\left(p^{1*}, p_2^2\right)/\partial p_2^2\right] \\ &= \Pi()\left\{y^2\left(p^{1*}, p_1^2\right)/(2t\phi)\right\}\left\{y^2\left(p^{1*}, p_2^2\right)/(2\mathrm{t})\right\} \overset{s}{=} \Pi(),\end{aligned}$$

where the terms after the equality are derived by substituting from the FOC and expanding the relevant partial derivatives. Claim 1 *Q.E.D.*

Claim 2. At $w^2 = c^2$, $\mathrm{F}_2(w^1(w^2), w^2) > 0$ in (14) (where $w^1(w^2)$ solves (16)).

Proof of Claim 2. First note that, if $\Pi = 0$ and $w^2 = c^2$, then (16) implies that $w^1 = c^1$ and, hence, $\Pi > 0$, a contradiction. Therefore, at $(w^1,w^2) = (w^1(c^2),c^2)$, $\Pi \neq 0$. With $\Pi \neq 0$, Claim 1 implies that the first set of right-hand terms in (17) is positive; with $w^2 = c^2$, the second set of right-hand terms in (17) is zero. Claim 2 *Q.E.D.*

Claim 3. There is a bounded $\hat{w}^2 > c^2$ such that $\mathrm{F}_2(w^1(\hat{w}^2),\hat{w}^2) < 0$ in (17).

Proof of Claim 3. Define $\hat{w}^2$ by $w^1(\hat{w}^2) = c^1$; that is, from (16),

$$\hat{w}^2 - c^2 = \Pi(p^{1*}, p^{2*}; c^1, c^2)\{y^2(p^{1*}, p^{2*})/\mathrm{t}\}/\{[y^2(p^{1*}, p^{2*})^2/t] - [\partial y^2(p^{1*}, p^{2*})/\partial p^2]\} > 0. \tag{A17}$$

Also from (16), we have

$$\Pi(p^{1*}, p^{2*}; c^1, \hat{w}^2) \overset{s}{=} \hat{w}^2 - c^2 > 0. \tag{A18}$$

Hence, by Claim 1 and Assumption 1, $0 < \partial p^2\,()/\partial p_2^2 < 1$ at $(w^1,w^2) = (c^1,\hat{w}^2)$, which implies (together with $\Pi > 0$ and $\hat{w}^2 > c^2$):

$$\mathrm{F}_2(c^1, \hat{w}^2) < \Pi()y^2() + (\hat{w}^2 - c^2)\mathrm{t}\,[\partial y^2()/\partial p^2] = -(w^1 - c^1)\mathrm{t}(\partial y^1()/\partial p^2) = 0, \tag{A19}$$

where the first inequality evaluates the right-hand side of (17) at $\partial p^2()/\,\partial {p^2}_2 = 1$; the first equality substitutes from (16); and the final equality is due to $w^1(\hat{w}^2) = c^1$. *Q.E.D.*

Claim 4 (*Proposition 3*). There is a $w^{2*} \in (c^2,\hat{w}^2)$ such that $(w^1,w^2) = (w^1(w^{2*}),w^{2*})$ solve (16) and (17).

Proof of Claim 4. Follows directly from Claim 2, Claim 3, continuity of $\mathrm{F}_2(w^1(w^2),w^2)$ in w^2, and the intermediate value theorem (IVT). *Q.E.D.*

Proof of Proposition 4. First note:

Claim 5. w^{1*} is above or below c^1, depending upon whether Π^{**} is positive or negative, and whether the two goods are complements $(\partial y^1(p^{1*}, p^{2*})/\partial p^2 < 0)$ or substitutes $(\partial y^1()/\partial p^2 > 0)$ as follows:

	Retail Goods Are	
	Complements	Substitutes
$\Pi^{**} > 0$	$w^{1*} > c^1$	$w^{1*} < c^1$
$\Pi^{**} < 0$	$w^{1*} < c^1$	$w^{1*} > c^1$

Proof of Claim 5. Substitute (16) into (17), giving us the following necessary condition for (w^{1*}, w^{2*}) to support the integrated optimum:

$$(w^1 - c^1)(\partial y^1(p^{1*}, p^{2*})/\partial p^2)\left(\partial p^2()/\partial p_2^2\right) = (w^2 - c^2)\left(1 - (\partial p^2()/\partial p_2^2)\right)\left[\left(\partial y^2(p^{1*}, p^{2*})/\partial p^2\right) - (y^2(p^{1*}, p^{2*})/\mathrm{t})\right]. \tag{A20}$$

By Assumption 1 $(\partial p^2()/\partial p_2^2 < 1)$ and Proposition 3 $(w^{2*} > c^2)$, the term on the right-hand side of (A20) is negative at the optimum. Hence, the term on the left-hand side of (A20) must be negative. Making use of Claim 1, this requirement yields Claim 5. *Q.E.D.*

Proposition 4(i) follows from Proposition 1 ($\Pi > 0$ at $w^2 = c^2$), $\mathrm{d}\Pi(.;w^1(w^2),w^2)/\mathrm{d}w^2 > 0$ (by $\delta > 0$ for complements), and $w^{2*} > c^2$ (Proposition 3), which together imply $\Pi^{**} > 0$ and hence (by Claim 5), $w^{1*} > c^1$. For part (ii), note:

Claim 6. If the goods are weak (strong) substitutes, $\Pi(p^{1*}, p^{2*}; w^1(c^2), c^2) < (>)\ 0$.

Proof of Claim 6. At $(w^1, w^2) = (c^1, c^2)$, $\Pi(.;c^1, c^2) > 0$ and $\mathrm{F}_1(c^1, c^2) < 0$ (from (16)); moreover, $\partial\mathrm{F}_1(w^1, c^2)/\partial w^1 = \delta >$ $(<)\ 0$ (for weak [strong] substitutes); hence, $\mathrm{F}_1(w^1, c^2) < 0$ for all $w^1 \leq (\geq)\ c^1$ and, in order to satisfy (16), $w^1(c^2) >$ $(<)\ c^1$. With $[\partial y^1()/\partial p^2] > 0$ (substitutes) and $w^1 > (<) c^1$, satisfaction of (16) requires that $\Pi(.;w^1(c^2), c^2)$ be negative (positive). *Q.E.D.*

From (A18), we have that $\Pi(.;w^1(\hat{w}^2),\hat{w}^2) > 0$ for $\hat{w}^2 > c^2$. With $\Pi(.;w^1(c^2), c^2) < 0$ (Claim 6 for weak substitutes) and $\Pi(.;w^1(\hat{w}^2),\hat{w}^2) > 0$, there is a $\hat{w}^{2+} \in (c^2, \hat{w}^2) : \Pi(.; w^1(\hat{w}^{2+}), \hat{w}^{2+}) = 0$ (by continuity of $\Pi(.;w^1(w^2), w^2)$ in

w^2 and the IVT). Moreover, at $w^2 = \hat{w}^{2+}$, $F_2(w^1(w^2), w^2)<0$ (because $\Pi() = 0$ and $w^2 = \hat{w}^{2+} > c^2$); hence, given Claim 2, continuity of $F_2(w^1(w^2),w^2)$ in w^2, and the IVT, $w^{2*} \in (c^2, \hat{w}^{2+})$ and $w^{1*} = w^1(w^{2*})$ solve (16) and (17). With $\Pi(.;w^1(\hat{w}^{2+}),\hat{w}^{2+}) = 0$, $w^{2*} < \hat{w}^{2+}$, and $d\Pi(.;w^1(w^2), w^2)/dw^2>0$ (by $\delta > 0$ for weak substitutes), we have $\Pi^{**}<0$ and hence (by Claim 5), $w^{1*} > c^1$.

For part (iii), $\Pi^{**}= \Pi(.;w^1(w^{2*}),w^{2*})>0$ follows from: (a) $\Pi(.;w^1(c^2),c^2) > 0$ (Claim 6 for strong substitutes); (b) $\Pi(.;w^1(\hat{w}^2),\hat{w}^2) > 0$ (from (A17) and (A18)); (c) $w^{2*} \in (c^2, \hat{w}^2)$ (Claim 4 of Proposition 3); and (d) $d\Pi(.;w^1(w^2),w^2)/dw^2 \leq 0$ (by $\delta \leq 0$ for strong substitutes). Hence, $w^{1*}< c^1$ follows from Claim 5. *Q.E.D.*

Proof of Corollary 3. The corollary follows directly from Propositions 2 and 3, and equation (18).

Proof of Proposition 5. With $\hat{w}^2$ as defined in (19), observe that $F_2(c^1,\hat{w}^2) < 0$ (Claim 3). Now consider $w^1< c^1$. Substituting for $(\hat{w}^2 - c^2)\gamma$ in F_2 of equation (17),

$$F_2(w^1, \hat{w}^2) = \left[\left(\partial p^2/\partial p_2^2\right) \Pi(p^{1*}, p^{2*}; w^1, c^2) - \Pi^*\right] y^2. \tag{A21}$$

Define $\omega = -w^1$ and expand the first right-hand term in (A21):

$$\left(\partial p^2/\partial p_2^2\right) \Pi(p^{1*}; p^{2*}; w^1,\ c^2) = A(\omega)/B(\omega), \tag{A22}$$

where $A(\omega) = (y^2)^2\ \Pi(.;-\omega = w^1, w^2)\ \Pi(.; -\omega = w^1, c^2)$, and

$$B(\omega) = \Pi(.; -\omega = w^1, w^2)[2(y^2)^2 + t(\partial y^2/\partial p^2)] - t^2 \left[2(\partial y^2/\partial p^2) + \sum_{i=1,2} (p^i - w^i)(\partial^2 y^i/\partial (p^2)^2)\right],$$

all evaluated at $(p^{1*}, p^{2*}, \hat{w}^2)$. Taking derivatives:

$$\partial A/\partial \omega = (y^2)^2[\Pi(.; -\omega = w^1, w^2) + \Pi(.; -\omega = w^1, c^2)]y^1 > 0, \tag{A23}$$

$$\partial B/\partial \omega = y^1[2(y^2)^2 + t(\partial y^2/\partial p^2)] + t^2[\partial^2 y^1/\partial (p^2)^2] = z, \tag{A24}$$

where the inequality in (A23) follows from $\Pi(.; -\omega = w^1, c^2) > 0$ (with $w^1 \leq c^1$ and $\Pi^* > 0$) and $\Pi(.; -\omega = w^1, w^2) > 0$ (from equation (16), $w^2 = \hat{w}^2 > c^2$ (equation (16′)), $w^1 \leq c^1$, $(\partial y^2/\partial p^2) < 0$, and with $\delta = 0$, $\partial y^1/\partial p^2 > 0$). Note that $\partial B/\partial \omega$ in (A24) is a constant (invariant to ω) z.

Claim 7. There is a $w^{1*} \in (-\infty,c^1)$: $F_2(w^{1*}, \hat{w}^2) > 0$.

Proof of Claim 7. Define $\omega_0 = -c^1$. There are three cases: (i) $z < 0$. Let $\Delta \equiv -B(\omega_0)/z > 0$, where the inequality is due to $B(\omega_0) > 0$ (by second-order conditions for the retailer's choice of p^2) and $z < 0$. Consider $\omega_1 = \omega_0+\Delta$. By construction, $B(\omega_1) = 0$ and $B(\omega) > 0$ for $\omega \in (\omega_0, \omega_1)$. Hence, $\lim_{\omega \to \omega 1}$ (A/B) $= \infty$, and by continuity, there is an $\omega^* \in (\omega_0, \omega_1)$: (A/B)> Π^*. (ii) $z = 0$. With $\lim_{\omega \to \infty} A(\omega) = \infty$ and $B(\omega) = B(\omega_0)>0$, there is an $\omega^* \in (\omega_0, \infty)$: (A/B)> Π^*. (iii) $z>0$. Now we have $\lim_{\omega \to \infty} A(\omega) = \lim_{\omega \to \infty} B(\omega) = \infty$. By L'Hopital's rule, $\lim_{\omega \to \infty} [A(\omega)/B(\omega)] = \lim_{\omega \to \infty} [\partial A/\partial \omega]/z = \infty$. Hence, for all cases, there is an $\omega^* \in (\omega_0, \infty)$: (A/B)> Π^*. The claim now follows from the definition of ω and equations (A21) and (A22). *Q.E.D.*

The proposition follows directly from Claims 3 and 7 and the IVT. *Q.E.D.*

Proof of Proposition 6. The claimed equilibrium properties of w and Π^{***} follow directly from equations (24) and (25). By second-order conditions and our premise that $\partial p^{1A+}/\partial p^{1B} < 1$ (note 27), we have (recalling notes 26 and 27),

$$dp^{1**}/dw \stackrel{s}{=} \partial p^{1A} + \partial w \stackrel{s}{=} (\partial y^{1B}/\partial p^{1A})\Pi^{**}_{2,2} - (\partial y^{1B}/\partial p^2)\Pi^{**}_{1A,2}, \tag{A25}$$

$$dp^{2**}/dw = (\partial p^2 + /\partial w) + (\partial p^2 + /\partial p^{1B})(dp^{1**}/dw), \tag{A26}$$

where

$$\partial p^2 + /\partial w \stackrel{s}{=} (\partial y^{1A}/\partial p^2)\Pi^{**}_{1A,1A} - (\partial y^{1B}/\partial p^{1A})\Pi^{**}_{1A,2} \tag{A27}$$

$$(\partial p^2 + /\partial p^{1B}) \stackrel{s}{=} \Pi^{**}_{1A,2}\Pi^{**}_{1A,1B} - \Pi^{**}_{1B,2}\Pi^{**}_{1A,1A}. \tag{A28}$$

For independent goods ($\partial y^{1i}/\partial p^2 = 0$, $i \in \{A,B\}$), the right-hand side of (A25) is negative (with $\partial y^{1B}/\partial p^2 = 0$, $\partial y^{1B}/\partial p^{1A} > 0$, and $\Pi^{**}_{2,2} < 0$ by second-order conditions) and the right-hand side of (A26) equals zero (with $\partial y^{1B}/\partial p^2 = 0$, and $\Pi^{**}_{1A,2} = \Pi^{**}_{1B,2} = 0$); hence, with $w > c$ in equilibrium, $p^{1**}(w) < p^{1**}(c) = p^{1*}$ and $p^{2**}(w)< p^{2**}(c) = p^{2*}$. For other cases, we assume that the conditions described in note 26 are satisfied, so that $dp^{1**}(w)/dw < 0$. In addition, we assume that $\Pi^{**}_{i,j} \stackrel{s}{=} \partial y^i/\partial p^j = 0$ for $j \neq i$, which will be true if third-order derivatives of u are sufficiently small. Hence, appealing to second-order conditions, $dp^{2**}(w)/dw < 0$ for substitutes and $dp^{2**}(w)/dw>0$ for complements. It follows for strong substitutes that $w < c$, $p^{1**}(w) > p^{1**}(c) = p^{1*}$ and $p^{2**}(w) > p^{2**}(c) = p^{2*}$; for weak substitutes that $w > c$, $p^{1**}(w) < p^{1**}(c) = p^{1*}$ and $p^{2**}(w) < p^{2**}(c) = p^{2*}$; and for complements that $w > c$, $p^{1**}(w) < p^{1**}(c) = p^{1*}$ and $p^{2**}(w) > p^{2**}(c) = p^{2*}$. *Q.E.D.*

References

Allain, M.L. and Chambolle, C. "Loss-Leaders Banning Laws as Vertical Restraints." *Journal of Agricultural & Food Industrial Organization,* Vol. 3 (2005), pp. 1–23.

Bagwell, K. and Ramey, G. "Advertising and Coordination." *Review of Economic Studies,* Vol. 61 (1994), pp. 153–172.

Bliss, C. "A Theory of Retail Pricing." *Journal of Industrial Economics,* Vol. 36 (1988), pp. 375–391.

Bolton, P. and Bonanno, G. "Vertical Restraints in a Model of Vertical Differentiation." *Quarterly Journal of Economics,* Vol. 103 (1988), pp. 555–570.

Bork, R.H. "The Rule of Reason and the Per Se Concept." *Yale Law Journal,* Vol. 75 (1966), pp. 373–475.

Bulow, J.I., Fischer, J.H., Creswell, J.S., Jr., and Taylor, C.T. "Midwest Gasoline Pricing and the Spring 2000 Price Spike." Mimeo, Stanford University, 2001.

Carbajo, J., de Meza, D., and Seidmann, D. "A Strategic Motivation for Commodity Bundling." *Journal of Industrial Economics,* Vol. 38 (1990), pp. 283–297.

Comanor, W. and Rey, P. "Competition Policy towards Vertical Restraints in Europe and the United States." *Empirica,* Vol. 24 (1997), pp. 37–52.

Crémer, J. and Riordan, M.H. "On Governing Multilateral Transactions with Bilateral Contracts." *RAND Journal of Economics,* Vol. 18 (1987), pp. 436–451.

DeGraba, P. "The Loss Leader Is a Turkey: Targeted Discounts from Multiproduct Competitors." *International Journal of Industrial Organization,* Vol. 24 (2006), pp. 613–628.

—— and Mohammed, R. "Intertemporal Mixed Bundling and Buying Frenzies." *RAND Journal of Economics,* Vol. 30 (1999), pp. 694–718.

Federal Trade Commission. *Slotting Allowances in the Retail Grocery Industry,* 2003. Available from www.ftc.gov/opa/2003/11/slottingallowance.htm.

Innes, R. and Hamilton, S.F. "Naked Slotting Fees for Vertical Control of Multi-Product Retail Markets." *International Journal of Industrial Organization,* Vol. 24 (2006), pp. 308–318.

Klein, B. and Murphy, K. "Vertical Restraints as Contract Enforcement Mechanisms." *Journal of Law and Economics,* Vol. 31 (1988), pp. 265–297.

Kühn, K.-U. "Nonlinear Pricing in Vertically Related Duopolies." *RAND Journal of Economics,* Vol. 28 (1997), pp. 37–62.

Lal, R. and Matutes, C. "Retail Pricing and Advertising Strategies." *Journal of Business,* Vol. 67 (1994), pp. 345–370.

Macleod, W.B. and Malcomson, J. "Contract Bargaining with Symmetric Information." *Canadian Journal of Economics,* Vol. 28 (1995), pp. 336–367.

Marion, B.W. "Changing Power Relationships in U.S. Food Industry: Brokerage Arrangements for Private Label Products." *Agribusiness,* Vol. 14 (1998), pp. 85–93.

Martimort, D. "Exclusive Dealing, Common Agency, and Multiprincipals Incentive Theory." *RAND Journal of Economics,* Vol. 27 (1996), pp. 1–31.

Marvel, H. and McCafferty, S. "Resale Price Maintenance and Quality Certification." *RAND Journal of Economics,* Vol. 15 (1984), pp. 340–359.

Marx, L.M. and Shaffer, G. "Predatory Accommodation: Below-Cost Pricing without Exclusion in Intermediate Products Markets." *RAND Journal of Economics,* Vol. 30 (1999), pp. 22–43.

—— and ——. "Upfront Payments and Exclusion in Downstream Markets." Mimeo, Duke University and University of Rochester, 2004.

Mathewson, G. and Winter, R. "An Economic Theory of Vertical Restraints." *RAND Journal of Economics,* Vol. 15 (1984), pp. 27–38.

—— and ——. "Tying as a Response to Demand Uncertainty," *RAND Journal of Economics,* Vol. 28 (1997), 566–583.

—— and ——. "The Law and Economics of Resale Price Maintenance," *Review of Industrial Organization,* Vol. 13 (1998), pp. 57–84.

McAfee, R.P., McMillan, J., and Whinston, M. "Multiproduct Monopoly, Commodity Bundling, and Correlation of Values." *Quarterly Journal of Economics,* Vol. 104 (1989), pp. 371–384.

Meyer, D. and Fischer, J. "The Economics of Price Zones and Territorial Restrictions in Gasoline Marketing." Mimeo, Federal Trade Commission, 2004.

Nalebuff, B. "Bundling as an Entry Barrier." *Quarterly Journal of Economics,* Vol. 119 (2004), pp. 159–187.

O'Brien, D.P. and Shaffer, G. "Vertical Control with Bilateral Contracts." *RAND Journal of Economics,* Vol. 23 (1992), pp. 299–308.

Posner, R.A. *Antitrust Law: An Economic Perspective*. Chicago: University of Chicago Press, 1976.

Reiffen, D. "On the Equivalence of Resale Price Maintenance and Quantity Restrictions." *International Journal of Industrial Organization,* Vol. 17 (1999), pp. 277–288.

Rey, P. and Tirole, J. "The Logic of Vertical Restraints." *American Economic Review,* Vol. 76 (1986), pp. 921–939.

—— and Vergé, T. "Resale Price Maintenance and Horizontal Cartel." Mimeo, University of Toulouse and University of Southampton, 2004

——, Thal, J., and Vergé, T. "Slotting Allowances and Conditional Payments." Mimeo, CREST-LEI and University of Toulouse, 2006.

RUBINSTEIN, A. "Perfect Equilibrium in a Bargaining Model." *Econometrica,* Vol. 50 (1982), pp. 97–109.

SHAFFER, G. "Slotting Allowances and Resale Price Maintenance: A Comparison of Facilitating Practices." *RAND Journal of Economics,* Vol. 22 (1991a), pp. 120–135.

——. "Capturing Strategic Rent: Full-Line Forcing, Brand Discounts, Aggregate Rebates, and Maximum RPM." *Journal of Industrial Economics,* Vol. 39 (1991b), pp. 557–575.

SHAKED, A. "Opting Out: Bazaars versus 'Hi-Tech' Markets." Mimeo, London School of Economics, 1987.

Supermarket News. "Masters of the House." March 13, 1995, pp. 39–42.

TELSER, L. "Why Should Manufacturers Want Free Trade?" *Journal of Law and Economics,* Vol. 3 (1960), pp. 86–105.

WHINSTON, M. "Tying, Foreclosure and Exclusion." *American Economic Review,* Vol. 80 (1990), pp. 837–859.

WINTER, R. "Vertical Control and Price versus Nonprice Competition." *Quarterly Journal of Economics,* Vol. 108 (1993), pp. 61–76.